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A growth model with externalities  
across foreign and domestic firms

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## **A growth model with externalities across foreign and domestic firms**

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*Abstract:* This paper presents a theoretical model which takes into account technological interdependence among economies and examines the impact of location effects in explaining growth. In a small open economy, final goods production combines the production processes of multinational enterprises (MNEs) and non MNEs firms, which compete for labor and capital inputs. Technological interdependence is assumed to work through spatial externalities across countries and vertical/horizontal spillovers from linkages between foreign and domestic firms. This augmented growth model yields a conditional convergence equation which is characterized by parameter heterogeneity across countries and spatial dependence.

*Keywords:* economic growth, multinational enterprises, technological spillovers, spatial dependence

*JEL Classification:* F21, F23, O33, O47

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## 1. Introduction

Technology diffusion plays a central role in the process of economic growth. The recent growth literature has highlighted the dependence of growth rates on the diffusion of ideas and new technologies, which can take place through a variety of channels of transmission. Besides trade flows of high-technology products, foreign direct investment by multinational enterprises (MNEs) is considered to be a major channel for the diffusion of advanced technologies.

Multinational activity is usually described with reference to ownership-specific advantages, internalization incentives, and location-specific advantages (Dunning, 1981). The presence of firm-specific assets, such as patents, managerial and organizational know-how, etc ..., enables foreign firms to outperform local firms. There are detailed studies showing that foreign investors exhibit higher levels of TFP than do their local counterparts (Blomstrom and Kokko, 1998).

Domestic firms in principle could benefit from horizontal technology spillovers, but foreign firms prevent leakages of their technology to preserve their special assets. In addition, in the short run, increased competition reduces profits, which in turn may decrease the incentive to engage in research. However, positive horizontal externalities can emerge, at least in the long run, through the increased competitive pressure in the local market. Furthermore, FDI could facilitate expansion of domestic firms through vertical linkages between the MNE and its local suppliers and customers.

In this paper we present a theoretical model which takes into account technological interdependence among economies and examines the impact of location effects in explaining growth. In a small open economy, final goods are produced by multinational enterprises (MNEs) and non MNEs firms. Technological interdependence is assumed to work through spatial externalities across countries and vertical/horizontal spillovers between foreign and domestic firms.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 describes the steady state and the transition process. Section 4 concludes.

## 2. The model

Let us consider a world with two countries, the host and the foreign country. The latter one may be interpreted as the set of other countries we call 'the rest of the world'.<sup>3</sup> Each country is described by a simple economy where domestic and foreign firms produce final goods. Domestic firms may operate at home only, or at multi-country level. In the latter case, some production is performed in the home country and the remaining in the foreign country. Both domestic and foreign firms' production processes combine labour and physical capital.<sup>4</sup> Domestic firms are single-country enterprises and multi-country enterprises (MNE), while foreign firms located in the host country are only MNE-type.

We assume that physical capital share is the same across all firms. This hypothesis reflects the common observation that the share of physical capital tends to be around

<sup>3</sup> This hypothesis can be easily generalized to a N-country setting.

<sup>4</sup> Human capital is not introduced to simplify notation. All results are confirmed for a general model with human and physical capital.

one third each of total factor payments across countries and therefore across foreign and domestic firms.

In the host country, the aggregate production function for the composite good  $Y$  is given by

$$Y(t) = Y_d^\eta(t) Y_{md}^\gamma(t) Y_f^{1-\eta-\gamma}(t) \quad (1)$$

where  $Y_d$ ,  $Y_{md}$  and  $Y_f$  denote production processes of domestic firms, domestic MNEs and foreign MNEs, respectively;  $\eta$  and  $\gamma$  are domestic production shares in single-country firms and domestic MNEs, respectively.<sup>5</sup>

The production of domestic single-country firms is given by the following Cobb-Douglas constant returns to scale production function

$$Y_d(t) = A_d(t) K_d^{\alpha_k} L_d^{1-\alpha_k}(t) \quad (2)$$

with  $0 < \alpha_k < 1$ .  $A_d$  represents the productivity level,  $L_d$  denotes the amount of labour used in the domestic production, and  $K_d$  is the amount of physical capital.

The domestic production of domestic MNEs is given by the following Cobb-Douglas constant returns to scale production function

$$Y_{dm}(t) = \frac{A_{dm}(t)}{F_{dm}} K_{dm}^{\alpha_k} L_{dm}^{1-\alpha_k}(t) \quad (3)$$

with  $0 < \alpha_k < 1$ .  $A_{dm}$  represents the productivity level,  $L_{dm}$  denotes the amount of labour used in the domestic production, and  $K_{dm}$  is the amount of physical capital. We introduce an additional term  $F_{dm}$  to account for all advantages/disadvantages coming from subsidiaries located abroad, influencing TFP at home. Specifically, the greater is  $F_{dm}$  the lower is the TFP level of domestic MNEs.

Foreign firms located in the host country, which necessarily have a multi-country structure, use the following Cobb-Douglas constant returns to scale production function

$$Y_f(t) = \frac{A_f(t)}{F_f} K_f^{\alpha_k} L_f^{1-\alpha_k}(t) \quad (4)$$

with  $0 < \alpha_k < 1$ .  $A_f$  represents the productivity level,  $L_f$  denotes the amount of labour used in the MNEs production, and  $K_f$  is the amount of physical capital. In production function (4) local advantages due to local factor endowment are considered, in terms of infrastructure, labor (skills and costs), and tariffs barriers as well as reverse spillovers, that is positive effects from domestic to foreign firms. All these factors increase or decrease the set of frictions of doing business in the domestic economy. Since our objective is to understand the effects of foreign production on growth and not the decision to invest abroad, we model these effects with the parameter  $F_f$ . The greater is  $F_f$  the lower is the TFP level of foreign MNEs.

It is useful to calculate total output per worker in the host country. It is easy to show that

$$y(t) = Y_d^\eta(t) Y_{dm}^\gamma(t) Y_f^{1-\eta-\gamma}(t) Q \quad (5)$$

With  $Q \equiv q_d^\eta q_{dm}^\gamma (1 - q_d - q_{dm})^{1-\eta-\gamma}$  and  $q_i \equiv L_i(t)/L(t)$ ,  $i = d, dm$ .  $Q$  is constant over time for simplicity. Taking into account (2), (3) and (4) we have

$$y(t) = \frac{A(t)}{F_{dm}^\gamma F_f^{1-\eta-\gamma}} k^{\alpha_k} Q \quad (6)$$

<sup>5</sup> The aggregator of foreign and domestic firms' products serves as an artefact that will allow us to capture composition and interaction effects among MNEs and non MNEs.

where the aggregate level of technology in the domestic economy is

$$A(t) = A_d^\eta(t) A_{dm}^\zeta(t) A_f^{1-\eta-\zeta}(t) \quad (7)$$

Analogous expressions describe the foreign economic system. We obtain them by adding a bar to all variables.

$$\bar{Y}(t) = \bar{Y}_d^\eta(t) \bar{Y}_{dm}^\zeta(t) \bar{Y}_f^{1-\eta-\zeta}(t) \quad (8)$$

$$\bar{Y}_d(t) = \bar{A}_d(t) \bar{K}_d^{\alpha_k}(t) \bar{L}_d^{1-\alpha_k}(t) \quad (9)$$

$$\bar{Y}_{dm}(t) = \frac{\bar{A}_{dm}(t)}{\bar{F}_{dm}} \bar{K}_{dm}^{\alpha_k}(t) \bar{L}_{dm}^{1-\alpha_k}(t) \quad (10)$$

$$\bar{Y}_f(t) = \frac{\bar{A}_f(t)}{\bar{F}_f} \bar{K}_f^{\alpha_k}(t) \bar{L}_f^{1-\alpha_k}(t) \quad (11)$$

$$\bar{y}(t) = \frac{\bar{A}(t)}{\bar{F}_d^\eta \bar{F}_{dm}^\zeta \bar{F}_f^{1-\eta-\zeta}} \bar{k}^{\alpha_k} \bar{Q} \quad (12)$$

with  $\bar{Q} \equiv \bar{q}_d^\eta \bar{q}_{dm}^\zeta (1 - \bar{q}_d - \bar{q}_{dm})^{1-\eta-\zeta}$ ,  $\bar{q}_i \equiv \bar{L}_i(t)/\bar{L}(t)$ ,  $i = d, dm$  and

$$\bar{A}(t) = \bar{A}_d^\eta(t) \bar{A}_{dm}^\zeta(t) \bar{A}_f^{1-\eta-\zeta}(t). \quad (13)$$

## 2.1 Total factor productivity and spillovers

In the previous section we have shown that the aggregate levels of technology in the domestic economy and abroad are given by equations (7) and (13). Now, a TFP specification for each type of firm is introduced. We assume that the level of technology in domestic firms  $A_d$  is

$$A_d(t) = \Omega(t) k_d^{\beta_k}(t) A^\theta(t) \bar{A}^\rho(t) \quad (14)$$

in domestic MNEs  $A_{dm}$  is

$$A_{dm}(t) = \Omega(t) \Gamma k_{dm}^{\beta_k}(t) \bar{A}^\rho(t) \quad (15)$$

and in foreign MNEs,  $A_f$  is

$$A_f(t) = \Omega(t) \Gamma k_f^{\beta_k}(t) \bar{A}^\rho(t) \quad (16)$$

The aggregate level of technology of type- $i$  firms,  $A_i$ ,  $i = d, dm, f$ , depends on three terms. First, as in the Solow model (Solow, 1956) we suppose that a part of technological progress is exogenous and identical to all firms across countries  $\Omega(t) = \Omega_0 e^{\mu t}$ , where  $\mu$  is its constant rate of growth. Second, the aggregate level of technology increases with physical capital per worker  $k_i(t) = K_i(t)/L_i(t)$ <sup>6</sup>. The parameter  $\beta_k$ , with  $0 < \beta_k < 1$ , describes the strength of home knowledge externalities generated by physical capital accumulation in the domestic economy (Romer, 1986). Finally, following Ertur and Koch (2007), we assume that a country's TFP level is influenced by neighboring countries' TFP levels. The latter factor influences domestic and foreign firms' TFP levels. The degree of technological interdependence is described by  $\rho$ , with  $0 < \rho < 1$ .

<sup>6</sup> We suppose that all knowledge is embodied in physical capital per worker and not in the level of capital in order to avoid scale effects (Jones, 1995).

Furthermore, positive horizontal (or knowledge diffusion) externalities can emerge for  $d$ -type firms through the increased competitive pressure of domestic and foreign MNEs, forcing them to use existing technology and resources more efficiently or to search for more efficient technologies. Spillovers can also occur through MNE's local workers whose increased skills could later benefit  $d$ -type firms. Such spillovers are modelled by the third term in equation (14):  $A_d$  depends on  $A$  and the degree of interdependence is described by  $\theta$ , with  $0 < \theta < 1$ .

Domestic and foreign MNEs, as leaders in technological and capital accumulation, show higher levels of TFP than non MNEs. This advantage is captured by the term  $\Gamma$  in (15) and (16),  $\Gamma > 1$ .

Symmetric expressions are introduced for the 'rest of the world':

$$\bar{A}_d(t) = \Omega(t) \bar{k}_d^{\beta_k}(t) \bar{A}^\theta(t) \bar{A}^\rho(t) \quad (17)$$

$$\bar{A}_{dm}(t) = \Omega(t) \Gamma \bar{k}_{dm}^{\beta_k}(t) \bar{A}^\rho(t) \quad (18)$$

$$\bar{A}_f(t) = \Omega(t) \Gamma \bar{k}_f^{\beta_k}(t) \bar{A}^\rho(t) \quad (19)$$

Substituting (14), (15) and (16) into (7), and (17), (18), and (19) into (13) we can write:

$$A(t) = \left[ \Omega(t) k^{\beta_k}(t) \Gamma^{(1-\eta)} \bar{A}^\rho(t) \right]^{\frac{1}{1-\theta\eta}} \quad (20)$$

$$\bar{A}(t) = \left[ \Omega(t) \bar{k}^{\beta_k}(t) \Gamma^{(1-\eta)} \bar{A}^\rho(t) \right]^{\frac{1}{1-\theta\eta}} \quad (21)$$

with

$$k(t) \equiv k_d^\eta(t) k_{dm}^\zeta(t) k_f^{1-\eta-\zeta}(t) \quad (22)$$

$$\bar{k}(t) \equiv \bar{k}_d^\eta(t) \bar{k}_{dm}^\zeta(t) \bar{k}_f^{1-\eta-\zeta}(t) \quad (23)$$

This formulation implies that production processes of domestic and foreign firms cannot be studied in separation but must be analysed as an interdependent system. Indeed, it is easy to show that TFP levels depend on the exogenous technological progress and by the parameter  $\Gamma$ , common to all countries and firms, and on the level of physical capital accumulated by all firms, which are located in the home country and abroad:

$$A(t) = \left[ \Omega^{1+\rho}(t) k^{\beta_k}(t) \bar{k}^{\beta_k \frac{\rho}{1-\theta\eta}}(t) \Gamma^{(1-\eta)+(1-\eta)\frac{\rho}{1-\theta\eta}} \right]^x \quad (24)$$

$$\bar{A}(t) = \left[ \Omega^{1+\rho}(t) \bar{k}^{\beta_k}(t) k^{\beta_k \frac{\rho}{1-\theta\eta}}(t) \Gamma^{(1-\eta)+(1-\eta)\frac{\rho}{1-\theta\eta}} \right]^{\bar{x}} \quad (25)$$

with

$$x \equiv \frac{1-\theta\eta}{(1-\theta\eta)(1-\theta\eta)-\rho^2} > 1 \quad \bar{x} \equiv \frac{1-\theta\eta}{(1-\theta\eta)(1-\theta\eta)-\rho^2} > 1$$

## 3. Capital accumulation and steady state

As in the Solow model, we assume that a constant fraction of output is saved to accumulate physical capital in each country. Total labour supply exogenously grows at rate  $n$  and  $\bar{n}$  in home and foreign country, respectively. We suppose also a constant and identical rate of depreciation of physical capital, denoted by  $\delta$ , across domestic and foreign firms at home and abroad.

The accumulation process of physical capital for domestic and foreign firms are described by the following differential equations

$$\dot{k}_i(t) = I_{ki}(t) - (\delta + n)k_i(t) \quad i = d, dm, f \quad (26)$$

$$\dot{\bar{k}}_i(t) = \bar{I}_{ki}(t) - (\delta + \bar{n})\bar{k}_i(t) \quad i = d, dm, f \quad (27)$$

It is easy to show that the aggregate law of accumulation is

$$\frac{\dot{k}(t)}{k(t)} = I_k(t) - (\delta + n) \quad (28)$$

$$\frac{\dot{\bar{k}}(t)}{\bar{k}(t)} = \bar{I}_k(t) - (\delta + \bar{n}) \quad (29)$$

with  $I_k(t) \equiv \eta I_{kd}(t) + \mathcal{A}_{kdm}(t) + (1 - \eta - \gamma)I_{kf}(t)$

$$n \equiv \eta n_d + \mathcal{A}_{dm} + (1 - \eta - \gamma)n_f$$

$$\bar{I}_k(t) \equiv \bar{\eta} \bar{I}_{kd}(t) + \bar{\mathcal{A}}_{kdm}(t) + (1 - \bar{\eta} - \bar{\gamma})\bar{I}_{kf}(t)$$

$$\bar{n} \equiv \bar{\eta} \bar{n}_d + \bar{\mathcal{A}}_{dm} + (1 - \bar{\eta} - \bar{\gamma})\bar{n}_f$$

In equilibrium aggregate savings equal aggregate investment

$$I_k(t) = s_k y(t) \quad (30)$$

$$\bar{I}_k(t) = \bar{s}_k \bar{y}(t). \quad (31)$$

This formulation implies diminishing returns to reproducible capital<sup>7</sup>. In the steady state  $\dot{k}(t)/k(t) = g$  and  $\dot{\bar{k}}(t)/\bar{k}(t) = \bar{g}$  with

$$g = \frac{x}{1 - \alpha_k - \beta_k x} \left[ \mu(1 + \rho) + \beta_k \frac{\rho}{1 - \theta \eta} \bar{g} \right] \quad (32)$$

and

$$\bar{g} = \frac{\bar{x}}{1 - \alpha_k - \beta_k \bar{x}} \left[ \mu(1 + \rho) + \beta_k \frac{\rho}{1 - \theta \eta} g \right]. \quad (33)$$

Since the production function per worker is characterised by decreasing returns, equations (6) and (12) imply that the physical capital-output ratio is constant and the capital stock converges to  $k^*$ . That is,

$$k^* = \frac{s_k}{G} y^* \quad (34)$$

with  $G = G_d^\eta G_{dm}^\gamma G_f^{1-\eta-\gamma}$  and  $G_i \equiv g_i + n_i + \delta$ ,  $i = d, dm, f$ . An analogous expression can be derived for the foreign economy.

$$\bar{k}^* = \frac{\bar{s}_k}{\bar{G}} \bar{y}^* \quad (35)$$

with  $\bar{G} = \bar{G}_d^\eta \bar{G}_{dm}^\gamma \bar{G}_f^{1-\eta-\gamma}$ .

Substituting (34) and (35) into domestic and foreign aggregate production functions (6) and (12) gives the following steady state income per capita in the home country

<sup>7</sup> Analytically,  $\partial(\dot{k}/k)/\partial k < 0$ .

$$y^*(t) = \left[ \frac{\Omega^{1+\rho}(t)}{F_{dm}^\gamma F_f^{1-\eta-\gamma}} \Gamma^{(1-\eta)+(1-\bar{\eta})\frac{\rho}{1-\theta\eta}} \right]^{\frac{x}{1-\alpha_k-\beta_k x}} \left( \frac{s_k}{G} \right)^{\frac{\alpha_k+\beta_k x}{1-\alpha_k-\beta_k x}} \left( \frac{\bar{s}_k}{\bar{G}} \right)^{\frac{\beta_k \bar{x}}{1-\alpha_k-\beta_k \bar{x}}} Q^{\frac{1}{1-\alpha_k-\beta_k x}} \quad (36)$$

Steady state income per capita depends on saving,  $s_k$ , population growth  $n$ , in the host country and in the 'rest of the world', exogenous technological progress  $\mu$ , the parameters  $\Gamma$ ,  $F_{dm}$ ,  $F_f$ .

Foreign steady state per capita product is equivalently

$$\bar{y}^*(t) = \left[ \frac{\Omega^{1+\rho}(t)}{F_{dm}^\gamma F_f^{1-\eta-\gamma}} \Gamma^{(1-\eta)+(1-\bar{\eta})\frac{\rho}{1-\theta\eta}} \right]^{\frac{\bar{x}}{1-\alpha_k-\beta_k \bar{x}}} \left( \frac{\bar{s}_k}{\bar{G}} \right)^{\frac{\alpha_k+\beta_k \bar{x}}{1-\alpha_k-\beta_k \bar{x}}} \left( \frac{s_k}{G} \right)^{\frac{\beta_k x}{1-\alpha_k-\beta_k x}} Q^{\frac{1}{1-\alpha_k-\beta_k \bar{x}}} \quad (37)$$

### 3.1 Convergence analysis

The specification of equations (36) and (37) is based on the assumption that a country is in its steady state. However, it is also possible to utilize the transition process to the steady state, approximated by the following equation:

$$\frac{d[\ln k(t)]}{dt} = g + \lambda[\ln k^* - \ln k(0)] \quad (38)$$

with  $\lambda \equiv G(1 - \alpha_k - \beta_k x)$

the aggregate output growth rate in the steady state is  $g$  (eq. 32), the speed of convergence is  $\lambda$ , the actual capital per worker at time  $t$  is  $\ln k(t)$ , and the steady state level of capital is  $\ln k^*$ , as given by equation (35). Equivalently for the foreign country we have

$$\frac{d[\ln \bar{k}(t)]}{dt} = \bar{g} + \bar{\lambda}[\ln \bar{k}^* - \ln \bar{k}(0)] \quad (39)$$

with  $\bar{\lambda} \equiv \bar{G}(1 - \alpha_k - \beta_k \bar{x})$

Equation (38) can be rewritten as follows:

$$\ln k(t) = \left(1 - e^{-\lambda t}\right) \left(\frac{g}{\lambda} + \ln k^*\right) + e^{-\lambda t} \ln k(0) \quad (39)$$

where  $\ln k(0)$  is capital per worker at some initial date. Subtracting  $\ln k(0)$  from both sides gives:

$$\ln k(t) - \ln k(0) = \left(1 - e^{-\lambda t}\right) \left(\frac{g}{\lambda} + \ln k^* - \ln k(0)\right) \quad (40)$$

or in terms of labour productivity

$$\ln y(t) - \ln y(0) = a_0 + a_1[\ln y^* - \ln y(0)] + a_2[\ln \bar{y}(t) - \ln \bar{y}(0)] + a_3[\ln \bar{y}^* - \ln \bar{y}(0)] \quad (41)$$

and, equivalently, for the foreign country we have

$$\ln \bar{y}(t) - \ln \bar{y}(0) = \bar{a}_0 + \bar{a}_1[\ln \bar{y}^* - \ln \bar{y}(0)] + \bar{a}_2[\ln y(t) - \ln y(0)] + \bar{a}_3[\ln y^* - \ln y(0)] \quad (42)$$

We can observe that foreign country growth rates influence growth at home and conversely growth at home influences foreign growth. In addition, the convergence processes at home and abroad are interdependent through the steady states and the initial conditions.

#### **4. Concluding remarks**

Foreign direct investment generates positive externalities through the adoption of foreign technology, and therefore plays an important role in promoting economic development. However, the empirical literature finds weak support to the idea that a country takes advantage of FDI externalities, since a country's capacity to take advantage of FDI externalities might be limited by the local conditions (Gorg and Greenaway, 2001). Our theoretical model shows that in the presence of technological interdependence, the appropriate empirical analysis of the effects of FDI on economic growth should consider parameter heterogeneity across countries and spatial dependence.

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