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# Time-reversal odd distribution functions in chiral models with vector mesons

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The so-called time-reversal odd distribution functions are known to be non-vanishing in QCD due to the presence of the link operator in the definition of these quantities. I show that T-odd distributions can be non-vanishing also in chiral models, if vector mesons are introduced as dynamical gauge bosons of a hidden local symmetry. Moreover, since the flavor dependence of these distributions is different in chiral models respect to non chiral ones, the phenomenological analysis of experimental data will be able to distinguish between these two classes of models.

## I. INTRODUCTION

The history of time-reversal odd distribution functions is interesting and rather complicated. Apparently, the first of these quantities has been initially introduced by Sivers to explain single-spin asymmetries [1, 2] and later used in several phenomenological analysis by Anselmino, Boglione and Murgia [3, 4]. Another T-odd distribution was introduced by Boer, Mulders and Tangerman and it was used to explain the angular dependence of the lepton pair in unpolarized Drell-Yan [5, 6, 7]. Both these quantities were considered to be forbidden in QCD since they apparently change sign under the combined parity and time-reversal transformation [8]. Their theoretical status remained therefore uncertain till the discovery by Brodsky, Hwang and Schimdt of an explicit mechanism which could originate single-spin asymmetries in QCD [9]. There, a gluon exchange between the struck quark and the target spectator was able to generate a phase difference between this amplitude and the lowest order contribution. Moreover, the existence of a non-vanishing transverse momentum  $k_{\perp}$  allows the asymmetry to be not power-law suppressed in  $Q^2$ , as long as  $k_{\perp}$  is small compared to Q. Collins recognized that the presence of the link operator, preserving the gauge invariance of the matrix element, was sufficient to make "T-odd" distribution functions non-vanishing in QCD [10] and, immediately after, Ji, Yuan and Belitsky [11, 12] clarified how the expansion of the link operator could explicitly provide a contribution of the type discussed in Ref. [9]. More recently, a few model calculations appeared of the Sivers function [13, 14, 15], all of them making use of the gluon exchange mechanism of Ref. [9] in order to generate the phase needed for a non-vanishing T-odd distribution.

An interesting question concerns the possibility of computing T-odd distributions using chiral models, in which quarks are interacting via the exchange of chiral fields instead of gluons. The main problem here is that apparently the link operator is not well defined, since chiral models are in general not considered to be gauge theories and therefore no gauge field is present. One could be tempted to naively substitute the chiral fields to the gluon field in the link operator, but this is clearly arbitrary, since the need for the link operator in the definition of the T-odd distribution is strictly connected with a local gauge symmetry, which is apparently absent in chiral models. On the other hand, one can remark that it would be rather surprising that in chiral models T-odd distribution functions are identically zero, since it would constitute, at least to my knowledge, the first example of a quantity, not directly involving gluons, which, while non-vanishing in QCD, cannot be estimated using a chiral lagrangian. This impasse can actually be circumvented if one recall that in the 80s vector mesons have been introduced in chiral lagrangians following two approaches, which at the end can be shown to be equivalent. In the first scheme, a hidden local symmetry is shown to be present in chiral lagrangians [16, 17, 18, 19, 20] and vector mesons are the gauge fields of this local symmetry. In the second approach, vector mesons are introduced as massive Yang-Mills fields of the chiral  $U(N_f)_L \otimes U(N_f)_R$ symmetry [21, 22, 23]. For simplicity, I will adopt the first scheme, although also the second one should lead to the same result. The main aim of the papers written in the 80s was to provide a model for vector meson phenomenology. Here, I will mainly be interested in the formal aspect of these models, in which a local gauge invariance is present, in order to show that T-odd distributions can be non-vanishing also in chiral models.

### **II. THE HIDDEN SYMMETRY**

The idea of a hidden local symmetry was proposed in supergravity theories [24, 25] and it states that any nonlinear sigma model based on the manifold G/H is gauge equivalent to another model with  $G_{\text{global}} \otimes H_{\text{local}}$  symmetry and the gauge bosons of the hidden local symmetry  $H_{\text{local}}$  are composite fields [26, 27, 28, 29]. In hadronic physics the nonlinear sigma model  $\text{SU}(2)_L \otimes \text{SU}(2)_R/\text{SU}(2)_V$  is extensively used as a model for low energy phenomenology and the diagonal subgroup is here  $H = \text{SU}(2)_V$ . In Ref.[16] the  $\rho$  meson was therefore interpreted as the gauge boson of the local hidden group H, and in [18] the gauge symmetry was extended to the  $\text{SU}(2)_V \otimes \text{U}(1)$  group, incorporating the phenomenology of the  $\omega$  meson in this scheme.

To recall what the hidden symmetry in chiral models is, let us start from the kinetic term of the lagrangian of the nonlinear sigma model, following Ref.[16]:

$$L = \left(f_{\pi}^2/4\right) \operatorname{Tr}\left(\partial_{\mu} U \partial^{\mu} U^{\dagger}\right) \,, \tag{1}$$

where

$$U(x) = \exp[2i\pi(x)/f_{\pi}] \tag{2}$$

with  $\pi \equiv \pi^a T^a$ . Here  $T^a$  are the generators of the SU(2) group and  $f_{\pi}$  is the pion decay constant. Under the global symmetry the field transforms as

$$U(x) \to g_L U(x) g_B^{\dagger} ,$$
 (3)

where  $g_L$  and  $g_R$  are elements of  $SU(2)_L$  and  $SU(2)_R$ , respectively. It is now possible to rewrite the lagrangian (1) so that it will exhibits a local symmetry, besides the global one. We rewrite the field U(x) in terms of two variables,  $\xi_L(x)$  and  $\xi_R(x)$ , as:

$$U(x) \equiv \xi_L^{\dagger}(x)\xi_R(x) \tag{4}$$

and we introduce the gauge field  $V_{\mu} \equiv V_{\mu}^{a}(x)T^{a}$ . Finally, we define the transformation rules of these fields under the group  $[SU(2)_{L} \otimes SU(2)_{R}]_{global} \otimes [SU(2)_{V}]_{local}$  as:

$$\begin{aligned} \xi_L(x) &\to h(x)\xi_L(x)g_L^{\dagger} \\ \xi_R(x) &\to h(x)\xi_R(x)g_R^{\dagger} \\ V_\mu(x) &\to ih(x)\partial_\mu h(x)^{\dagger} + h(x)V_\mu(x)h(x)^{\dagger} . \end{aligned}$$

$$\tag{5}$$

We can now define a covariant derivative as:

$$D_{\mu}\xi_{L,R}(x) \equiv [\partial_{\mu} - iV_{\mu}(x)]\xi_{L,R}(x), \qquad (6)$$

and we can recognize that it is possible to write two invariants under  $[SU(2)_L \otimes SU(2)_R]_{global} \otimes [SU(2)_V]_{local}$ , namely:

$$L_V = -\left(f_\pi^2/4\right) \operatorname{Tr}\left(D_\mu \xi_L \cdot \xi_L^{\dagger} + D_\mu \xi_R \cdot \xi_R^{\dagger}\right)^2 \tag{7}$$

$$L_A = -\left(f_{\pi}^2/4\right) \operatorname{Tr}\left(D_{\mu}\xi_L \cdot \xi_L^{\dagger} - D_{\mu}\xi_R \cdot \xi_R^{\dagger}\right)^2 \,. \tag{8}$$

Any linear combination of (7) and (8) is equivalent to the original lagrangian (1), as it is easy to check choosing e.g. the gauge  $\xi_L^{\dagger} = \xi_r \equiv \xi = \exp(-i\pi f_{\pi})$ , for which  $L_A = L$  and  $L_V = 0$ .

As already mentioned, the previous analysis was extended in Ref. [18] so that the  $\omega$  meson can also be interpreted as a gauge field.

Up to now no dynamics has been attributed to the gauge field  $V_{\mu}$  and it can therefore be eliminated in terms of the chiral fields by solving field equations. The reason to introduce a dynamics associated with the gauge field is twofold. From one side it is possible to obtain a very successful description of the vector meson phenomenology by identifying the gauge field with the  $\rho$  meson. Moreover, one can speculate that the kinetic term is originated by quantum effects. Finally, the mass of the vector mesons is obtained via the Higgs mechanism, in which the unphysical scalar modes in  $\xi_{L,R}$  are absorbed into the  $\rho$  meson. For simplicity, I will also assume the existence of a kinetic term for the vector mesons, although this is not a necessary condition in order to get non-vanishing T-odd distributions, but in this way I will be able to easily reproduce the result obtained in Ref. [9] where a massive photon field was assumed to be exchanged between the struck quark and the spectator diquark.

# III. INTRODUCING QUARKS

The hidden symmetry mechanism was first introduced in models as the Skyrme one, in which only chiral fields are present. It is anyway possible, and relatively straightforward, to extend this idea including the quark sector (see e.g. [17, 19, 30, 31]). In this way we can obtain models which have a more direct contact with QCD and can be used to evaluate matrix elements of quark operators as, e.g., quark distribution functions. Following in particular Ref. [17],

quarks can be introduced assuming that they transform as a fundamental representation of  $H_{\text{local}}$  and as singlet of  $G_{\text{global}}$  (the so-called "constituent gauge" of Ref. [30]). The quark kinetic term

$$L_q = \bar{\psi} i \gamma^\mu \partial_\mu \psi \tag{9}$$

can be made invariant under the group  $[SU(2)_V]_{local}$ , under which quarks transform as

$$\psi(x) \to h(x)\psi, \tag{10}$$

by introducing the covariant derivative already defined in the previous section

$$D_{\mu}\psi(x) \equiv [\partial_{\mu} - iV_{\mu}(x)]\psi(x).$$
(11)

In this way, an "effective QCD" lagrangian can be defined as

$$L_{\text{eff}} = L_A + aL_V + \bar{\psi}i\gamma^{\mu}D_{\mu}\psi + \dots \tag{12}$$

in which vector mesons are introduced as the gauge fields of a local gauge transformation.

As in any gauge theory, when dealing with bilocal operator a link operator need to be introduced in order to preserve local gauge invariance. As usual, the link is defined to be:

$$W(x_1, x_2) \equiv \text{P.O.} \exp\left(i \int_{x_1}^{x_2} \mathrm{d}s_{\mu} V^{\mu}\right).$$
 (13)

Therefore, when considering e.g. a quark distribution function, the gauge invariance of the theory requires the existence of the link operator. At this point the contact with QCD is re-established and the matrix element defining the partonic distribution can be evaluated within the effective lagrangian in the same way as it can be computed in QCD.

### IV. T-ODD DISTRIBUTIONS

We can now return to the original question, namely the possibility of computing T-odd distributions using chiral models. The argument of Collins [10] is that the definition of the distribution functions requires the existence of the link operator to preserve  $SU(3)_c$  gauge invariance. When deep-inelastic scattering is considered, the factorization scheme dictates the use of future-pointing Wilson lines, while for Drell-Yan the Wilson lines are past-pointing. Therefore under time-reversal the distribution functions appropriate for Drell-Yan transform into the distribution functions appropriate for DIS, but for a sign. It is clear that the problem of the existence of T-odd distributions in QCD is then solved, because the definition itself of these quantities explicitly contains a specific direction in time. Therefore, no problem connected with the breaking of time reversal symmetry exists, as long as a link operator can be unambiguously defined. This is precisely what chiral lagrangian with vector mesons can provide, as shown in the previous section, if vector mesons are introduced as gauge fields of a local symmetry.

To be more explicit, let me remark that the calculation of the Sivers function provided in [9], was actually based on the exchange of a massive photon instead of a gluon, and the mass was introduced to eliminate infrared divergences. When using vector-meson exchange instead of gluons the mass of the mesons provides a natural infrared regulator. But for that, the calculation proceeds strictly parallel to the one of Ref. [9] and a non-vanishing Sivers distribution can therefore be obtained.

### V. CONCLUSIONS

I will now discuss the phenomenological implications of the possibility of computing T-odd distributions, and particularly the Sivers function, using chiral models. This possibility was discussed in the past using sigma models [32, 33, 34, 35], but unfortunately in those papers no link operator was introduced and therefore the correct way to circumvent the time-reversal problem was not found. It is nevertheless interesting to remark that a phenomenological relation was derived, showing that, at leading order in the  $1/N_c$  expansion the Sivers function for the up quark is equal and opposite to the Sivers function for the down quark:

$$\Delta_0^T f_u(x, \mathbf{k}_\perp) = -\Delta_0^T f_d(x, \mathbf{k}_\perp) \,. \tag{14}$$

This same relation was later obtained in a model independent way by Pobylitsa [36], unfortunately also there without indicating the origin of the link operator. In this paper I have provided an example of a chiral model in which the link operator is clearly defined. Since the possibility of introducing vector mesons in chiral models is largely independent on the details of the model at hand, one can conclude that relation (14) can indeed be obtained in those models, with both sides of the relation non-vanishing. More explicitly, the main difference between the calculation of the Sivers function in chiral models and e.g. the calculation of Ref. [9] is in the wave function of the proton which provides the spin decomposition. In chiral models the spin structure is obtained as an expansion in  $1/N_c$  and from there the relation (14) can be derived in a rather direct way [36]. It is interesting to notice that in non-chiral models the Sivers function for the down quarks turns out to be much smaller than that for the up quarks [14, 15], opening the possibility to discriminate between various model previsions using already collected data and the ones obtainable in future experiments [37, 38, 39, 40, 41].

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