

6. Links between the INVALSI Mathematics test and teaching practices: an exploratory study

by Ferdinando Arzarello, Federica Ferretti

The chapter shows the first results of an interdisciplinary project aimed at investigating the link between the Mathematical INVALSI tests and the teaching and learning processes of Mathematics, in particular with didactic practices. The research project is conducted by the INVALSI Group – Disciplinary Didactics of the SIRD – Italian Didactic Research Society. The group is composed by disciplinary experts (from the Universities of Turin, the Free University of Bolzano, the University of Milan and the University of Bari) and educationalist (from the University of Bologna, the University of Turin and the Sapienza University of Rome) coordinated by Prof. Arzarello and Prof. Vannini. The interdisciplinary study consisted in the construction of a tool for detecting teachers’ attitudes towards INVALSI, both towards the INVALSI as an Institute, with its aims and working methods, and towards the INVALSI Mathematical tests and their effects on teaching. The aim is to understand which are the “tools” that the teachers have and, above all, use to read and interpret the INVALSI standardized assessment and which “tools” are available to identify possible effects of INVALSI tests on Mathematics teaching. The aim of the research is to identify professional development teachers’ needs at national level within schools and to propose guidelines for improving practices regarding the use of INVALSI surveys. To answer the research hypotheses, a questionnaire was designed and administered to investigate teachers’ beliefs regarding the knowledge and skills investigated by the INVALSI tests, their closeness to teaching practices in Mathematics and the role that they assume within the context school. In detail, the questionnaire consists of two areas of variables, one specifically for Mathematics Education and one relating to the aspects of Education. In detail Mathematics Education variables are aimed at investigating how much the Mathematical contents and abilities detected with the INVALSI tests are – more or less close to daily personal teaching

practices – lived as coherent/inconsistent with the national guidelines and are recognized or not consistent with the intentions of INVALSI – considered useful for influencing/innovating personal teaching practice. Questions about teachers' ability to read INVALSI data were in the questionnaire; these variables represent the focus of the survey and they are the dependent variables. The independent variables are those related to the aspects of Education and Didactics that refer to constructs aimed at detecting the attitudes of teachers and all scholastic subjects. The research that is taking place is a correlation type; the first data collected are being analyzed.

Il capitolo mostra i primi risultati di un progetto interdisciplinare volto ad indagare il legame tra le prove INVALSI di Matematica con i processi di insegnamento e apprendimento della Matematica, in particolare con le pratiche didattiche. Il progetto di ricerca è condotto dal Gruppo INVALSI -Didattiche Disciplinari della SIRD – Società Italiana di Ricerca Didattica, formato da esperti disciplinaristi (appartenenti alle Università di Torino, alla Libera Università di Bolzano, all'Università Statale di Milano e all'Università di Bari) e pedagogisti (appartenenti all'Università di Bologna, all'Università di Torino e alla Sapienza Università di Roma) coordinato dal prof. Arzarello e dalla prof.ssa Vannini. Il lavoro interdisciplinare è costituito nella costruzione di uno strumento di rilevazione degli atteggiamenti degli insegnanti nei confronti dell'INVALSI, sia nell'INVALSI come Istituto, con le sue finalità e modalità di lavoro, sia nei confronti delle prove INVALSI di Matematica e delle loro ricadute sulla didattica. Lo scopo è quello di comprendere quali sono gli "strumenti" che gli insegnanti hanno e, soprattutto, utilizzano per leggere e interpretare le rilevazioni INVALSI e di quali "strumenti" dispongono per individuare possibili ricadute delle prove sulla didattica della Matematica. L'obiettivo della ricerca è quello di identificare i bisogni formativi a livello nazionale all'interno delle scuole e proporre linee guida per il miglioramento delle prassi per quanto riguarda l'uso delle rilevazioni INVALSI. Per rispondere alle ipotesi di ricerca è stato progettato e somministrato un questionario volto a indagare le convinzioni degli insegnanti per quanto riguarda le conoscenze e le competenze indagate dalle prove INVALSI, la loro vicinanza alle pratiche didattiche in Matematica e ruolo che assumono all'interno del contesto scolastico. In dettaglio, il questionario è costituito da due ambiti di variabili, uno specificamente di Didattica della Matematica e uno relativo agli aspetti di Didattica generale. In dettaglio variabili di Didattica della Matematica sono finalizzate a rilevare quanto i contenuti e le abilità matematiche rilevate con le prove INVALSI siano: a) più o meno vicini alle pratiche didattiche personali quo-

tidiane; b) vissute come coerenti/incoerenti con le Indicazioni nazionali e siano riconosciute o meno in modo coerente con le intenzioni di INVALSI; c) ritenute utili per influenzare/innovare la pratica didattica personale. Sono state inoltre proposte domande sulla capacità di lettura dei dati INVALSI da parte degli insegnanti; queste variabili rappresentano il focus dell'indagine e sono le variabili dipendenti. Le variabili indipendenti sono quelle relative agli aspetti di Didattica generale che fanno riferimento a costrutti volti alla rilevazione degli atteggiamenti di insegnanti e di tutti i soggetti scolastici. L'indagine che si sta svolgendo è di tipo correlazione; i primi dati raccolti sono in fase di analisi.

1. Rationale

This chapter shows the first results of an interdisciplinary research project aimed at investigating the links between INVALSI tests of Mathematics and Mathematics teaching and learning processes, especially with respect to teachers' didactic practices. The research project is conducted by the "Gruppo INVALSI – Didattica e Saperi Disciplinari" of the SIRD (Italian Society of Didactic Research), which includes experts in Mathematics and in Pedagogy. The coordinators of the group are Ferdinando Arzarello (*Mathematics*: University of Turin) and Ira Vannini (*Pedagogy*: University of Bologna)¹. The aim of the study is to investigate Mathematics teachers' beliefs about INVALSI surveys, in particular about the way they read and interpret INVALSI surveys data, and in what measure INVALSI tests effectively impact on their Mathematics teaching practices.

In line with the goals of the SIRD, a broader objective of our research is to identify training needs at national level within schools, and to propose guidelines for the improvement of practices regarding the use of INVALSI tests. In details, our study is part of a broader line of research aimed at improving a close link between standardized assessments and Mathematics education. Its aim is to find an effective way to merge standardized assessments' results, methods, theoretical frameworks and tools – that are designed in order to impact at a systemic level – into actions of teachers and schools (Doig, 2006;

¹ The researchers of the project are: Barbara Balconi (University of Milan), Giorgio Bolondi (Free University of Bozen-Bolzano), Eleonora Faggiano (University of Bari), Federica Ferretti (University of Ferrara), Violetta Lonati (University of Milan), Daniela Maccario (University of Turin), Annarita Monaco (Teacher, Rome), Ottavio Rizzo (University of Milan), Roberto Trincherò (University of Turin), Valentina Vaccaro (INVALSI, Rome).

Looney, 2011). In order to fully acknowledge the potentials and educational aims of standardized assessment we need effective theoretical tools to interpret the quantitative data they provide and the macro-phenomena that emerge from the complexity of educational systems. The use of the standardized assessment can truly improve the teaching and learning of Mathematics only if it is able to give refined, culturally wide-ranging and operational information to policy makers, teacher training programs, curriculum developers, principals and teachers (De Lange, 2007). Our research moves exactly within this stream of thought: in particular, its first part investigates teachers' beliefs on standardized assessment and tools and on the way they actually use to read and interpret standardized tests and data.

1.1. The INVALSI test and Mathematics education

As we underlined, our research is part of a study within a broader strand of international research regarding the link between Large Scale Assessment (LSA) results and Mathematics Education Research (De Lange, 2007) and, in particular, the central role that analysis of standardized assessment data may have for teachers' professional development.

Specifically, INVALSI provides annually the data results, based on a statistically significant national sample, for every single item of all INVALSI tests and in our research these data have been used. The framework adopted by INVALSI assessment tests (INVALSI, 2018) is strictly connected to the Italian National Guidelines, includes aspects of mathematical modelling adopted in PISA research, and is developed according to results provided by Mathematics education research. These facts show the link between INVALSI tests and results from research in Mathematics education. The INVALSI publishes the results of the national sample of each item and gives back to the schools the results relative to each grade; the results are issued annually and can provide important information for categorizing students' errors at a macro level. As already shown in various research studies (e.g. Ferretti and Bolondi, 2019), the results of the INVALSI surveys highlight didactic macro-phenomena that can provide very useful information on learning/teaching processes. As we will see in the examples presented below, one of the focuses of our investigation is to study if and how teachers are aware and properly understand the macro-phenomena emerging from INVALSI assessments.

2. The research

As hinted above, the general aim of our research is to identify training needs at a national level and to propose guidelines for the improvement of Mathematics practices with the use of INVALSI tests. The general objectives of our study are:

- to investigate teachers' beliefs regarding the knowledge and skills investigated by the INVALSI tests;
- to investigate the relation between INVALSI Mathematics tests and Mathematics teaching and learning processes in the classroom, in particular about the didactic practices adopted by teachers.
- More specifically, the aim is to understand:
- (level 1) which tools teachers actually use to read and interpret INVALSI tests and data;
- (level 2) which means are available to researchers in order to identify the possible effects of INVALSI tests on mathematics education in our schools.

To answer the research questions, a questionnaire was designed and administered. Before the final administration to a large sample of Italian teachers, a try-out was performed; some of the results of the try-out are presented in the following paragraphs. The questionnaire aimed at investigating aspects such as teachers' awareness of the learning objectives detected by the INVALSI tests, their conceptions of errors in Mathematics, their use of tests in daily teaching, their misconceptions about standardized tests, their idea of assessment and, in particular, of formative assessment. It is built basing on variables from two areas: one specifically for Mathematics Education and one relating to aspects of General Education. The questions were prepared considering teachers' ability to read INVALSI Mathematics data as well as using constructs aimed at detecting more general teachers' attitudes and practices. The interdisciplinary work led to the construction of a tool apt to detect the attitudes of teachers towards INVALSI in two main directions: both of INVALSI as an Institute with its aims and methods of work, and of INVALSI Mathematics tests and their impact on teaching practices.

Mathematics Education variables are aimed at detecting to what extent the Mathematical contents and abilities identified within INVALSI tests are:

- more or less “close” to daily personal teaching practices;
- experienced as consistent/inconsistent with the National Guidelines and whether they are recognized as consistent or not with the intentions of INVALSI;
- deemed useful for influencing/innovating personal teaching practices.

The try-out of the questionnaire consisted in its administration to 85 primary school Mathematics teachers. This first part of the analysis mainly investigates the types of correlation between the involved variables. In the following section we will present some highlights of the try-out of the questionnaire, providing examples from the section of Mathematics education.

3. Examples

This paragraph illustrates three examples of reflections inherent to 3 INVALSI Mathematics items, which emerged after the administration of the questionnaire. In addition to the questions focused on 7 items of the INVALSI tests which highlight didactic macro phenomena, at the end of the Mathematics education section there are some transversal questions. Two of these questions ask how suitable INVALSI items are for assessing students learning and how commonly they are used in assessment practices. The following analyses will also illustrate how the investigated items position with respect to these two questions.

3.1. Decimals

The first of the questions investigated is related to the following item, administered in the Mathematics INVALSI test for grade 5 Italian students in the s.y. 2008/09.

<p>10. To which number do they correspond "12 tens, 7 tenths and 2 thousandths"?</p> <p><input type="checkbox"/> A. 12,702.</p> <p><input type="checkbox"/> B. 120,702.</p> <p><input type="checkbox"/> C. 12,72.</p> <p><input type="checkbox"/> D. 120,72.</p>

Fig. 1 – Item 10, grade 5 Mathematics INVALSI test 2009

Teachers are asked to estimate the degree of difficulty of the question, in terms of “how difficult do they find this question at the end of the fifth grade”.

The following figure (Fig. 2) shows the results with reference to 82 collected responses (facilissima = *very easy*, difficilissima = *very difficult*).

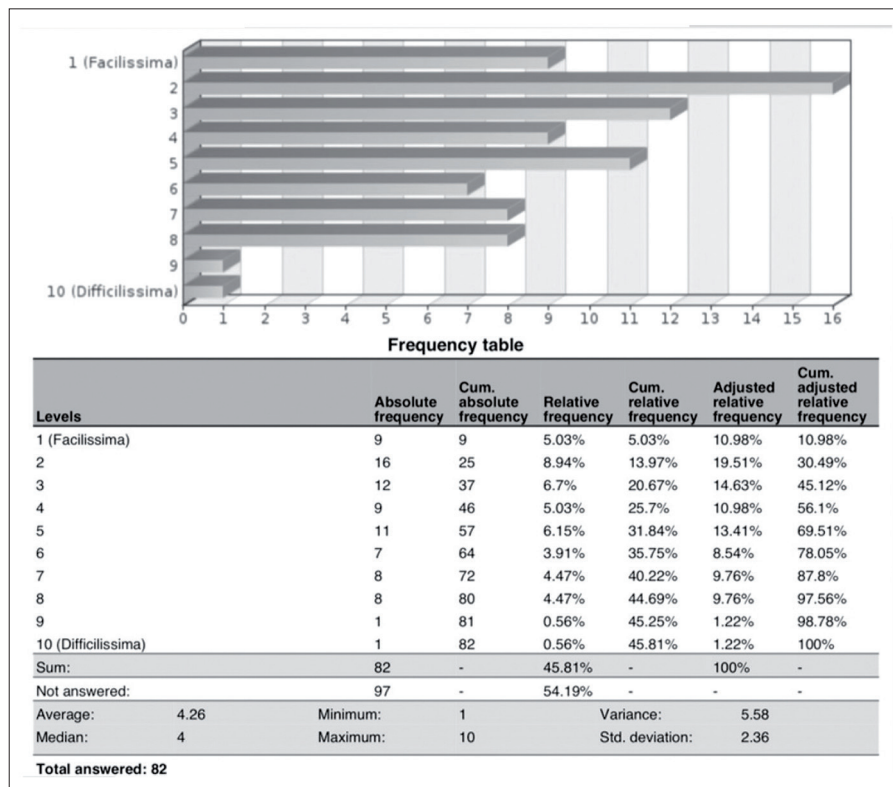


Fig. 2 – The results in reference to the “Decimal” question

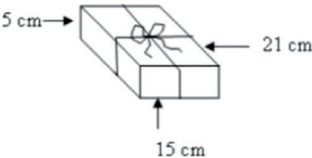
The question requires a *conversion* transformation between two different semiotic registers (Duval, 2006) and the low percentage of correct answers at national level (33%) highlights that managing this *conversion* is difficult for many Italian students. The perception of the difficulty of the question by teachers is very different from the national trend: looking at the cumulative probability we note that almost 70% answered “5”. The data suggest a failure to perceive the degree of difficulty of a question. Reflections inherent in this item suggest that it could be very interesting also to investigate this non-correspondence between a perceived difficulty and a national difficulty also in other questions. In the try-out it is the only item in which we do not inform about the percentages of correct answers. This fake perception of the difficulty level of the item poses also an interesting question to investigate: it

may be indeed significant to scrutinize whether, and to what extent, this fake perception is connected with the fact that the item is one of those considered “most suitable for assessing learning” and one of the “most commonly used in assessment tests”.

3.2. Ribbon bow

Other interesting results emerged from the following INVALSI question, administered to students of the fifth primary class in the s.y. 2008/09.

17. **Alessandra buys a book at the supermarket; at home she prepares a package similar to this:**



How many cm of ribbon did she use in all, knowing that 30 cm was needed to make the bow?

A. 41.

B. 71.

C. 112.

D. 122.

Fig. 3 – Item 17, grade 5 Mathematics INVALSI test 2009

In the questionnaire, the percentage score for correct responses at national level (14.7%) was given.

Various possible causes of students’ errors were then reported and teachers were asked to indicate which, by their opinion, was the main cause of students’ difficulties.

The sentences were:

- because the students don’t have sufficient spatial visualization skills;
- because the students didn’t do enough manipulative activities;
- because the students didn’t understand the task;
- because the students made wrong calculations;
- because the students didn’t read the text carefully;

- because the students thought only about performing calculations;
- other (specify).

The graphs in the follow figure (Fig. 4), show the percentage of choice for each sentence.

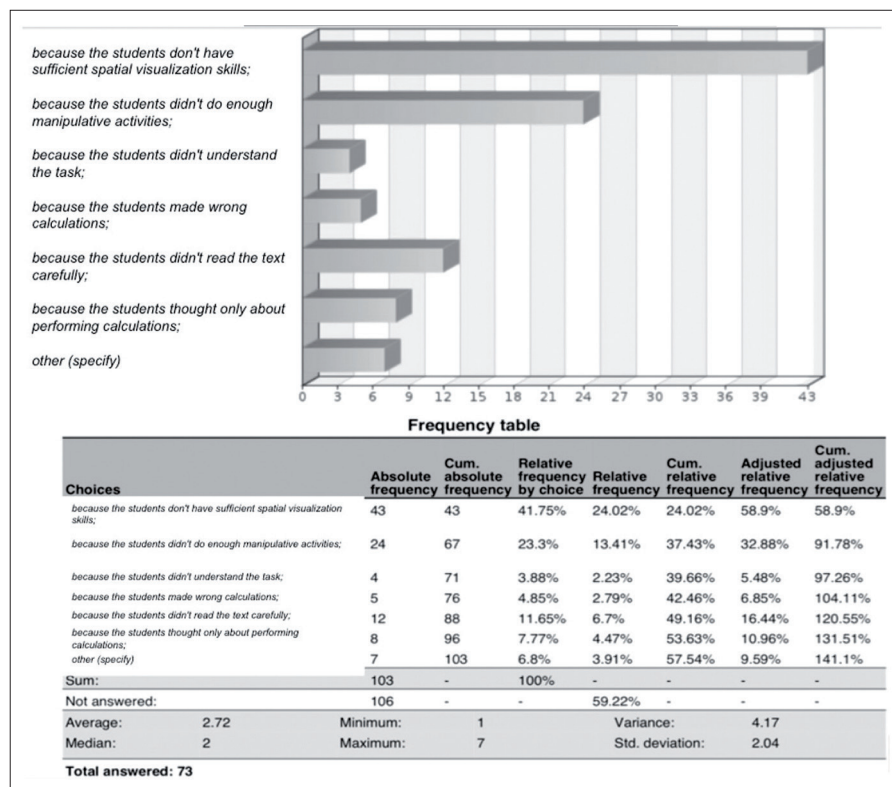


Fig. 4 – The results in reference to the “Ribbon Bow” question

Almost 60% of teachers attribute the low percentage of correct answers at national level to the fact that students do not have sufficient spatial visualization capacity and 33% to the fact that students did not do enough manipulative activities.

At national level, the correct answer (D) was selected by 14.7% (one of the worst performances in closed-ended answers since 2008) while option A was chosen by 28.9% and option B by 41.5%.

Option A shows the number 41 which corresponds exactly to the sum of the numbers shown in the figure (5+15+21) while option B (71) corresponds to the sum of the numbers in the figure and the only number expressed in

figures in the text (41+30). The causes of the difficulties, framed by the *didactic contract* construct in the sense of Brousseau (1988), are quite evident and they do not correspond or correspond only partially to those recognized by teachers. We are therefore faced with a lack of awareness of the causes of the error. It will certainly be interesting to consider this issue in depth, to try to investigate whether the causes of these interpretative difficulties are linked to difficulties in interpreting students' difficulties or to difficulties in understanding the goal of the question.

Regarding the transversal questions, this item is considered by teachers as one of the least "suitable" for assessing learning and one of the questions "least used in assessment tests".

3.3. Cotton balls

Relevant results are those inherent to the question concerning the following item, given in the INVALSI grade 5 test in the s.y. 2012/13.

D11. To make 4 crochet tablecloths the grandmother uses 6 cotton balls.

a. How many balls of the same type must she use to prepare 20 tablecloths?
Answer:

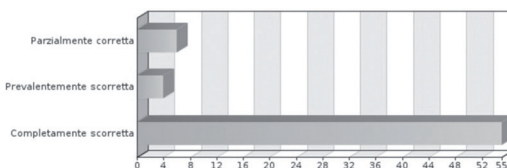
b. Write how you found the answer.

Fig. 5 – Item 11, grade 5 Mathematics INVALSI test 2013

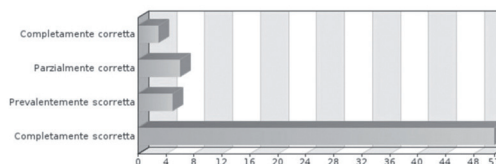
The following table shows some answers from grade 5 students (Primary School). Some possible answers plausibly provided by the students (some taken from Arzarello, 2018) and others designed by the group of researchers were then proposed. The teachers were asked to indicate how they would rate them. The following table shows the answers in reference to each proposed sentence (Completamente corretta = *Completely correct*, Parzialmente corretta = *Partially correct*, Prevalentemente scorretta = *Mainly incorrect*, Completamente scorretta = *Completely incorrect*).

Tab. 1 – Results of each sentence – Cotton balls item

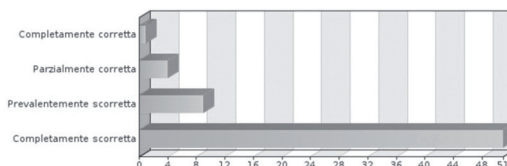
a. “First, I calculated $6-4 = 2$; after the 20, I added 2 and I got 22”



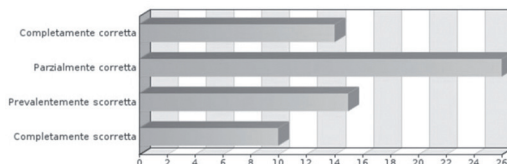
b. “I multiplied 6 balls for 4 placemats and I got 24”



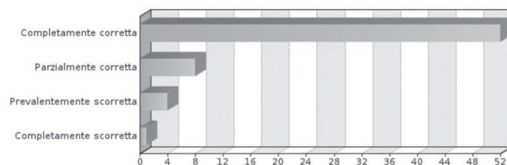
c. “I have multiplied 6 balls by 20, for 20 placemats”



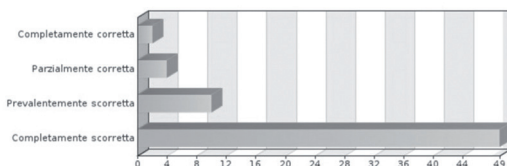
d. “Since, for 4 placemats, we need 6 balls of yarn (so 2 more), for 20 place mats I just add 10”



e. “The grandmother uses a ball and a half to prepare a placemat so I did this = $20 \times 1.5 = 30$ (in one corner of the sheet is the multiplication in a column)”



f. “Because I calculated $20+6+4$ ”



One of the most interesting data is certainly the “noise” in the answers inherent to our solution *d*, in which the strategy inherent to the correct an-

swer uses the mixed addition-multiplication model and does not explain all the steps.

In fact, while the teachers evaluate incorrect the first three answers (as they in fact are), there is a lot of confusion as far as the fourth is concerned. Why do almost 40% of teachers not accept d as correct? Because not all the steps are made explicit or because they do not recognize it as a possible correct strategy?

Finally, the last option is the only one for which there is low coherence in answers to the two final questions that we have always asked in each item. A consequent suggestion for changes in the text is to modify it, maintaining the combination right answer-wrong strategy. In fact, together with item 2 (ribbon bow), this item is considered by teachers to be the least “suitable for assessing learning” and one of the questions “least used in assessment tests”.

4. Discussion

In the paper we have presented the first findings from the try-out of a questionnaire aimed at investigating the way primary school Mathematics teachers read and interpret INVALSI surveys data.

To discuss what we have found until now, we make an analogy between the small context of an usual mathematics classroom and the general national school context, which our research is considering. In the analysis of classroom interactions it is common to see what Anna Sfard calls an “incommensurable discourse” (Sfard, 2008) between the teacher and the students: they use the same words but with a different meaning and moreover are not aware of the difference; a conflict is generated, which, if not overcome, can have serious consequences for successful teaching/learning processes in the classroom.

Something similar happened to us when we analyzed the answers to our questionnaire: also here we found incommensurable languages. They are the sign of what we call a *three-fold meta-didactical conflict*: it has been possible to realize its existence thanks to the questionnaire. As its name suggests, this conflict has three components and is meta-didactic since it concerns discourses about didactic processes like assessment, students’ competencies and mistakes, etc., and not about the thought mathematics concepts themselves, as it is in the case of usual epistemic or didactic conflicts reported in the literature (e.g. by Brousseau, 2002 or Sfard, 2008). We will now sketchily describe it.

A first component of the conflict concerns the fact that many teachers perceive the difficulty of a question from the INVALSI survey very differently

from the national trend (see example 3.1). A second component concerns the fact that many teachers interpret the difficulties of students (see example 3.2) or evaluate their answers (see example 3.3) to the INVALSI tests in a way that is completely different from what unquestionably appears from the data of the survey. A third component is a consequence of the previous two and concerns the contradictory way according to which teachers interpret the rationale of the INVALSI tests (for example, see how they couple the dyads suitable/not suitable Vs most used/not used in the examples with respect to what appears in the survey data).

Of course, it is possible that the three components may be only the epiphenomenon of a deeper conflict, whose nature at the moment we have not yet understood, but until now we can speak only of a three-fold conflict, since its three components in any case appear deeply intertwined.

We think that our current hypothesis about this conflict will be confirmed or refuted through the administration of the questionnaire to a wider court of subjects and a deep analysis of the related data. In case of confirmation, it will be possible to refine the same analysis of the conflict, deciding about its three-fold or different nature, and also clarifying its deep structure and nature, e.g. with respect the knowledge and beliefs of teachers. Basing on the analogy from the conflicts in the classroom, for which a successful strategy for overcoming them is generated by a clear understanding of their nature, it will be precisely from a clean picture of the structure and dynamics of our conflict that it will be possible to design suitable guidelines for getting rid of it and obtaining a real improvement of practices regarding the use of INVALSI tests in the school. But this will be possible only with a further step of our research, after the completion of the current one with the analysis of a suitable number of subjects.

References

- Arzarello F. (2018), “Insegnare, apprendere e valutare in matematica: un’interazione delicata”, *L’insegnamento della matematica e delle scienze integrate*, 41A-B (2), pp.135-159.
- Brousseau G. (2002), *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970-1990*, Kluwer Academic Publishers, New York/Boston/Dordrecht/London/Moscow.
- De Lange J. (2007), “Large-Scale Assessment and Mathematics Education”, in F. K. Lester Jr. (ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, National Council of Teachers of Mathematics (NCTM), Charlotte (NC), pp. 1111-1142.

- Doig B. (2006), “Large-scale mathematics assessment: looking globally to act locally”, *Assessment in Education: Principles, Policy & Practice*, 13 (3), pp. 265-288.
- Duval R. (1995), *Sémiois et pensée humaine. Registres sémiotiques et apprentissages intellectuels*, Peter Lang, Berne.
- Ferretti F., Bolondi G. (2019), “This cannot be the result! The didactic phenomenon ‘the age of the earth’”, *International Journal of Mathematical Education in Science and Technology*, 52, 2, pp. 194-207.
- Looney J.W. (2011), *Integrating Formative and Summative Assessment: Progress Toward a Seamless System?*, *OECD Education Working Papers*, 58, OECD Publishing.
- Sfard A. (2008), *Thinking as communicating: human development, the growth of discourses, and mathematizing*, Cambridge University Press, Cambridge (UK).