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Observation of a cusp-like structure in the $\pi^0\pi^0$ invariant mass distribution from $K^\pm \to \pi^\pm\pi^0\pi^0$ decay and determination of the $\pi\pi$ scattering lengths

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Abstract

We report the results from a study of a partial sample of $\sim 2.3 \times 10^7$ $K^\pm \to \pi^\pm \pi^0 \pi^0$ decays recorded by the NA48/2 experiment at the CERN SPS, showing an anomaly in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution in the region around $M_{00} = 2m_+$, where m_+ is the charged pion mass. This anomaly, never observed in previous experiments, can be interpreted as an effect due mainly to the final state charge exchange scattering process $\pi^+\pi^- \to \pi^0\pi^0$ in $K^\pm \to \pi^\pm\pi^+\pi^-$ decay [N. Cabibbo, Phys. Rev. Lett. 93 (2004) 121801]. It provides a precise determination of $a_0 - a_2$, the difference between the $\pi\pi$ scattering lengths in the isospin $I = 0$ and $I = 2$ states. A best fit to a rescattering model [N. Cabibbo, G. Isidori, JHEP 0503 (2005) 21] corrected for isospin symmetry breaking gives $(a_0 - a_2)m_+ = 0.268 \pm 0.010(\text{stat}) \pm 0.004(\text{syst})$, with additional external uncertainties of ± 0.013 from branching ratio and theoretical uncertainties. If the correlation between a_0 and a_2 predicted by chiral symmetry is taken into account, this result becomes $(a_0 - a_2)m_+ = 0.264 \pm 0.006(\text{stat}) \pm 0.004(\text{syst}) \pm 0.013(\text{ext})$.

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1. Introduction

The NA48/2 experiment at the CERN SPS is searching for direct CP violation in $K^\pm$ decay to three pions. The experiment uses simultaneous K^+ and K^- beams with a momentum of 60 GeV/c propagating along the same beam line. Data have been collected in 2003–2004, providing samples of $\sim 4 \times 10^9$ fully reconstructed $K^\pm \to \pi^\pm\pi^+\pi^-$ and $\sim 10^8$ $K^\pm \to \pi^\pm\pi^0\pi^0$ decays. Here we report the results from a study of a partial sample of $\sim 2.3 \times 10^7$ $K^\pm \to \pi^\pm\pi^0\pi^0$ decays recorded in 2003, showing an anomaly in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution in the region around $M_{00} = 2m_+$, where m_+ is the charged pion mass. This anomaly, never observed in previous experiments, can be interpreted as an effect due mainly to the final state charge exchange scattering process $\pi^+\pi^- \to \pi^0\pi^0$ in $K^\pm \to \pi^\pm\pi^+\pi^-$ decay [1]. A best fit to a rescattering model [2] provides a precise determination of $a_0 - a_2$, the difference between the S-wave $\pi\pi$ scattering lengths in the isospin $I = 0$ and $I = 2$ states.

2. Beam and detectors

The two simultaneous beams are produced by 400 GeV protons impinging on a 40 cm long Be target. Particles of opposite charge with a central momentum of 60 GeV/c and a momentum band of $\pm 3.8\%$ produced at zero angle are selected by a system of dipole magnets forming an "achromat" with null total deflection, focusing quadrupoles, muon sweepers and collimators. With 7×10^{11} protons per burst of ~ 4.5 s duration incident on the target the positive (negative) beam flux at the entrance of the decay volume is 3.8×10^7 (2.6×10^7) particles per pulse, of which $\sim 5.7\%$ ($\sim 4.9\%$) are K^+ (K^-). The decay volume is a 114 m long vacuum tank with a diameter of 1.92 m for the first 66 m, and 2.4 m for the rest.

Charged particles from $K^\pm$ decays are measured by a magnetic spectrometer consisting of four drift chambers [3] and a large-aperture dipole magnet located between the second and third chamber. Each chamber has eight planes of sense wires, two horizontal, two vertical and two along each of two orthogonal 45° directions. The spectrometer is located in a tank filled with helium at atmospheric pressure and separated from the decay volume by a thin (0.0031 radiation lengths, X_0) Kevlar window. A 16 cm diameter vacuum tube centered on the beam axis runs the length of the spectrometer through central holes in the Kevlar window, drift chambers and calorimeters. Charged particles are magnetically deflected in the horizontal plane by an angle corresponding to a transverse momentum kick of 120 MeV/c. The momentum resolution of the spectrometer is $\sigma(p)/p = 1.02\% \oplus 0.044\% p$ (p in GeV/c), as derived form the known properties of the spectrometer and checked with the measured invariant mass resolution of $K^\pm \to \pi^\pm\pi^+\pi^-$ decays. The magnetic spectrometer is followed by a scintillator hodoscope consisting of two planes segmented into horizontal and vertical strips and arranged in four quadrants.

A liquid krypton calorimeter (LKr) [4] is used to reconstruct $\pi^0 \to \gamma\gamma$ decays. It is an almost homogeneous ionization chamber with an active volume of ~ 10 m^3 of liquid krypton, segmented transversally into 13248 2 cm $\times$ 2 cm projective cells by a system of Cu–Be ribbon electrodes, and with no longitudinal segmentation. The calorimeter is 27 X_0 thick and has an energy resolution $\sigma(E)/E = 0.032/\sqrt{E} \oplus 0.09/E \oplus 0.0042$ (E in GeV). The space resolution for single electromagnetic shower can be parametrized as $\sigma_x = \sigma_y = 0.42/\sqrt{E} \oplus 0.06$ cm for each transverse coordinate x, y.

A neutral hodoscope consisting of a plane of scintillating fibers is installed in the LKr calorimeter at a depth of $\sim 9.5X_0$. It is divided into four quadrants, each consisting of eight bundles of vertical fibers optically connected to photomultiplier tubes.

3. Event selection and reconstruction

The $K^\pm \to \pi^\pm\pi^0\pi^0$ decays are selected by a two level trigger. The first level requires a signal in at least one quadrant of the scintillator hodoscope in coincidence with the presence of energy depositions in LKr consistent with at least two photons. At the second level, a fast on-line processor receiving the drift chamber information reconstructs the momentum of charged particles and calculates the missing mass under the assumption that the particle is a $\pi^\pm$ originating from the decay of a 60 GeV/c $K^\pm$ travelling along the nominal beam axis. The requirement that the missing mass is not consistent with the π^0 mass rejects most of the main $K^\pm \to \pi^\pm\pi^0$ background. The typical rate of this trigger is $\sim 15\,000$ per burst.

Events with at least one charged particle track having a momentum above 5 GeV/c, and at least four energy clusters in LKr, each consistent with a photon and above an energy threshold of 3 GeV, are selected for further analysis. In addition, the relative track and photon timings must be consistent with the same event within the experimental resolution (~ 1.5 ns). The distance between any two photons in LKr is required to be larger than 10 cm, and the distance between each photon and the impact point of any track on LKr must exceed 15 cm. Fiducial cuts on the distance of each photon from the LKr edges and centre are also applied in order to ensure full containment of the electromagnetic showers and to remove effects from the beam pipe. Finally, the distance between the charged particle track and the beam axis at the first drift chamber is required to be larger than 12 cm.

At the following step of the analysis we check the consistency of the surviving events with the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay hypothesis. We assume that each possible pair of photons originates from $\pi^0 \to \gamma\gamma$ decay and we calculate the distance D_{ik} between the π^0 decay vertex and the LKr:

$$D_{ik} = \frac{\sqrt{E_i E_k[(x_i - x_k)^2 + (y_i - y_k)^2]}}{m_0},$$

where E_i, E_k are the energies of the ith and kth photon, respectively, x_i, y_i, x_k, y_k are the coordinates of the impact point on LKr, and m_0 is the π^0 mass. Among all photon pairs, the two with the smallest D_{ik} difference are selected as the best combination consistent with the two π^0 mesons from $K^\pm \to \pi^\pm\pi^0\pi^0$ decay, and the distance of the $K^\pm$ decay vertex from the LKr is taken as the arithmetic average of the two D_{ik} values (it can be demonstrated that this choice gives the best

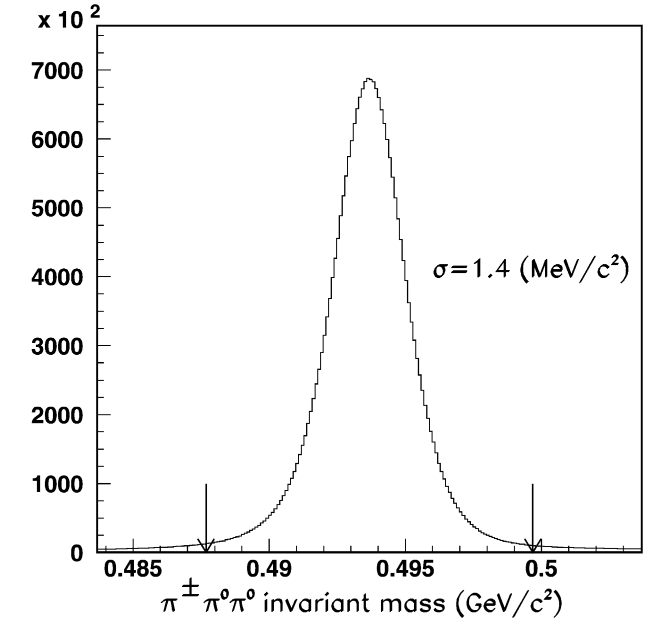

Fig. 1. Invariant mass distribution of reconstructed $\pi^\pm\pi^0\pi^0$ candidate events. The arrows indicate the selected mass interval.

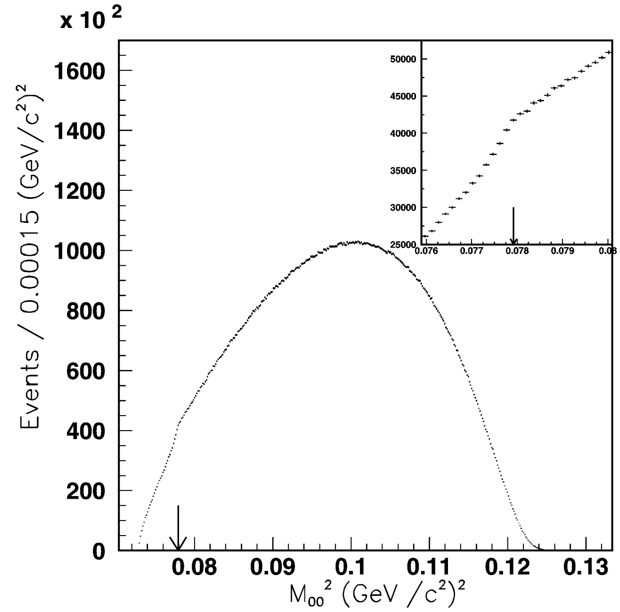

Fig. 2. Distribution of M_{00}^2, the square of the $\pi^0\pi^0$ invariant mass. The insert is an enlargement of a narrow region centered at $M_{00}^2 = (2m_+)^2$ (this point is indicated by the arrow). The statistical error bars are also shown in these plots.

$\pi^0\pi^0$ invariant mass resolution near threshold). Fig. 1 shows the invariant mass distribution of the system consisting of the two π^0 and a reconstructed charged particle track, assumed to be a $\pi^\pm$. This distribution is dominated by the $K^\pm$ peak, as expected. The non-Gaussian tails originate from unidentified $\pi^\pm \to \mu^\pm$ in flight or wrong photon pairing. The final event selection requires that the $\pi^\pm\pi^0\pi^0$ invariant mass differs form the $K^\pm$ mass by at most ± 6 MeV. This requirement is satisfied by 2.287×10^7 events. The fraction of events with wrong photon pairing in this sample is $\sim 0.25\%$, as estimated by a high-statistics fast Monte Carlo simulation of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays which takes into account the momentum distribution of the three pions, $\pi^0 \to \gamma\gamma$ decay kinematics and the effect of the detector acceptance and resolution.

4. Cusp anomaly in the $\pi^0\pi^0$ invariant mass distribution

Fig. 2 shows the distribution of the square of the $\pi^0\pi^0$ invariant mass, M_{00}^2, for the final event sample. This distribution is displayed with a bin width of 0.00015 $(\text{GeV}/c^2)^2$, with the 51st bin centered at $M_{00}^2 = (2m_+)^2$ (as discussed below, the bin width is chosen to be smaller than the M_{00}^2 resolution). A sudden change of slope near $M_{00}^2 = (2m_+)^2 = 0.07792$ $(\text{GeV}/c^2)^2$ is clearly visible. Such an anomaly has not been observed in previous experiments.

The Dalitz plot distribution for $K^\pm \to \pi^\pm\pi^0\pi^0$ decays is usually parametrized by a series expansion in the Lorentz-invariant variable $u = (s_3 - s_0)/m_+^2$, where $s_i = (P_K - P_i)^2$ $(i = 1, 2, 3)$, $s_0 = (s_1 + s_2 + s_3)/3$, P_K (P_i) is the K (π) four-momentum, and $i = 3$ corresponds to the $\pi^\pm$ [5]. In our case $s_3 = M_{00}^2$, and $s_0 = (m_K^2 + 2m_0^2 + m_+^2)/3$. We have used this parametrization in a fast Monte Carlo simulation of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays with the same detector parameters used in previous NA48 analyses [6]. This simulation takes into account most detector effects, including the trigger efficiency and the presence of a small number (<1%) of "dead" LKr cells. For any given value of the generated $\pi^0\pi^0$ invariant mass the simulation provides the detection probability and the distribution function for the reconstructed value of M_{00}^2. This allows the transformation of any theoretical distribution into an expected distribution which can be compared directly with the measured one.

Fig. 3(a) shows the expected M_{00} resolution (r.m.s.) as a function of M_{00}^2, together with examples of M_{00}^2 distributions for five generated M_{00}^2 values. The M_{00} resolution is the best at small M_{00}^2 values, varying between ~ 0.4 MeV$/c^2$ near $M_{00} = 2m_0$, and ~ 1.4 MeV$/c^2$ at the end of the M_{00} allowed range. It is 0.56 MeV$/c^2$ at $M_{00} = 2m_+$. A plot of the overall detector acceptance as a function of the generated M_{00}^2 value, as predicted by the Monte Carlo simulation (see Fig. 3(b)), shows no structure in the M_{00}^2 region where the sudden change of slope is observed in the data.

We have tried to fit the distribution of Fig. 2 in the interval $0.074 < M_{00}^2 < 0.097$ $(\text{GeV}/c^2)^2$ using the distribution predicted by the Monte Carlo simulation with a matrix element as given in Ref. [1]:

$$\mathcal{M}_0 = 1 + \frac{1}{2} g_0 u. \tag{1}$$

In this fit the free parameters are g_0 and an overall normalization constant. Because of the anomaly at $M_{00}^2 = (2m_+)^2$, it is impossible to find a reasonable fit to the distribution of Fig. 2 (the best fit gives $\chi^2 = 9225$ for 149 degrees of freedom). However, fits with acceptable χ^2 values are obtained if the lower edge of the fit interval is raised few bins above $M_{00}^2 = (2m_+)^2$. As an example, a fit in the interval $0.07994 < M_{00}^2 < 0.097$ $(\text{GeV}/c^2)^2$, with the lower edge only

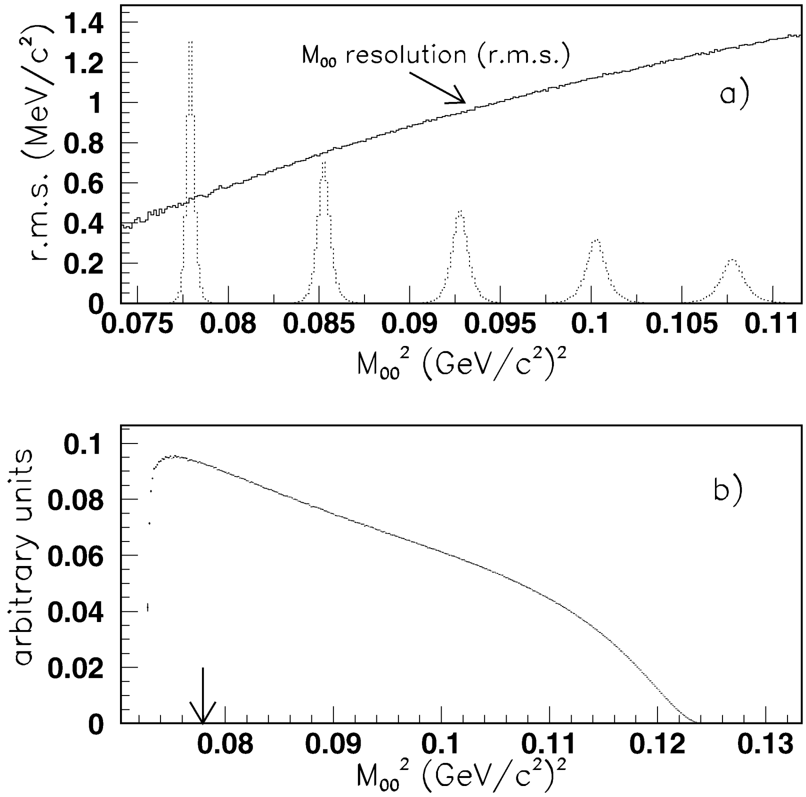

Fig. 3. (a) Expected M_{00} resolution (r.m.s. in MeV/c^2) versus generated M_{00}^2 (full line histogram), together with M_{00}^2 distributions for five generated values of M_{00}^2; (b) Acceptance versus M_{00}^2 (see text). The point $M_{00}^2 = (2m_+)^2$ is indicated by the arrow.

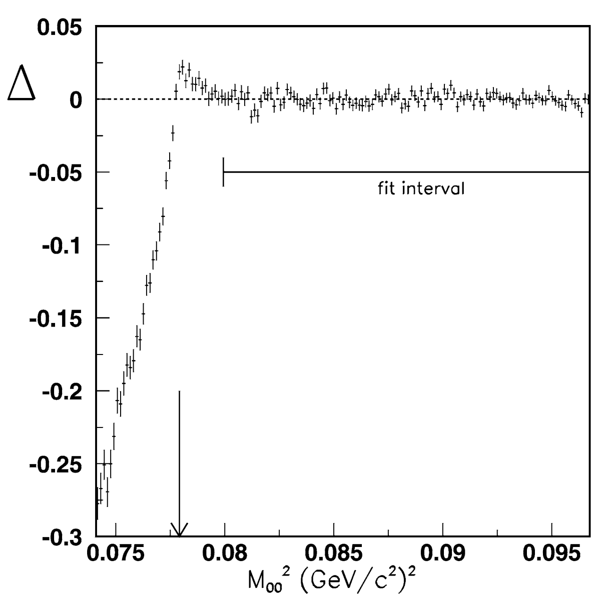

Fig. 4. $\Delta \equiv$ (data − fit)/data versus M_{00}^2. The point $M_{00}^2 = (2m_+)^2$ is indicated by the arrow. Also shown is the M_{00}^2 region used in the fit.

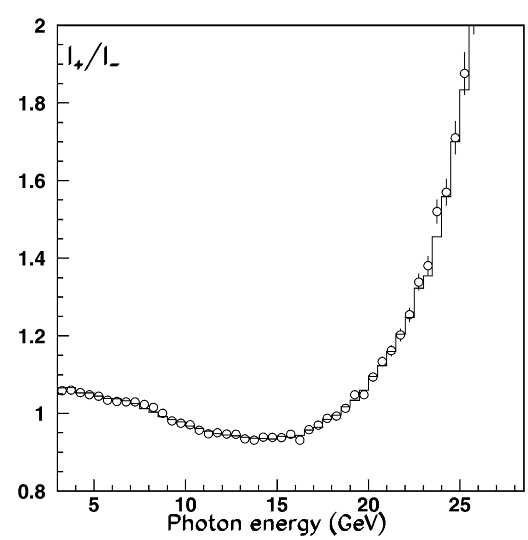

Fig. 5. Data (points with error bars) and Monte Carlo (histogram) comparison of the ratio of normalized photon energy distributions I_+/I_- between events with $M_{00}^2 > (2m_+)^2$ and $M_{00}^2 < (2m_+)^2$ (see text).

0.002 (GeV/c^2)^2 above $(2m_+)^2$, gives $\chi^2 = 133.6$ for 110 degrees of freedom. This fit gives $g_0 = 0.683 \pm 0.001$ (statistical error only), in reasonable agreement with the present world average, $g_0 = 0.638 \pm 0.020$ [5] (it should be noted, however, that the matrix element used here has not the same form as that used in Ref. [5]). The quality of this fit is illustrated in Fig. 4, which displays the quantity $\Delta \equiv$ (data − fit)/data as a function of M_{00}^2 for the fit region $0.07994 < M_{00}^2 < 0.097$ (GeV/c^2)^2 and also for $M_{00}^2 < 0.07994$ (GeV/c^2)^2, where the prediction with the same parameters is extrapolated.

Fig. 4 shows that, in the region $M_{00}^2 < (2m_+)^2$, the data fall below the prediction based on the same parameters obtained from the fit region. The total number of events in the

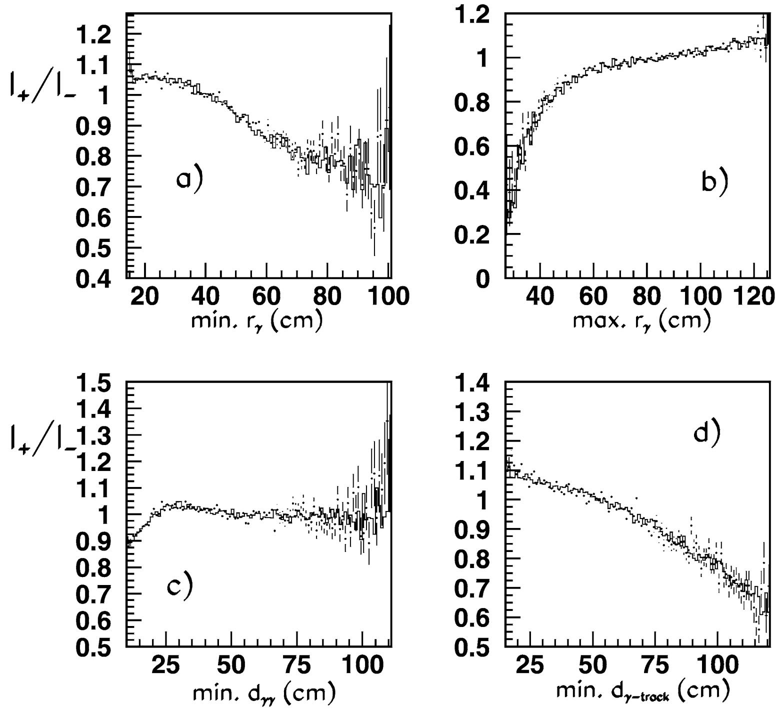

Fig. 6. Data (points with error bars) and Monte Carlo (histogram) comparison of ratios I_+/I_- of normalized distance distributions between events with $M_{00}^2 > (2m_+)^2$ and $M_{00}^2 < (2m_+)^2$ (see text), (a) min. r_γ: distance (cm) between LKr centre and closest photon; (b) max. r_γ: distance (cm) between LKr centre and farthest photon; (c) min. $d_{\gamma\gamma}$: minimum distance (cm) between photons at LKr; (d) $d_{\gamma\text{-track}}$: minimum distance (cm) between photons and tracks at LKr.

first 50 bins of the data is 7.261×10^5, while the extrapolated prediction gives 8.359×10^5 events.

In order to investigate the origin of this "deficit" of events in the data we have studied the event shape distributions in two 20 bins wide intervals, one just below and the other just above $M_{00}^2 = (2m_+)^2$. Since M_{00}^2 is computed using only information from the LKr calorimeter, we consider only photon cluster parameters. We denote the distributions of measured photon energy and distances in these two intervals as I_- and I_+, respectively, and we compare the I_+/I_- ratios with those predicted by the simulation after normalizing I_- and I_+ to the same area.

These ratios (see Figs. 5 and 6) show that the shapes of all distributions for the two M_{00}^2 intervals, as measured in the data, are in excellent agreement with the Monte Carlo predictions. In addition, no difference is observed between K^+ and K^- nor between the data taken with opposite direction of the spectrometer magnetic field. The simulation also shows that the M_{00}^2 distribution of the events affected by wrong photon pairing has no local structures over the whole M_{00}^2 range. We conclude that the Monte Carlo simulation describes correctly the M_{00}^2 dependence of the detection efficiency in the region around $M_{00}^2 = (2m_+)^2$, and the "deficit" of events in the data in the region $M_{00}^2 < (2m_+)^2$ is due to a real physical effect.

5. Interpretation and determination of the $\pi\pi$ scattering lengths

The sudden change of slope observed in the M_{00}^2 distribution at $M_{00}^2 = (2m_+)^2$ (see Fig. 2) suggests the presence of a threshold "cusp" effect from the decay $K^\pm \to \pi^\pm\pi^+\pi^-$ contributing to the $K^\pm \to \pi^\pm\pi^0\pi^0$ amplitude through the charge exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$. The presence of a cusp at $M_{00}^2 = (2m_+)^2$ in $\pi^0\pi^0$ elastic scattering due to the effect of virtual $\pi^+\pi^-$ loops has been discussed first by Meissner et al. [7]. For the case of $K^\pm \to \pi^\pm\pi^0\pi^0$ decay Cabibbo has proposed a simple rescattering model [1] describing the $K^\pm \to \pi^\pm\pi^0\pi^0$ decay amplitude as the sum of two terms:

$$\mathcal{M}\big(K^\pm \to \pi^\pm\pi^0\pi^0\big) = \mathcal{M}_0 + \mathcal{M}_1,$$

where $\mathcal{M}_0$ is the "unperturbed amplitude" of Eq. (1), and $\mathcal{M}_1$ is the contribution from the $K^\pm \to \pi^\pm\pi^+\pi^-$ decay amplitude through $\pi^+\pi^- \to \pi^0\pi^0$ charge exchange, with the renormalization condition $\mathcal{M}_1 = 0$ at $M_{00}^2 = (2m_+)^2$. The contribution $\mathcal{M}_1$ is given by

$$\mathcal{M}_1 = -2a_x m_+ \mathcal{M}_+ \sqrt{1 - \left(\frac{M_{00}}{2m_+}\right)^2}, \tag{2}$$

where a_x is the S-wave $\pi^+\pi^-$ charge exchange scattering length (threshold amplitude), and $\mathcal{M}_+$ is the known $K^\pm \to \pi^\pm\pi^+\pi^-$ decay amplitude at $M_{00} = 2m_+$. $\mathcal{M}_1$ changes from real to imaginary at $M_{00} = 2m_+$ with the consequence that $\mathcal{M}_1$ interferes destructively with $\mathcal{M}_0$ in the region $M_{00} < 2m_+$, while it adds quadratically above it. In the limit of exact isospin symmetry $a_x = (a_0 - a_2)/3$, where a_0 and a_2 are the S-wave $\pi\pi$ scattering lengths in the $I = 0$ and $I = 2$ states, respectively.

In this simple rescattering model there is only one additional parameter, $a_x m_+$. A fit to the M_{00}^2 distribution in the interval $0.074 < M_{00}^2 < 0.097$ (GeV/c^2)^2 using $a_x m_+$ as a free parame-

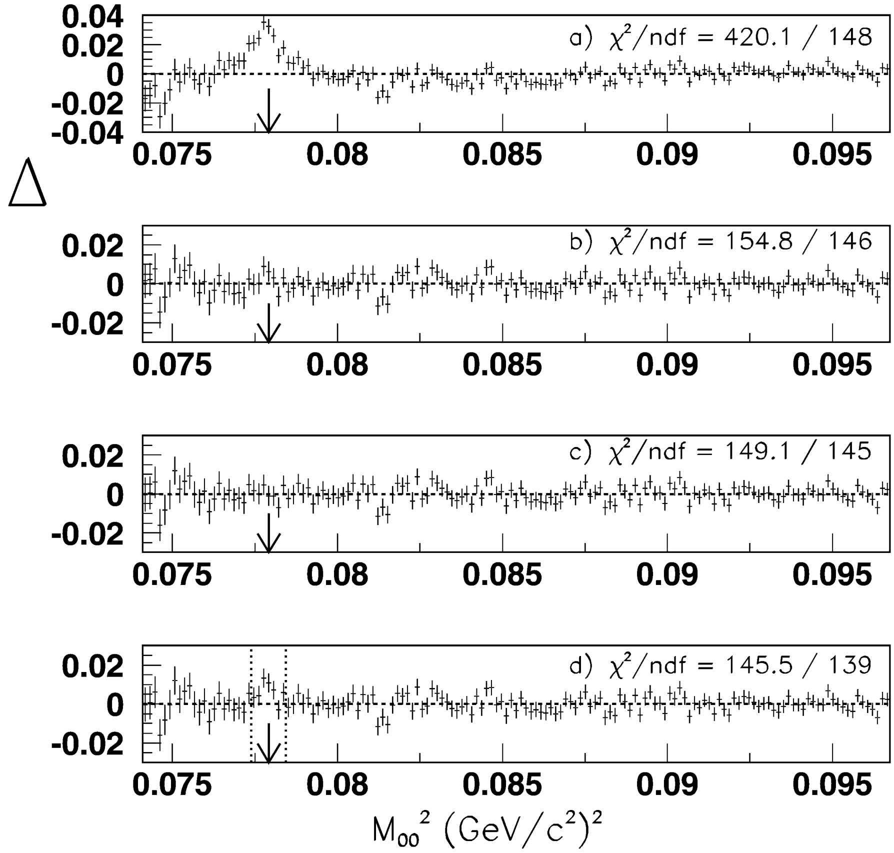

Fig. 7. $\Delta = (\text{data} - \text{fit})/\text{data}$ versus M_{00}^2 for various theoretical models: (a) using the simple charge-exchange model of Ref. [1]; (b) fit to the rescattering model of Ref. [2]; (c) fit to the model of Ref. [2] including pionium formation; (d) fit to the model of Ref. [2] excluding a 7 bin wide interval centered at $M_{00}^2 = (2m_+)^2$. The two vertical dotted lines in (d) show the interval excluded form the fit. The point $M_{00}^2 = (2m_+)^2$ is indicated by the arrow.

ter gives $\chi^2 = 420.1$ for 148 degrees of freedom. The quality of this fit is illustrated in Fig. 7(a) which displays the quantity Δ defined in Section 4 as a function of M_{00}^2. One can see that this model provides a much better but still unsatisfactory description of the data. In particular, the data points are systematically above the fit in the region near $M_{00}^2 = (2m_+)^2$.

Recently, Cabibbo and Isidori [2] have proposed a more complete formulation of the model which takes into account all rescattering processes at the one-loop and two-loop level. In this formulation the matrix element for $K^\pm \to \pi^\pm\pi^0\pi^0$ decay includes several additional terms which depend on five S-wave scattering lengths, denoted by a_x, a_{++}, a_{+-}, a_{+0}, a_{00}, and describing $\pi^+\pi^- \to \pi^0\pi^0$, $\pi^+\pi^+ \to \pi^+\pi^+$, $\pi^+\pi^- \to \pi^+\pi^-$, $\pi^+\pi^0 \to \pi^+\pi^0$, $\pi^0\pi^0 \to \pi^0\pi^0$ scattering, respectively. In the limit of exact isospin symmetry these scattering lengths can all be expressed as linear combinations of a_0 and a_2.

At tree level, omitting one-photon exchange diagrams, isospin symmetry breaking contributions to the elastic $\pi\pi$ scattering amplitude can be expressed as a function of one parameter $\epsilon = (m_+^2 - m_0^2)/m_+^2 = 0.065$ [8]. In particular, the ratio between the threshold amplitudes a_x, a_{++}, a_{+-}, a_{+0}, a_{00} and the corresponding isospin symmetric ones—evaluated at the $\pi^\pm$ mass—is equal to $1-\epsilon$ for $\pi^+\pi^+ \to \pi^+\pi^+$, $\pi^+\pi^0 \to \pi^+\pi^0$, $\pi^0\pi^0 \to \pi^0\pi^0$, $1+\epsilon$ for $\pi^+\pi^- \to \pi^+\pi^-$, and $1+\epsilon/3$ for $\pi^+\pi^- \to \pi^0\pi^0$. These corrections have been applied to the rescattering model of Ref. [2] in order to extract a_0 and a_2 from the fit to the data.

In the model of Ref. [2] the matrix element for $K^\pm \to \pi^\pm\pi^0\pi^0$ decay includes terms which depend on both independent kinematic variables (M_{00} and M_{+0}, the invariant mass of the $\pi^\pm\pi^0$ pair) requiring, therefore, a fit to the two-dimensional Dalitz plot. We have performed an approximate fit to this model by calculating these terms at the average value of M_{+0}^2 for each value of M_{00}^2. This fit has five free parameters: $(a_0 - a_2)m_+$, $a_2 m_+$, g_0, a quadratic term of the form $0.5h'u^2$ added in Eq. (1) and an overall normalization constant. The quality of the fit ($\chi^2 = 154.8$ for 146 degrees of freedom) is shown in Fig. 7(b). A better fit ($\chi^2 = 149.1$ for 145 degrees of freedom, see Fig. 7(c)) is obtained by adding to the model a term describing the expected formation of $\pi^+\pi^-$ atoms ("pionium") decaying to $\pi^0\pi^0$ at $M_{00} = 2m_+$. The best fit value for the rate of $K^\pm \to \pi^\pm$ + pionium decay, normalized to the $K^\pm \to \pi^\pm\pi^+\pi^-$ decay rate, is $(1.61 \pm 0.66) \times 10^{-5}$, in reasonable agreement with the predicted value $\sim 0.8 \times 10^{-5}$ [9].

The rescattering model of Ref. [2] does not include radiative corrections, which are particularly important near $M_{00} = 2m_+$, and contribute to the formation of $\pi^+\pi^-$ atoms. For this reason we prefer to exclude from the final fit a group of seven consecutive bins centered at $M_{00} = 2m_+$. The quality of this fit ($\chi^2 = 145.5$ for 139 degrees of freedom) is illustrated in

Table 1
Parameter best fit from two independent analyses (statistical error only)

Parameter	Analysis A	Analysis B	Arithmetic average
$(a_0 - a_2)m_+$	0.269 ± 0.010	0.268 ± 0.010	0.268 ± 0.010
$a_2 m_+$	-0.053 ± 0.020	-0.030 ± 0.022	-0.041 ± 0.022
g_0	0.643 ± 0.004	0.647 ± 0.004	0.645 ± 0.004
h'	-0.055 ± 0.010	-0.039 ± 0.012	-0.047 ± 0.012

Fig. 7(d), which shows the small excess of events from pionium formation in the bins excluded from the fit. Table 1 lists the best fit values of the parameters, as obtained by two independent analyses which use different event selection criteria and different Monte Carlo simulations to take into account acceptance and resolution effects (the analysis described so far is denoted as Analysis A; Analysis B uses a simulation of the detector based on GEANT [10]). We take the arithmetic average of these values as the measurement of these parameters, and one half of the difference between the two values as a systematic uncertainty from acceptance calculations. In both analyses changing the selection criteria never leads to variations of the best fit parameters larger than these uncertainties.

6. Other systematic uncertainties on the best fit parameters

In addition to the systematic uncertainties associated with differences of the two analyses, the following potential sources of systematic errors have been considered (see Table 2).

6.1. *Variation of the trigger efficiency over the M^2_{00} fit interval*

The trigger efficiency has been measured using a sample of “minimum bias” events recorded continuously by a trigger requiring only the presence of a signal in at least two quadrants of the neutral hodoscope (during data taking the rate of this trigger was downscaled by a large factor). Within statistical errors the dependence of the trigger efficiency on M^2_{00} is found to be consistent with the constant value $\epsilon_{\mathrm{tr}} = 0.928 \pm 0.001$ for $(2m_0)^2 < M^2_{00} < 0.097\ (\mathrm{GeV}/c^2)^2$. An equally good fit to the trigger efficiency is obtained using a 3rd degree polynomial. Varying the polynomial coefficients, so that the χ^2 increases by an amount corresponding to $\pm 1\sigma$, $(a_0 - a_2)m_+$, $a_2 m_+$, g_0 and h' change as shown in Table 2.

6.2. *Dependence on the upper edge of the fit interval*

The upper edge of the M^2_{00} fit interval has been varied from 0.094 to 0.107 $(\mathrm{GeV}/c^2)^2$, resulting in variations of the best fit parameters with respect to the default upper bound $M^2_{00} = 0.097\ (\mathrm{GeV}/c^2)^2$ as listed in Table 2.

6.3. *Dependence on the position of the $K^\pm$ decay vertex*

As an additional check of the acceptance calculation, the $K^\pm \to \pi^\pm \pi^0 \pi^0$ events have been subdivided into two independent samples with the distance D of the reconstructed $K^\pm$ decay vertex from the LKr in the intervals $48 < D < 88$ m, and $88 < D < 136$ m, respectively. The best fit parameter values obtained from separate fits to the two samples agree within statistics, providing no evidence for a possible systematic uncertainty associated with the position of the kaon decay vertex.

6.4. *Dependence on the $K^\pm$ charge sign*

The $K^\pm \to \pi^\pm \pi^0 \pi^0$ events consist of 1.470×10^7 K^+ and 0.817×10^7 K^-. Separate fits to these two samples give statistically consistent values for all best fit parameters. The two values of the slope parameter g_0 are $g_0 = 0.638 \pm 0.005$ for K^+ and $g_0 = 0.653 \pm 0.006$ for K^-, which disagree by $\sim 1.9\sigma$. We take one half of their difference (0.008) as a systematic uncertainty on the value of g_0 obtained by the fit to the full $K^\pm$ sample.

6.5. *Dependence on the distance between the $\pi^\pm$ track and the nearest photon*

The $\pi^\pm$ interaction in LKr may produce multiple energy clusters which are located, in general, near the impact point of the $\pi^\pm$ track and in some cases may be identified as photons. In order to study the effect of these fake photons on the best fit parameters we have repeated the analysis by varying the cut on the minimum distance d between each photon and the track impact point on LKr (both analyses A and B require $d > 15$ cm). Varying d between 10 and 25 cm changes $(a_0 - a_2)m_+$ by ± 0.002, while leaving the other parameters unchanged. We take this variation as a systematic uncertainty on $(a_0 - a_2)m_+$.

6.6. *Effect of LKr resolution and non-linear response at low photon energies*

The effect of possible uncertainties in the parameters describing the LKr energy resolution has been simulated by adding a Gaussian noise with r.m.s. value of 0.06 GeV to the measured photon energies. An additional uncertainty may arise form the correction applied to the measured photon energies to account for the LKr non-linear response at low photon energies (typically <2% at 3 GeV and becoming negligible above

Table 2
Systematic uncertainties

Parameter	Acceptance calculation	Trigger efficiency	Fit interval	K^+/K^- difference	$\pi^\pm$–γ distance	LKr response	Total
$(a_0 - a_2)m_+$	0.001	0.001	0.0025	–	0.002	0.001	± 0.004
$a_2 m_+$	0.012	0.005	0.006	–	–	–	± 0.014
g_0	0.002	0.002	0.002	0.008	–	–	± 0.009
h'	0.009	0.003	0.006	–	–	–	± 0.011

10 GeV). The parameters describing this correction have been varied within limits chosen so that the measured π^0 mass for symmetric photon pairs does not depend on the π^0 energy. Varying both the LKr resolution and non-linearity correction parameters according to these procedures changes $(a_0 - a_2)m_+$ by ± 0.001, while leaving the other parameters unchanged. We take this variation as an additional systematic uncertainty on $(a_0 - a_2)m_+$.

Table 2 lists all the systematics uncertainties discussed above. These are added in quadrature to obtain the total experimental systematic error on the values of the best fit parameters.

6.7. "External" uncertainties

A crucial parameter in the model of Refs. [1,2] is the ratio $R = A_{++-}/A_{+00}$ between the weak amplitudes of $K^\pm \to \pi^\pm\pi^+\pi^-$ and $K^\pm \to \pi^\pm\pi^0\pi^0$ decay. The value extracted from the measured decay branching ratio [5] is $R = 1.972 \pm 0.023$. Varying R withing its error changes $(a_0 - a_2)m_+$ by ± 0.003, while leaving the other parameters unchanged. An additional theoretical error of $\pm 5\%$ on $(a_0 - a_2)m_+$, or ± 0.013 is estimated in Ref. [2] as the result of neglecting higher-order terms and radiative corrections in the rescattering model. These uncertainties have no significant effect on $a_2 m_+$.

Taking into account all systematic and external uncertainties we quote:

$$(a_0 - a_2)m_+ = 0.268 \pm 0.010(\text{stat}) \pm 0.004(\text{syst}) \pm 0.013(\text{ext}), \tag{3}$$

$$a_2 m_+ = -0.041 \pm 0.022(\text{stat}) \pm 0.014(\text{syst}). \tag{4}$$

The two statistical errors from the fit are strongly correlated, with a correlation coefficient of -0.858. We note that this analysis offers the first direct determination of a_2, though not as precise as that of $a_0 - a_2$.

Preliminary results obtained under the assumption of exact isospin symmetry have been reported earlier [11].

7. Fit using the correlation between a_0 and a_2 predicted by chiral symmetry

It has been shown that analyticity and chiral symmetry provide a constraint between a_0 and a_2 [12]:

$$a_2 m_+ = (-0.0444 \pm 0.0008) + 0.236(a_0 m_+ - 0.22) - 0.61(a_0 m_+ - 0.22)^2 - 9.9(a_0 m_+ - 0.22)^3.$$

Using this constraint in the fit to the rescattering model of Ref. [2] we obtain

$$a_0 m_+ = 0.220 \pm 0.006(\text{stat}) \pm 0.004(\text{syst}) \pm 0.011(\text{ext}), \tag{5}$$

which corresponds to

$$(a_0 - a_2)m_+ = 0.264 \pm 0.006(\text{stat}) \pm 0.004(\text{syst}) \pm 0.013(\text{ext}). \tag{6}$$

8. Summary and conclusions

The $\pi^0\pi^0$ invariant mass (M_{00}) distribution measured from a sample of 2.287×10^7 $K^\pm \to \pi^\pm\pi^0\pi^0$ fully reconstructed decays collected by the NA48/2 experiment at the CERN SPS shows an anomaly at $M_{00} = 2m_+$. This anomaly has been observed for the first time in this experiment thanks to the large statistical sample and the excellent M_{00} resolution. It can be described by a rescattering model [1,2] dominated by the contribution from the decay $K^\pm \to \pi^\pm\pi^+\pi^-$ through the charge-exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$. These data have been used, therefore, to determine the difference $a_0 - a_2$ between the $I = 0$ and $1 = 2$ S-wave $\pi\pi$ scattering lengths. Our result (see Eq. (3)) is in very good agreement with theoretical calculations performed in the framework of Chiral Perturbation Theory (ChPT) [13], which predict $(a_0 - a_2)m_+ = 0.265 \pm 0.004$. A different theoretical calculation based on a direct analysis of $\pi\pi$ scattering data without using chiral symmetry [14] leads to a somewhat different value with a larger uncertainty, $(a_0 - a_2)m_+ = 0.278 \pm 0.016$, which also agrees with our result.

Previous determination of the $\pi\pi$ scattering lengths have relied on a variety of methods, such as the measurement of $K^+ \to \pi^+\pi^-e^+\nu_e$ decay [15], also being studied by the NA48/2 collaboration, or the measurement of the lifetime of the $\pi^+\pi^-$ atom [16]. Our value of a_0 (see Eq. (5)) is in good agreement with the result of experiment 865 at BNL [15], $a_0 m_+ = 0.216 \pm 0.013(\text{stat}) \pm 0.002(\text{syst}) \pm 0.002(\text{theor})$, also obtained using constraints based on analyticity and chiral symmetry. Our value of $a_0 - a_2$ is also in good agreement with the first measurement of the lifetime of the $\pi^+\pi^-$ atom [16], which corresponds to $|a_0 - a_2|m_+ = 0.264^{+0.033}_{-0.020}$ (it should be noted that the latter result provides only a determination of $|a_0 - a_2|$, while our measurement of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays is also sensitive to its sign).

To conclude, the study of a large sample of $K^\pm \to \pi^\pm\pi^0\pi^0$ decays with excellent resolution on the $\pi^0\pi^0$ invariant mass has provided a novel, precise determination of $a_0 - a_2$, independent of other methods and with different systematics uncertainties. In the near future the expected increase of the event sample by about a factor of 5 from the analysis of all the 2003–2004 data will further reduce the statistical error of our measurement. To be useful, this will require an improvement of the rescattering model to include higher-order terms and also radiative corrections.

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