

# Matching microscopic and macroscopic responses in glasses

M. Baity-Jesi,<sup>1</sup> E. Calore,<sup>2</sup> A. Cruz,<sup>3,4</sup> L.A. Fernandez,<sup>5,4</sup> J.M. Gil-Narvion,<sup>4</sup> A. Gordillo-Guerrero,<sup>6,4</sup>  
D. Iñiguez,<sup>4,7</sup> A. Maiorano,<sup>8,4</sup> E. Marinari,<sup>9</sup> V. Martin-Mayor,<sup>5,4</sup> J. Monforte-Garcia,<sup>4</sup>  
A. Muñoz-Sudupe,<sup>5,4</sup> D. Navarro,<sup>10</sup> G. Parisi,<sup>9</sup> S. Perez-Gaviro,<sup>11,4</sup> F. Ricci-Tersenghi,<sup>9</sup>  
J.J. Ruiz-Lorenzo,<sup>12,4</sup> S.F. Schifano,<sup>13</sup> B. Seoane,<sup>14,4</sup> A. Tarancon,<sup>3,4</sup> R. Tripiccione,<sup>2</sup> and D. Yllanes<sup>15,4</sup>

(Janus Collaboration)

<sup>1</sup>*Institut de Physique Théorique, Université Paris Saclay, CEA, CNRS, F-91191 Gif-sur-Yvette, France*

<sup>2</sup>*Dipartimento di Fisica e Scienze della Terra, Università di Ferrara e INFN, Sezione di Ferrara, I-44122 Ferrara, Italy*

<sup>3</sup>*Departamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain*

<sup>4</sup>*Instituto de Biocomputación y Física de Sistemas Complejos (BIFI), 50018 Zaragoza, Spain*

<sup>5</sup>*Departamento de Física Teórica I, Universidad Complutense, 28040 Madrid, Spain*

<sup>6</sup>*Departamento de Ingeniería Eléctrica, Electrónica y Automática, U. de Extremadura, 10071, Cáceres, Spain*

<sup>7</sup>*Fundación ARAID, Diputación General de Aragón, Zaragoza, Spain*

<sup>8</sup>*Dipartimento di Fisica, Sapienza Università di Roma, I-00185 Rome, Italy*

<sup>9</sup>*Dipartimento di Fisica, Sapienza Università di Roma, INFN,*

*Sezione di Roma 1, and CNR-Nanotec, I-00185 Rome, Italy*

<sup>10</sup>*Departamento de Ingeniería, Electrónica y Comunicaciones and I3A, U. de Zaragoza, 50018 Zaragoza, Spain*

<sup>11</sup>*Centro Universitario de la Defensa, Carretera de Huesca s/n, 50090 Zaragoza, Spain*

<sup>12</sup>*Departamento de Física and Instituto de Computación Científica Avanzada (ICCAEx),*

*Universidad de Extremadura, 06071 Badajoz, Spain*

<sup>13</sup>*Dipartimento di Matematica e Informatica, Università di Ferrara e INFN, Sezione di Ferrara, I-44122 Ferrara, Italy*

<sup>14</sup>*Laboratoire de Physique Théorique, École Normale Supérieure & Université de Recherche Paris Sciences et Lettres,  
Pierre et Marie Curie & Sorbonne Universités, UMR 8549 CNRS, 75005 Paris, France*

<sup>15</sup>*Department of Physics and Soft Matter Program, Syracuse University, Syracuse, NY, 13244*

(Dated: April 26, 2017)

We first reproduce on the Janus and Janus II computers a milestone experiment that measures the spin-glass coherence length through the lowering of free-energy barriers induced by the Zeeman effect. Secondly we determine the scaling behavior that allows a quantitative analysis of a new experiment reported in the companion Letter [S. Guchhait and R. Orbach, *Phys. Rev. Lett.* **118**, 157203 (2017)]. The value of the coherence length estimated through the analysis of microscopic correlation functions turns out to be quantitatively consistent with its measurement through macroscopic response functions. Further, non-linear susceptibilities, recently measured in glass-forming liquids, scale as powers of the same microscopic length.

*Introduction.* It has long been suspected that the exceedingly slow dynamics that disordered and glassy systems (spin glasses, super-spin glasses, colloids, polymers, etc.) exhibit upon cooling is due to the increasing size of the cooperative regions [1], which one would like to describe in terms of a correlation length  $\xi$ . The standard way of accessing  $\xi$  is measuring the structure factor in a neutron-scattering experiment. Unfortunately, this approach is unsuitable for experiments on glassy systems, because their structure factors show no trace of a growing length scale.

Yet, for example for spin-glass systems, the replica method provides a “microscopic” approach to obtain the correlation functions of the overlap field [2–15], which decay with a correlation length  $\xi_{\text{mic}}$ . Unfortunately, these correlation functions are only easy to access through numerical simulations, since computing replicas requires direct access to the microscopic configurations.

In spite of the above difficulties it has been possible to develop effective techniques to measure  $\xi$  in real experiments. The state-of-the-art techniques are based on non-linear responses to external perturbations. Very often

these measurements are carried out in a non-equilibrium regime. If the temperature is low enough,  $\xi$  grows sluggishly but also indefinitely (unless the sample has a film geometry [16, 17]). For spin glasses and super-spin glasses, the magnetic response to an external magnetic field is accurately measured with a SQUID. A delicate analysis of this response yields a “macroscopic” correlation length, which we denote by  $\xi_{\text{mac}}$ , as a function of time. In the case of glass-forming liquids, one can study the dielectric polarizability.

Here we implement numerically, for the first time, on the Ising spin glass, the seminal experimental protocol introduced in [18], which is now a crucial protocol for spin glass experiments [16, 19]. Thanks to our dedicated computers Janus [20] and Janus II [21], the system size and the time scales reached in our simulation allow us to assert the mutual consistency of the correlation lengths obtained through macroscopic response,  $\xi_{\text{mac}}$ , and the length scale  $\xi_{\text{mic}}$  derived from the direct measurement of the overlap correlation function.

Our analysis unveils a scaling law describing how the magnetic response depends both on the applied magnetic

field  $H$  and on the size  $\xi_{\text{mic}}$  of the magnetic domains. Remarkably, this scaling law is already very useful in the analysis of the experiment by Guchhait and Orbach described in the companion Letter [22].

The reader is probably aware of the long ongoing controversy about the nature of the spin-glass phase. The Replica Symmetry Breaking theory [23] predicts a spin-glass transition in a field [24], while the droplet model predicts that the magnetic field (no matter how small) avoids the transition [25–28]. In particular, the dynamics of a spin-glass in a field has been analyzed within the context of the droplet model [29]. However, it has been difficult for experiments to distinguish both theories [18, 30–33], because the two predict a barrier-height that depends on the length scale  $\xi_{\text{mic}}$ . Fortunately, our analysis completely avoids this controversy.

Finally, we link our results to the physics of glass-forming liquids through a study of the non-linear susceptibilities  $\chi_3$  (see below). To date it has not yet been possible to reproduce the delicate experimental protocol of Ref. [18] for supercooled liquids or glasses. However,  $\chi_3$  [34] (and also  $\chi_5$  [35]) can be measured and do grow. We find that in our spin-glass simulation  $\chi_3$  has a well-defined scaling form as a power of  $\xi_{\text{mic}}$ .

*Model and protocol.* We study the Edwards-Anderson model in a three-dimensional,  $D = 3$  cubic lattice of linear size  $L$ , with periodic boundary conditions. Our  $N = L^D$  Ising spins,  $\sigma_{\mathbf{x}} = \pm 1$ , interact with their lattice nearest neighbors through the Hamiltonian

$$\mathcal{H} = - \sum_{\langle \mathbf{x}, \mathbf{y} \rangle} J_{\mathbf{x}, \mathbf{y}} \sigma_{\mathbf{x}} \sigma_{\mathbf{y}} - H \sum_{\mathbf{x}} \sigma_{\mathbf{x}}. \quad (1)$$

The couplings  $J_{\mathbf{x}, \mathbf{y}}$  take the values  $\pm 1$  with 50% probability. In the absence of a magnetic field,  $H = 0$ , this model undergoes a spin-glass transition at the critical temperature  $T_c = 1.102(3)$  [36]. The value of the dimensionless magnetic field  $H$  used in the numerical simulation can be matched to the physical one. For the Ising spin glass  $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$  we find  $H_{\text{experimental}} \approx 50 \text{ kG} \times H$  [37]. This matching is likely to be strongly dependent on the material under consideration.

We describe succinctly our simulation protocol (for details see the analysis of the aging linear response in [38]). We consider a large system (with  $L = 80$  or  $160$ , large enough to avoid relevant finite-size effects). The initial random spin configuration is placed instantaneously at the working temperature  $T = 0.7 \approx 0.64T_c$  and left to relax for a time  $t_w$ , with  $H = 0$ . At time  $t_w$ , the magnetic field is turned on and we start recording the magnetization density,  $m = \sum_{\mathbf{x}} \sigma_{\mathbf{x}} / N$ . We write  $m(t + t_w, t_w; H)$  to emphasize that the system is perennially out of equilibrium (and, hence,  $t_w$ -dependent). In the following the symmetry under the inversion of the magnetic field,  $m(t + t_w, t_w; H) = -m(t + t_w, t_w; -H)$ , will be crucial.

*Scaling.* As the system relaxes at the working temperature for a time  $t_w$ , the size of the glassy domains

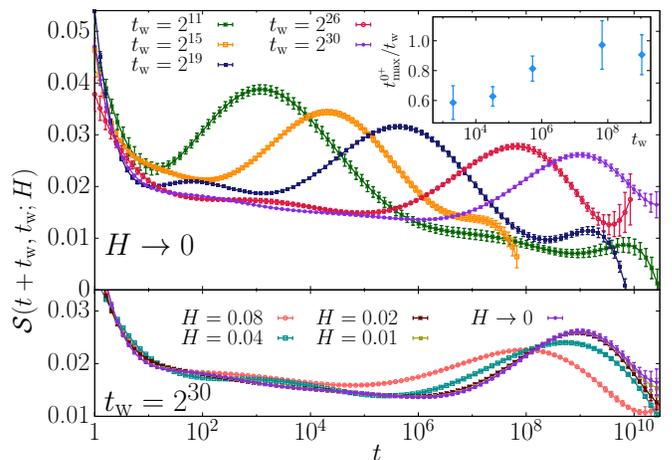


FIG. 1. The function  $\mathcal{S}(t + t_w, t_w; H)$ , Eq. (5) versus the time  $t$  elapsed after switching on the external magnetic field  $H$ . In the **top** panel we show the  $H \rightarrow 0$  extrapolation for several waiting times  $t_w$  (one unit of computer time roughly corresponds to one picosecond of physical time [39]). **Bottom:**  $\mathcal{S}(t + t_w, t_w; H)$  as a function of  $t$  for our largest waiting time  $t_w = 2^{30}$  and for different values of  $H$ . **Inset:** The peak position ( $H \rightarrow 0$ ), in units of  $t_w$ , depends on  $t_w$  only for  $t_w < 10^6$ .

grows. The overlap correlation function  $C_4(r, t_w)$  [40] decays with the distance  $r$  as  $C_4(r, t_w) = f_c(r/\xi_{\text{mic}}(t_w))r^{-\theta}$  [3, 10, 11]. The cut-off function  $f_c(x)$  decays faster than exponentially at large  $x$ . The exponent  $\theta = 0.38(2)$  [11, 41] will be crucial in our analysis. The microscopic coherence length grows with time as  $\xi_{\text{mic}}(t_w) \propto t_w^{1/z(T)}$ , with  $z(T = 0.7) = 11.64(15)$  [11].

In equilibrium conditions and for large  $\xi_{\text{mic}}$ , there is a well developed scaling theory for the magnetic response to an external field, see, e.g., [42, 43]. However, dynamic scaling [44] suggests borrowing the equilibrium formulae, and replacing the equilibrium  $\xi_{\text{mic}}$  by the aging  $\xi_{\text{mic}}(t + t_w)$  (as obtained at  $H = 0$ ). This bold approach has been successfully tested for spin glasses close to  $T_c$  [13, 14] (and, to a small extent, also for glass-forming liquids [35]), thanks to the relation

$$m(t + t_w, t_w; H) = \xi_{\text{mic}}^{y_h - D} \mathcal{F}\left(H[\xi_{\text{mic}}(t + t_w)]^{y_h}, \mathcal{R}_{t, t_w}\right), \quad (2)$$

where  $y_h$  is a scaling dimension that we will now determine,  $\mathcal{R}_{t, t_w} \equiv \xi_{\text{mic}}(t + t_w)/\xi_{\text{mic}}(t_w)$ , and the scaling function  $\mathcal{F}(x, \mathcal{R})$  is odd on its first argument for symmetry reasons. As we will show below, see Fig. 1–inset, we shall be interested in the regime  $t \approx t_w$  where the approximation  $\mathcal{R}_{t, t_w} \approx 1$  is safe [38]. Therefore,  $\xi_{\text{mic}}(t_w)$  will be the relevant length scale from now on.

The (generalized) susceptibilities  $\chi_1, \chi_3, \chi_5, \dots$  are defined from the Taylor expansion

$$m(H) = \chi_1 H + \frac{\chi_3}{3!} H^3 + \frac{\chi_5}{5!} H^5 + \mathcal{O}(H^7), \quad (3)$$

where we omitted the  $t$  and  $t_w$  dependencies of  $m$  and

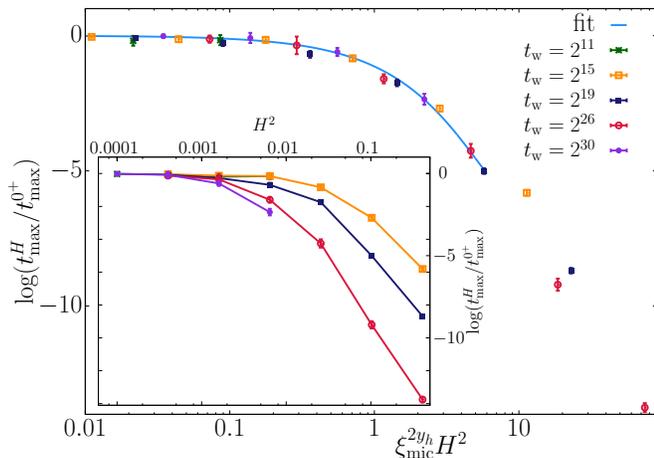


FIG. 2. The Zeeman energy follows the scaling form suggested in Eq. (6). We show a fit to  $F_{\text{Zeeman}}(x) = c_1 x + c_2 x^2$ . **Inset:** the data of the main panel do not collapse when plotted as a function of  $H^2$ .

of the susceptibilities to simplify our notation. Matching Eqs. (2) and (3), we find the scaling behavior  $\chi_{2n-1} \propto [\xi(t_w)]^{2y_h n - D}$ . At least in equilibrium,  $\chi_3$  is connected to the space-integral of the microscopic correlation function  $C_4(r, t_w)$  [45]. We thus conclude that

$$2y_h = D - \frac{\theta}{2}. \quad (4)$$

Taking  $\theta$  from [11, 41], we find  $2y_h = 2.81(1)$ . Although  $2y_h$  is sometimes referred to as the fractal dimension of the glassy domains [6, 19, 35, 46], we regard it as just a scaling dimension [47] (the droplet model prediction is  $2y_h = D$ ).

*Simulating the experiment.* The main quantity used in the experiment of [18] is

$$\mathcal{S}(t + t_w, t_w; H) = \frac{\partial}{\partial \log t} \left[ \frac{m(t + t_w, t_w; H)}{H} \right]. \quad (5)$$

This quantity, shown in Fig. 1, has a local maximum at time  $t_{\text{max}}^{(H)}$ . The time scale  $t_{\text{max}}^{(H)}$  was interpreted by Joh et al. as representative of the free-energy barriers  $\Delta(t_w; H)$  that are relevant at time  $t_w$ :  $t_{\text{max}}^{(H)} \propto \exp[\Delta/k_B T]$  [18] (see also the numerical computation in Ref. [29]).

$\mathcal{S}(t + t_w, t_w; H)$  depends on two time scales,  $t$  and  $t_w$ , as it is typical of aging systems [48]. However, we want to use  $\mathcal{S}$  to extract information from the single-time  $\xi_{\text{mac}}(t_w)$ . The paradox is solved in the inset to Fig. 1, where we show that, when  $t_w$  is large enough, the ratio  $t_{\text{max}}^{(0+)}/t_w$  becomes independent of  $t_w$ : we are, in these conditions, in the asymptotic regime. This regime is also reached, at significantly shorter  $t_w$ , with Gaussian couplings [29].

The maximum  $t_{\text{max}}^{(H)}$  decreases upon increasing  $H$ , see Fig. 1–bottom. This reflects the lowering of the barriers  $\Delta$  due to the Zeeman effect of the (glassy) magnetic

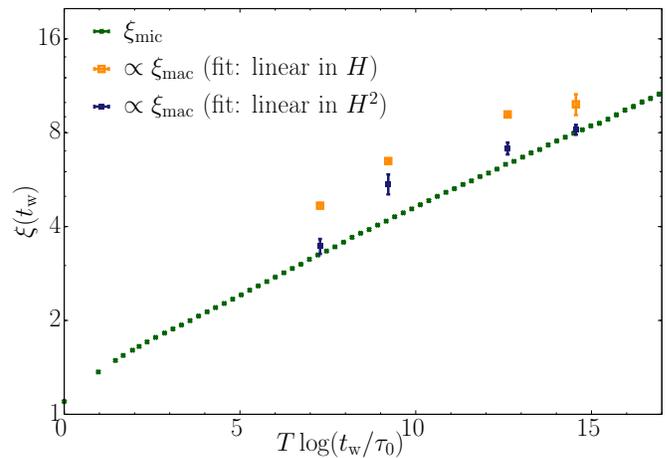


FIG. 3. The time growth of the correlation length  $\xi_{\text{mic}}$ , as obtained from the microscopic correlation function  $C_4(r, t_w)$  [10, 11, 38], is compared to the length  $\xi_{\text{mac}}$  obtained from a fit linear in  $H^2$ , see Eq. (7). The microscopic time scale  $\tau_0 = 1$  corresponds to a single lattice sweep in our Monte Carlo simulation (see caption to Fig. 1). We also show the results obtained with Eq (9), which are sensible as well. The temperature-dependent scaling variable,  $T \log(t_w/\tau_0)$ , is common in the experimental literature (e.g., see Ref. [19]).

domains [18]. From Eq. (2), and given the  $H \leftrightarrow -H$  symmetry, it is natural to expect the Zeeman effect to be described through a smooth scaling function

$$\log \frac{t_{\text{max}}^{(H)}}{t_{\text{max}}^{0+}} = F_{\text{Zeeman}}(x), \quad x = H^2 [\xi_{\text{mic}}(t_w)]^{D - \frac{\theta}{2}}, \quad (6)$$

where  $t_{\text{max}}^{0+}$  is the extrapolation to  $H = 0$  of  $t_{\text{max}}^{(H)}$ . As Fig. 2 shows, this scaling holds for values of the scaling variable as large as  $x \approx 6$ : we have a very good scaling for close to three orders of magnitude. Up to that value, we find that the scaling function can be parameterized as  $F_{\text{Zeeman}}(x) = c_1 x + c_2 x^2$ . In other words, for small  $H$  we expect the Zeeman energy to be proportional to  $H^2$  with sizable corrections of order  $H^4$ . To the best of our knowledge, the explicit scaling form in Eq. (6) has never been used in the analysis of experimental data. Yet the authors of the original experiment [18] fitted their data at fixed  $t_w$  to

$$\log \frac{t_{\text{max}}^{(H)}}{t_{\text{max}}^{0+}} = A N_f(t_w) H^2, \quad (7)$$

where  $A$  is a  $t_w$ -independent constant.  $N_f(t_w)$  was interpreted as the number of spins in a correlated domain, and hence

$$\xi_{\text{mac}}(t_w) = [N_f(t_w)]^{1/D}. \quad (8)$$

Eqs. (7) and (8) can be seen as the first-order expansion of Eq. (6). In fact, the smallness of exponent  $\theta$  implies that the small correction  $[\xi_{\text{mac}}(t_w)]^{\theta/2}$  can easily go unobserved.

Fig. 3 shows  $\xi_{\text{mac}}(t_w) = [N_f(t_w)]^{1/(D-\theta/2)}$  [we obtained  $N_f(t_w)$  from the fit to Eq. (7)]. Since different determinations of the correlation length should coincide only up to a multiplicative constant of order one, we have not fitted for  $A$ , choosing instead  $A = 1$ . It is clear that  $\xi_{\text{mac}}(t_w)$  and  $\xi_{\text{mic}}(t_w)$  have the same behavior.

Finally, let us remark that in Ref. [49] it was suggested that Ising spin glasses should have a Zeeman energy of order  $H$ . On theoretical grounds, this is not possible for protocols respecting the symmetry  $H \leftrightarrow -H$ . However, we found that for  $1 < x < 4$  a best fit to the form  $F_{\text{Zeeman}}(x) = d_1 + d_2\sqrt{x}$  gives an acceptable value of  $\chi^2$ , but one gets that  $d_1 \neq 0$ , that implies an unphysical value for the  $H \rightarrow 0$  extrapolation. Only a careful control of the limit of vanishing field (see the companion Letter by Guchhait and Orbach [22]), reveals that the true behavior for small  $H$  is proportional to  $H^2$ . In practice, the transient behavior of  $F_{\text{Zeeman}}(x)$  implies that one could fit the data to the form

$$\log \frac{t_{\text{max}}^{(H)}}{t_{\text{max}}^{0+}} = A' \sqrt{N_f(t_w)H^2}, \quad (9)$$

and then extract  $\xi_{\text{mac}}(t_w) = [N_f(t_w)]^{1/(D-\theta/2)}$  (again,  $A' = 1$ ). Although Eq. (9) is incorrect for small values of  $H$ , the scaling law Eq. (6) implies that one will still obtain a reasonable determination of  $\xi_{\text{mac}}$ , as we indeed find (see Fig. 3, where we also show  $\xi_{\text{mac}}(t_w)$  obtained from this approach).

*Non-linear susceptibilities.* At variance with spin glasses [18], the detection of a large correlation length accompanying the glass transition is still an open problem for supercooled liquids [50]. It is now clear that linear responses are not up to the task [34, 51], so higher-order non-linear responses are currently under investigation [34, 35, 52]. However, even in the more familiar context of spin glasses the connection between  $\chi_3(t + t_w; t_w)$  and  $\xi_{\text{mic}}(t_w)$  needs to be clarified.

To make some progress, we extract generalized susceptibilities such as  $\chi_3$  through Eq. (3). Fig. 4-top shows that  $\chi_3(t + t_w, t_w)$  has a  $t_w$ -independent regime for  $t \ll t_w$  (the time-translational invariant regime [48], see also [53]). Yet, it displays a peak as a function of  $t$ , whose position and height are strongly  $t_w$ -dependent. In fact, we empirically find (see Fig. 4-bottom) the following scaling behavior for large enough values of  $t$  and  $t_w$

$$\chi_3(t + t_w, t_w) = [\xi_{\text{mic}}(t_w)]^{D-\theta} G(t/t_w). \quad (10)$$

The prefactor  $[\xi_{\text{mic}}(t_w)]^{D-\theta}$  follows from Eqs. (2), (3) and (4). Deriving the details of the function  $G(t/t_w)$  will require further work.

*Conclusions.* Using the dedicated computers Janus and Janus II, we have studied the aging magnetic response of an Ising spin-glass to an applied field. In this way, we have simulated a milestone experiment [18], and we have shown that the glassy correlation length

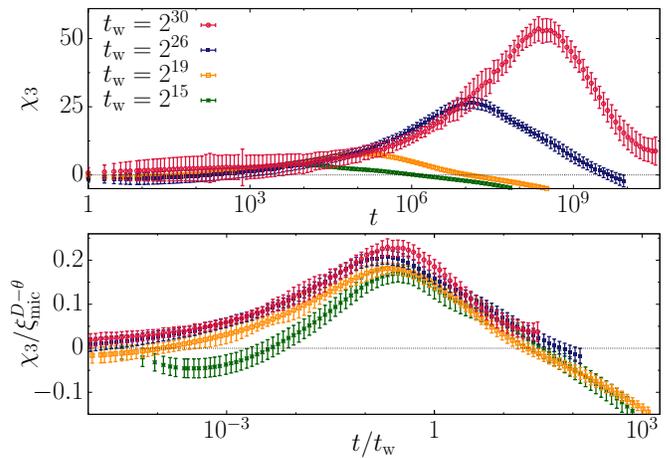


FIG. 4. The non-linear susceptibility  $\chi_3$  is shown as a function of  $t$  for several values of  $t_w$  (**top**), as obtained from Eq. (3). The difficulty lies in balancing systematic errors (numerical data obtained with high fields underestimate  $\chi_3$ ) with statistical errors (which are larger for small values of  $H$ ). Our compromise, shown here, tries to obtain statistical and systematic errors of comparable size. In the **bottom** panel we show the  $G(t/t_w)$  scaling function (10). Note that scaling corrections are visible only for the smallest waiting times (and, even in those cases, they only appear for small  $t/t_w$ ).

extracted from this macroscopic response is numerically consistent with its microscopic determination from overlap correlation functions. Furthermore, we have unveiled scaling laws that relate the magnetic response to the applied field and the correlation length. We expect that this scaling analysis will be useful in future experiments on film geometry. Our scaling analysis has been relevant for the study of the experiment reported in the companion Letter [22]. The agreement with experiments is even more impressive when one notices that we are comparing numerical time scales of the order of the millisecond to experimental time scales of the order of the hour: this looks like a very nice piece of evidence for invariance in time scales.

Although the delicate experimental study of Ref. [18] has not yet been carried out for glass-forming liquids, the (dielectric polarizability analogue of) the non-linear susceptibilities are measured in current experiments [34, 35]. We have shown that these susceptibilities scale as powers of the microscopically-determined correlation lengths.

*Acknowledgments.* We warmly thank Ray Orbach and Samareh Guchhait for sharing with us their data prior to publication [22], and for a most fruitful exchange of ideas.

We thank the staff of BIFI supercomputing center for their assistance. We thank M. Pivanti for his contribution to the early stages of the development of the Janus II computer. We also thank Link Engineering (Bologna, Italy) for their precious role in the technical aspects related to the construction of Janus II. We thank EU, Gov-

ernment of Spain and Government of Aragon for the financial support (FEDER) of Janus II development. This work was partially supported by Ministerio de Economía, Industria y Competitividad (MINECO) (Spain) through Grants No. FIS2012-35719-C02, No. FIS2013-42840-P, No. FIS2015-65078-C2, No. FIS2016-76359-P, and No. TEC2016-78358-R, by the Junta de Extremadura (Spain) through Grant No. GRU10158 (partially funded by FEDER) and by the DGA-FSE (Diputación General de Aragón – Fondo Social Europeo). This project has received funding from the European Union’s Horizon 2020 research and innovation program under the Marie Skłodowska–Curie Grant No. 654971. This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (Grant No. 694925). D. Y. acknowledges support by Grant No. NSF-DMR-305184 and by the Soft Matter Program at Syracuse University. M. B. J. acknowledges financial support from ERC Grant No. NPRGGLASS.

- 
- [1] G. Adam and J. H. Gibbs, *The Journal of Chemical Physics* **43**, 139 (1965).
- [2] H. Rieger, *J. Phys. A* **26**, L615 (1993).
- [3] E. Marinari, G. Parisi, J. Ruiz-Lorenzo, and F. Ritort, *Phys. Rev. Lett.* **76**, 843 (1996).
- [4] J. Kisker, L. Santen, M. Schreckenberg, and H. Rieger, *Phys. Rev. B* **53**, 6418 (1996).
- [5] E. Marinari, G. Parisi, F. Ricci-Tersenghi, and J. J. Ruiz-Lorenzo, *J. Phys. A* **33**, 2373 (2000).
- [6] L. Berthier and J.-P. Bouchaud, *Phys. Rev. B* **66**, 054404 (2002).
- [7] L. Berthier and A. P. Young, *Phys. Rev. B* **69**, 184423 (2004).
- [8] S. Jiménez, V. Martín-Mayor, and S. Pérez-Gaviro, *Phys. Rev. B* **72**, 054417 (2005).
- [9] L. C. Jaubert, C. Chamon, L. F. Cugliandolo, and M. Picco, *J. Stat. Mech.* **2007**, P05001 (2007).
- [10] F. Belletti, M. Cotallo, A. Cruz, L. A. Fernandez, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, A. M. Sodupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J. J. Ruiz-Lorenzo, S. F. Schifano, D. Sciretti, A. Tarancon, R. Tripiccion, J. L. Velasco, and D. Yllanes (Janus Collaboration), *Phys. Rev. Lett.* **101**, 157201 (2008), arXiv:0804.1471.
- [11] F. Belletti, A. Cruz, L. A. Fernandez, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, J. Monforte, A. Muñoz Sodupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J. J. Ruiz-Lorenzo, S. F. Schifano, D. Sciretti, A. Tarancon, R. Tripiccion, and D. Yllanes (Janus Collaboration), *J. Stat. Phys.* **135**, 1121 (2009), arXiv:0811.2864.
- [12] C.-W. Liu, A. Polkovnikov, A. W. Sandvik, and A. P. Young, *Phys. Rev. E* **92**, 022128 (2015).
- [13] L. A. Fernández and V. Martín-Mayor, *Phys. Rev. B* **91**, 174202 (2015).
- [14] M. Lulli, G. Parisi, and A. Pelissetto, *Phys. Rev. E* **93**, 032126 (2016).
- [15] M. Manssen and A. K. Hartmann, *Phys. Rev. B* **91**, 174433 (2015), arXiv:1411.5512.
- [16] S. Guchhait and R. Orbach, *Phys. Rev. Lett.* **112**, 126401 (2014).
- [17] S. Guchhait, G. G. Kenning, R. L. Orbach, and G. F. Rodriguez, *Phys. Rev. B* **91**, 014434 (2015).
- [18] Y. G. Joh, R. Orbach, G. G. Wood, J. Hammann, and E. Vincent, *Phys. Rev. Lett.* **82**, 438 (1999).
- [19] S. Nakamae, C. Crauste-Thibierge, D. L’Hôte, E. Vincent, E. Dubois, V. Dupuis, and R. Perzynski, *Appl. Phys. Lett.* **101**, 242409 (2012).
- [20] F. Belletti, M. Cotallo, A. Cruz, L. A. Fernandez, A. Gordillo, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, A. Muñoz Sodupe, D. Navarro, S. Perez-Gaviro, J. J. Ruiz-Lorenzo, S. F. Schifano, D. Sciretti, A. Tarancon, R. Tripiccion, and J. L. Velasco (Janus Collaboration), *Comp. Phys. Comm.* **178**, 208 (2008), arXiv:0704.3573.
- [21] M. Baity-Jesi, R. A. Baños, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion, A. Gordillo-Guerrero, D. Iniguez, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, J. Monforte-Garcia, A. Muñoz Sodupe, D. Navarro, G. Parisi, S. Perez-Gaviro, M. Pivanti, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccion, and D. Yllanes (Janus Collaboration), *Comp. Phys. Comm* **185**, 550 (2014), arXiv:1310.1032.
- [22] S. Guchhait and R. L. Orbach, *Phys. Rev. Lett.* **118**, 157203 (2017).
- [23] E. Marinari, G. Parisi, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, and F. Zuliani, *J. Stat. Phys.* **98**, 973 (2000), arXiv:cond-mat/9906076.
- [24] J. R. L. de Almeida and D. J. Thouless, *J. Phys. A: Math. Gen.* **11**, 983 (1978).
- [25] W. L. McMillan, *J. Phys. C: Solid State Phys.* **17**, 3179 (1984).
- [26] A. J. Bray and M. A. Moore, in *Heidelberg Colloquium on Glassy Dynamics*, Lecture Notes in Physics No. 275, edited by J. L. van Hemmen and I. Morgenstern (Springer, Berlin, 1987).
- [27] D. S. Fisher and D. A. Huse, *Phys. Rev. Lett.* **56**, 1601 (1986).
- [28] D. S. Fisher and D. A. Huse, *Phys. Rev. B* **38**, 373 (1988).
- [29] H. Takayama and K. Hukushima, *Journal of the Physical Society of Japan* **73**, 2077 (2004), <http://dx.doi.org/10.1143/JPSJ.73.2077>.
- [30] F. Lefloch, J. Hammann, M. Ocio, and E. Vincent, *EPL (Europhysics Letters)* **18**, 647 (1992).
- [31] K. Jonason, E. Vincent, J. Hammann, J. P. Bouchaud, and P. Nordblad, *Phys. Rev. Lett.* **81**, 3243 (1998).
- [32] K. Jonason, P. Nordblad, E. Vincent, J. Hammann, and J.-P. Bouchaud, *Eur. Phys. J. B* **13**, 99 (2000).
- [33] J.-P. Bouchaud, V. Dupuis, J. Hammann, and E. Vincent, *Phys. Rev. B* **65**, 024439 (2001).
- [34] L. Berthier, G. Biroli, J.-P. Bouchaud, L. Cipelletti, D. El Masri, D. L’Hôte, F. Ladieu, and M. Pierno, *Science* **310**, 1797 (2005).
- [35] S. Albert, T. Bauer, M. Michl, G. Biroli, J.-P. Bouchaud, A. Loidl, P. Lunkenheimer, R. Tourbot, C. Wiertel-Gasquet, and F. Ladieu, *Science* **352**, 1308 (2016), arXiv:1606.04079.
- [36] M. Baity-Jesi, R. A. Baños, A. Cruz, L. A. Fernandez,

- J. M. Gil-Narvion, A. Gordillo-Guerrero, D. Iniguez, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, M. Pivanti, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccion, and D. Yllanes (Janus Collaboration), *Phys. Rev. B* **88**, 224416 (2013), arXiv:1310.2910.
- [37] At the critical temperature, we matched the value of  $\langle(m)/H\rangle_H / \langle(m)/H\rangle_{H\rightarrow 0}$ , computed numerically [54], with the same quantity measured experimentally in  $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$  [55] ( $\langle m \rangle$  is the equilibrium value of the magnetization density).
- [38] M. Baity-Jesi, E. Calore, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion, A. Gordillo-Guerrero, D. Iniguez, A. Maiorano, E. Marinari, V. Martín-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccion, and D. Yllanes, *Proceedings of the National Academy of Sciences* **114**, 1838 (2017).
- [39] J. A. Mydosh, *Spin Glasses: an Experimental Introduction* (Taylor and Francis, London, 1993).
- [40]  $C_4(r, t_w) = \overline{[q_i(t_w)q_{i+r}(t_w)]}$  where  $q_i(t_w)$  is the local overlap defined as  $q_i = \sigma_i \tau_i$ , where  $\sigma_i$  and  $\tau_i$  are two real replicas which evolve with the same (random) couplings.  $[\overline{(\dots)}]$  denotes the average over initial spin configurations and  $\overline{(\dots)}$  is the average over the random couplings.
- [41] R. Alvarez Baños, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martín-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccion, and D. Yllanes (Janus Collaboration), *J. Stat. Mech.* **2010**, P06026 (2010), arXiv:1003.2569.
- [42] G. Parisi, *Statistical Field Theory* (Addison-Wesley, 1988).
- [43] D. J. Amit and V. Martín-Mayor, *Field Theory, the Renormalization Group and Critical Phenomena*, 3rd ed. (World Scientific, Singapore, 2005).
- [44] Y. Ozeki and N. Ito, *J. Phys. A: Math. Theor.* **40**, R149 (2007).
- [45] K. Binder and A. P. Young, *Rev. Mod. Phys.* **58**, 801 (1986).
- [46] J. D. Stevenson, J. Schmalian, and P. G. Wolynes, *Nat Phys* **2**, 268 (2006).
- [47] Indeed, a satisfactory geometric construction would build spin clusters with a number of spins scaling as the relevant susceptibility, i.e.,  $[\xi(t_w)]^{D-\theta}$  in our case. For ferromagnetic systems, these domains are the Fortuin-Kasteleyn (FK) clusters [56]. Unfortunately, it is still unknown how to satisfactorily generalize FK clusters to glassy systems (for instance, the straightforward generalization of FK clusters to spin glasses are space-filling — i.e., percolating — well above  $T_c$  [57]). Yet, see Refs. [58–62] for some recent work.
- [48] E. Vincent, J. Hammann, M. Ocio, J.-P. Bouchaud, and L. F. Cugliandolo, in *Complex Behavior of Glassy Systems*, Lecture Notes in Physics No. 492, edited by M. Rubí and C. Pérez-Vicente (Springer, 1997).
- [49] F. Bert, V. Dupuis, E. Vincent, J. Hammann, and J.-P. Bouchaud, *Phys. Rev. Lett.* **92**, 167203 (2004).
- [50] S. Karmakar, C. Dasgupta, and S. Sastry, *Annu. Rev. Condens. Matter Phys.* **5**, 255 (2014).
- [51] G. B. J. P. Biroli, A. Cavagna, T. S. Grigera, and P. Verrocchio, *Nature Physics* **4**, 771 (2008), arXiv:cond-mat/0607817.
- [52] C. Brun, F. Ladieu, D. L’Hôte, G. Biroli, and J.-P. Bouchaud, *Phys. Rev. Lett.* **109**, 175702 (2012).
- [53] G. Biroli and P. Urbani, *Nature Physics* **12**, 1130 (2016), arXiv:1601.06724.
- [54] M. Baity-Jesi, R. A. Baños, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion, A. Gordillo-Guerrero, D. Iniguez, A. Maiorano, M. F., E. Marinari, V. Martín-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, M. Pivanti, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccion, and D. Yllanes, *J. Stat. Mech.* **2014**, P05014 (2014), arXiv:1403.2622.
- [55] H. Aruga Katori and A. Ito, *Journal of the Physical Society of Japan* **63**, 3122 (1994), <http://dx.doi.org/10.1143/JPSJ.63.3122>.
- [56] C. M. Fortuin and P. W. Kasteleyn, *Physica (Utrecht)* **57**, 536 (1972).
- [57] A. Coniglio, F. di Liberto, G. Monroy, and F. Peruggi, *Phys. Rev. B* **44**, 12605 (1991).
- [58] J. Houdayer, *The European Physical Journal B - Condensed Matter and Complex Systems* **22**, 479 (2001).
- [59] J. Houdayer and A. K. Hartmann, *Phys. Rev. B* **70**, 014418 (2004).
- [60] T. Jörg, *Progress of Theoretical Physics Supplement* **157**, 349 (2005).
- [61] J. Machta, C. M. Newman, and D. L. Stein, *Journal of Statistical Physics* **130**, 113 (2008).
- [62] Z. Zhu, A. J. Ochoa, and H. G. Katzgraber, *Phys. Rev. Lett.* **115**, 077201 (2015).