Taking Account of Uncertainty in Demand Growth When Phasing the Construction of a Water Distribution Network

E. Creaco[1]; M. Franchini[2]; and T. M. Walski[3]

Abstract: As is well known, water systems grow gradually over long periods of time, and the life of piping tends to be much longer than the planning horizon used for pipe sizing. Furthermore, the uncertainty about future demands grows with the length of the time horizon. The design of water-distribution systems should therefore be performed in phases, to follow the gradual network growth, and taking account of the uncertainty connected with demand growth. The design approach proposed in this paper to consider these aspects is able to identify, on prefixed time steps or intervals, the necessary upgrades of the construction where each upgrade consists of installing pipes in new sites or in parallel to pipes that already exist, in order to render the network able to satisfy user demand with acceptable service pressure over the different phases of its life. Uncertainty in demand growth is considered by expressing the growth rate by means of a discrete random variable with assigned probability mass function. Optimization of phasing of construction is then performed by considering two objective functions: present-worth cost of the construction (to be minimized), and minimum-pressure surplus over time (to be maximized), which is represented as a discrete random variable with a derived probability distribution as a consequence of the assumption made on the water demand, which randomly grows from phase to phase of the construction. Within this framework, a specific criterion to rank discrete random variables is presented here. The application of the methodology to a case study shows that optimizing phasing of construction while accounting for uncertainty in demand growth leads to the network being sized more conservatively, so that the network construction obtained turns out to be more flexible to adapt itself to various conditions of demand growth over time. **DOI: 10.1061/(ASCE)WR.1943-5452.0000441.**

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Introduction

In the scientific literature, the problem of optimal water distribution network design has been studied in several hundred papers by using either methodologies based on single-objective optimization (see for example, Alperovits and Shamir 1977; Quindry et al. 1979; 1981; Goulter et al. 1986; Fujiwara et al. 1987; Kessler and Shamir 1989, 1991; Fujiwara and Khang 1990, 1991; Bhave and Sonak 1992; Simpson et al. 1994; Loganathan et al. 1995; Savic and Walters 1997; Wu and Simpson 2001; Eusuff and Lansey 2003; Maier et al. 2003; Krapivka and Ostfeld 2009; Haghighi et al. 2011) or multiobjective optimization (Gessler and Walski 1985; Todini 2000; Wu et al. 2002; Prasad et al. 2003; Bentley Systems 2006; Farmani et al. 2006; Creaco and Franchini 2012). In contrast to the first approach, which is simply aimed at minimizing investment costs, the second has the advantage of taking account of reliability, expressed by means of such compact indexes as pressure surplus (Gessler and Walski 1985) or resilience (Todini 2000) or by means of performance indices (Gargano and Pianese 2000; Tanyimboh et al. 2001; Ciaponi 2009), as an objective function to maximize while minimizing the investment cost during the optimization process. However, a drawback of all the design methodologies mentioned above lies in the fact that they do not take account of the practical problem of phasing (also called *sequencing*) of construction, *which is particularly relevant to distribution mains*. In fact, all these approaches were developed on the restrictive assumption that design is performed *statically* by referring to one or more theoretical operating conditions corresponding to a single design date [fixed water demands corresponding to the heaviest loading condition for the water distribution system, usually the hour of maximum demand for a future scenario(s) positioned at the end of the assumed life cycle for the water distribution system] and all the construction is done in a single phase such that there is no gradual growth/build-out in the system. This assumption clearly refers to a theoretical situation because real water-distribution systems are usually subject to expansion or modifications related to the social, commercial, and industrial evolution of the area.

The idea of structuring the design of infrastructures in phases, i.e., *dynamically* in a bid to follow the expansion of urban centers, has been faced by Beh et al. (2011a, b, 2012) and Mortazavi et al. (2012) in the context of water supply options at the regional scale, by Kang and Lansey (2012) in the context of water reclamation systems and by Lansey et al. (1992), Basupi and Kapelan (2012, 2013), and Creaco et al. (2013) in the context of water-distribution systems. In the latter case, by applying a multiobjective optimization methodology, Creaco et al. (2013) showed that resorting to phasing of construction yields some advantages, in that:

- It allows engineers to design the short term upgrades, which are supposed to guarantee a prefixed level of reliability, while keeping an idea of the long-term network growth and expansion; and
- For a long time horizon, it turns out to be cost effective; in fact, by partially deferring construction, the community is able to put

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aside resources that can be more effectively allocated to alternative uses.

However, the methodology developed by Creaco et al. (2013) relied on the following restrictive assumptions:

- Demand growth is known with certainty;
- Network expansion layout is known with certainty;
- Costs and discount rate are known with certainty;
- There are no pumps and therefore energy costs are not accounted for (this will mostly affect large transmission mains);
- Analysis of shutdowns and valving are not included;
- There is only a single pressure zone, so decisions about boundaries are not included;
- Only peak demand conditions are considered in the design;
- Leakage costs, as were defined for instance by *Pezzinga and Pititto* (2005), are not taken into account; and
- Decrease in pipe resistance due to aging is not considered.

Among the restrictive assumptions listed above, those related to uncertainty (indeed already considered by Basupi and Kapelan 2012, 2013) play an important role. This spurred us to generalize the methodology of Creaco et al. (2013) in order to take into account uncertainty in demand growth in the framework of phasing of construction. With respect to the probabilistic optimization approach developed by Basupi and Kapelan (2012, 2013), the methodology proposed here deals differently with the uncertainty in water demand. Whereas Basupi and Kapelan (2012, 2013) assumed nodal demands to follow a distribution function with mean value equal to the deterministic projection, here ~~the water demand is assumed to growth systematically but with~~ uncertainty~~,~~ expressing the parameters of the demand-growth model by means of a (discrete) random variable of given probability mass function.

It is clear that the uncertainty in the layout expansion also plays an important role, particularly on a long time horizon, and thus it would deserve to be considered along with the uncertainty in the demand growth. ~~However, this~~ second source of uncertainty is not considered here for space reasons~~,~~ and in order to avoid a too-heavy and conceptually complex presentation~~,~~ it will be the subject of other investigations. In other words, a known spatial expansion is assumed here. Incidentally, this marks a further difference with respect to Basupi and Kapelan (2012, 2013) where a fixed pipe network is instead considered.

In the following sections, the methodology is described and applied to a case study made up of a real network; the benefits derived from its application are then highlighted and conclusions are drawn.

Methodology

Overview of the Phasing of Construction

The application of the methodology proposed requires the whole construction period T to be subdivided into n phases of length Δt, with n being an integer number of phases. Though the methodology can be applied to any combination of time step sizes, the adoption of a single time step Δt makes its description more straightforward since it entails that at the generic year $(k-1)\Delta t$ [at the beginning of the generic phase, with k being an integer number within the range $(1{:}n)$], the network is supposed to be upgraded considering the demand and layout predicted at year $k\Delta t$ (i.e., at the end of the current phase or, equivalently, at the beginning of the subsequent phase). In order to guarantee the efficiency of the network in the whole construction time, a number of upgrades equal to n (i.e., equal to the number of phases) has to be performed on the generic connection (i.e., installation of pipe or multiple pipes) between two network nodes.

As to the demand that has to be *allocated to network nodes*, various scenarios, each of which featuring its own occurrence probability, can be considered in the methodology. In the next subsection, first the demand variation model is described; then, the assessment of the occurrence probability of the corresponding scenario follows.

Definition of Demand Model and Scenarios

For the sake of simplicity and explanatory purposes, a linear model is considered here to represent the demand variation within each construction phase of the generic mth scenario. In the model, the demand-growth rate $A_{j,k}$, L/s/year, is considered to be constant for the generic jth node during the generic kth construction phase. Another parameter that characterizes network nodes in the model is the initial (peak) hour demand, i.e., the value of the peak hour demand at the year $t_{0,j} = k_{0,j}\Delta t$ when the node j starts to exist ($k_{0,j}$ is the time index relative to the node j first appearance and can assume values $0, 1, 2, \ldots, n-1$); this parameter is indicated with symbol $D_{j,0}$. Both $A_{j,k}$ and $D_{j,0}$ may change from node to node. As an example, for a node j, which starts to exist at year 0, the demand at the end of the first phase will be equal to $D_{j,1} = D_{j,0} + A_{j,1}(t_1 - t_0)$; at the end of the second phase, it will be equal to $D_{j,2} = D_{j,1} + A_{j,2}(t_2 - t_1) = D_{j,0} + A_{j,1}(t_1 - t_0) + A_{j,2}(t_2 - t_1)$, and so on with the successive phases. For another generic node j, which starts to exist at year $2\Delta t$, the demand at the end of the third phase will be equal to $D_{j,3} = D_{j,0} + A_{j,3}(t_3 - t_2)$; at the end of the fourth phase, demand will be equal to $D_{j,4} = D_{j,3} + A_{j,4}(t_4 - t_3) = D_{j,0} + A_{j,3}(t_3 - t_2) + A_{j,4}(t_4 - t_3)$. In a bid to generalize, for the *generic node*, the demand at the end of the kth phase will be equal to

$$D_{j,k} = D_{j,0} + \sum_{l=k_{0,j}+1}^{k} A_{j,l}(t_l - t_{l-1}), \qquad n \geq k \geq l > k_{0,j} \quad (1)$$

In Eq. (1) index k can assume value $1, 2, \ldots, n$ for the generic jth node which has been present in the network since time $t = 0$ (in this case $k_{0,j} = 0$); otherwise, k can assume values $k_{0,j} + 1, k_{0,j} + 2, \ldots, n$ if the node j appears in the layout at time $t_{0,j} = k_{0,j}\Delta t$ (in this case $k_{0,j} > 0$).

Summing up, the characterization of the generic water demand scenario entails defining, for the generic jth network node, the value $D_{j,0}$ and the values $A_{j,k}$ ($k = k_{0,j} + 1, \ldots, n$ where $k_{0,j}$ can assume values $0, 1, 2, \ldots, n-1$).

In an effort to simplify the problem, a unique representative value of the demand-growth rate $\bar{A}_k$ is defined for the generic kth phase. Starting from $\bar{A}_k$, the demand-growth rate value $A_{j,k}$ of the jth node is defined by taking into account the fact that the demand of the older nodes may head toward saturation and, then, the demand-growth rate of the older nodes may turn out to be lower than that of the younger nodes. As a consequence of this, $A_{j,k}$ can be evaluated as a function of the age of the node itself at the beginning of the kth phase (nodal age $= (k - k_{0,j})\Delta t$), by using the following relationship:

$$A_{j,k} = \bar{A}_k - r_d(k - k_{0,j})\Delta t, \qquad n \geq k > k_{0,j} \quad (2)$$

where r_d, L/s/year^2, is the age-related decrease rate for the demand-growth rate. In light of the structure of Eq. (2), r_d has to be selected in such a way as to avoid negative values of $A_{j,k}$.

As a consequence of the simplification above, after a value of the age-related decrease rate r_d has been defined for the whole network and the initial demands $D_{j,0}$ have been set for the various nodes, the characterization of each demand-growth scenario then requires only n values of the representative demand-growth rate $\bar{A}_k$, each of which valid for the whole network and associated with a single construction phase, to be defined. To consider the uncertainty associated with demand growth, growth rate $\bar{A}_k$ is assumed to be a discrete random variable. The choice of a discrete random variable instead of a continuous random variable enables a better and simpler characterization of the possible levels of the growth rate (low, medium, high, etc,) which are *pragmatically* used by practitioners. From an operative point of view, this entails that at the kth phase $\bar{A}_k$ takes on a certain number v of discrete values $[\bar{a}_{1,k}, \bar{a}_{2,k}, \ldots, \bar{a}_{\nu,k}]$, to each of which a certain probability value $p_{i,k}$ [i.e., $p_{1,k}, p_{2,k}, \ldots, p_{v,k}$], with $\sum_{i=1}^{\nu} p_{i,k} = 1$, is associated.

The possible combinations of the v discrete values $a_{i,k}$ ($i = 1{:}\nu$; $k = 1{:}n$) in the n construction phases are as numerous as $n_s = v^n$ and this represents the highest number of demand scenarios, to each of which a probability $P_m = \prod_{k=1}^{n} p_{i,k}$ will be associated under the assumption that the demand-growth rate at the kth phase is independent from that at the $(k-1)$th phase. The sum of the occurrence probabilities of the various scenarios will then be $\sum_{m=1}^{n_s} P_m = 1$.

It is worth stressing that all the scenarios mentioned above do not include future fire-protection requirements because phasing of construction mainly concerns the transmission pipes (as clearly shown by the case study described in the numerical example), whereas fire-protection constraints create more problems in small distribution pipes.

Decisional Variables and Objective Functions

At the generic year $(k-1)\Delta t$ (i.e., at the beginning of the kth phase/interval) in order to supply water with acceptable service pressure in the network within the next Δt years, it is necessary to add $n_{p,k}$ pipes, among which $n_{p1,k}$ have to be inserted in new sites (where no pipes were present earlier) in order to reach new demanding nodes and $n_{p2,k}$ have to be laid in parallel to previously existing pipes. Since the whole construction period is divided into n upgrade phases, the decisional variables of the network upgrade problem are then the diameters to be adopted for the $n_p = \sum_{k=1}^{n} n_{p,k} = \sum_{k=1}^{n} n_{p1,k} + \sum_{k=1}^{n} n_{p2,k}$ pipes, to be chosen in a prefixed set of n_D diameters. At year 0 (i.e., at the beginning of the construction period), a certain number n_{p0} of pipes *may* already be present in the network (i.e., n_{p0} is a value assigned *a priori*). Furthermore, the numbers $n_{p1,1}, n_{p1,2}, \ldots, n_{p1,n}$ of pipes to be inserted in new sites at the beginning of the first, second, … and nth upgrade phase *are assumed to be known*.

For the generic upgrade, the corresponding cost $C_{k\Delta t}$ is equal to

$$C_{k\Delta t} = \sum_{j1=1}^{n_{p1,k}} c_{j1} L_{j1} + \sum_{j2=1}^{n_{p2,k}} c_{p,j2} L_{j2} \tag{3}$$

where $\sum_{j1=1}^{n_{p1,k}} c_{j1} L_{j1}$ is the part of the cost associated with the installation of pipes at new sites (with L and c being the length of the pipes which have to be introduced and the unit cost associated with the diameters of the pipes to be laid in new sites); $\sum_{j2=1}^{n_{p2,k}} c_{p,j2} L_{j2}$ is the part of the cost associated with the installation of pipes in parallel to existing pipes, where the unit cost c_p can be increased with respect to c in order to take account of the fact that laying a pipe parallel to another pipe may be more expensive than installing the same pipe in a new site.

A multiobjective optimization can then be performed to assess the n_p diameters in order to obtain optimal network upgrade configurations, which represent a trade-off between cost and reliability.

The whole present-worth construction cost C, first objective function of the optimization process, can be evaluated as the sum of the present-worth values of the costs $C_{k\Delta t}$ [calculated by Eq. (3)] of the n upgrades, that is

$$C = \sum_{k=1}^{n} \frac{C_{k\Delta t}}{(1+R)^{(k-1)\cdot\Delta t}} \tag{4}$$

where R is the discount rate.

The second objective function of the optimization process, representative of network reliability (capacity), is connected with the pressure surplus IS that the network shows to have over the entire construction time (surplus is intended to be the pressure head excess with respect to the minimum value for demand satisfaction. Its expression will follow below).

In particular, for each scenario m ($m = 1{:}n_s$), featuring a given series of demand-growth coefficient values and an occurrence probability P_m, n values (one for each phase) of the pressure surplus IS are obtained by a *demand-driven* network simulation model (Todini and Pilati 1988); for each phase, the surplus $\mathrm{IS}_k = \min_j(h_{k,j} - h_{\mathrm{des},k,j})$ can be calculated, where $h_{k,j}$ indicates the pressure head value obtained at the jth network demanding node by considering nodal demands at time $k\Delta t$ and $h_{\mathrm{des},k,j}$ is the requested (desired) pressure head at the jth node at time $k\Delta t$ (j ranges between 1 and nn_k, number of nodes in the network during the kth phase). As a matter of fact, the following formula can be used to evaluate the minimum surplus relative to the mth scenario

$$\mathrm{IS}_{\min,m} = \min_k \mathrm{IS}_{k,m} \quad k = 1{:}n \tag{5}$$

The fact that each scenario (with occurrence probability P_m) features its value of $\mathrm{IS}_{\min,m}$ entails that the whole set of $\mathrm{IS}_{\min,m}$ values, with $m = 1{:}n_s$ (associated with a certain design solution—diameter selection—and then with a cost), represents a discrete random variable with discrete probability distribution derived from the occurrence probability P_m of the various demand-growth scenarios. It is worth underlining that the expression of $\mathrm{IS}_{\min,m}$ as a discrete random variable is descended from the representation of the demand-growth rate $\bar{A}_k$ as a discrete random variable (see previous subsection). As a matter of fact each of the n_s values of $\mathrm{IS}_{\min,m}$ can be associated with the occurrence probability of the corresponding scenario. The second objective function of the optimization process is then a discrete random variable $\mathrm{IS}_{\min}$ whose probability mass function is derived from the occurrence probability of the various scenarios.

In this study, the multiobjective optimization, aimed at minimizing the cost C and maximizing the discrete random variable $\mathrm{IS}_{\min}$ is performed by means of a modified version of the NSGAII genetic algorithm (Deb et al. 2002). This modified version makes it possible to encode genes made up of integer numbers (Creaco et al. 2010, 2013; Alvisi et al. 2011) rather than real numbers. The integer nature of the numbers is obtained and preserved not by rounding real numbers but thanks to ad hoc and effective implementations of the procedures for initializing the individuals and for the genetic operators.

As is well known, in genetic algorithms, individuals representing the solutions are compared and ranked in terms of objective functions; in this work the traditional module of NSGAII (Deb et al. 2002) was used to compare individuals in terms of the first objective function, which is the cost, i.e., a crisp number. A different approach, instead, was set up for the second objective function, the discrete random variable $\mathrm{IS}_{\min}$. Indeed, for $\mathrm{IS}_{\min}$, the problem is how to rank (for instance in ascending order) discrete random variables of known probability mass function. A criterion or a method is needed to assess if a (discrete) random variable is larger than another. The comparison criterion set up in order to compare two discrete random variables IS^1_{min} and IS^2_{min}, yielded by NSGAII in correspondence to two individuals, requires the probabilities $P_{\geq}$ and $P_{\leq}$ to be assessed for the former to be *larger than or equal to*, or *lower than or equal to* the latter. The comparison is made with reference to each scenario m ($m = 1{:}n_s$) and $P_{\geq}$ can be calculated as

$$P_{\geq} = \sum_{m=1}^{n_s} P_m \cdot \alpha_m \tag{6}$$

where $\alpha_m = 1$ if $IS^1_{min,m} \geq IS^2_{min,m}$; otherwise $\alpha_m = 0$. In other words, we fix a scenario m and with reference to that scenario we evaluate if the crisp value $IS^1_{min,m}$ is greater than or equal to the crisp value $IS^2_{min,m}$. P_m is the probability associated to the scenario m. As a matter of fact, in Eq. (6) each of the n_s discrete values of the random variable IS^1_{min} is compared with the corresponding element (i.e., referring to the same water demand scenario) of the random variable IS^2_{min}.

Similarly, the probability $P_{\leq}$ for the former to be inferior or equal to the latter is equal to

$$P_{\leq} = \sum_{m=1}^{n_s} P_m \cdot \beta_m \tag{7}$$

where $\beta_m = 1$ if $IS^1_{min,m} \leq IS^2_{min,m}$; otherwise $\beta_m = 0$.

In the end, the former individual is deemed to be superior, equal, or inferior to the latter individual in terms of second objective function if $P_{\geq}$ is larger than, equal to or lower than $P_{\leq}$.

As a matter of fact, by comparing the series of values $IS^1_{min,m}$ and $IS^2_{min,m}$ (with $m = 1{:}n_s$) relative to the first and second individual of the genetic algorithm, respectively, the application of the criterion based on Eqs. (6) and (7) yields the values $P_{\geq}$ and $P_{\leq}$, which express the probabilities for the former individual to be *larger than or equal to*, or rather *lower than or equal to* the latter; if for instance $P_{\geq} = 0.65$ and $P_{\leq} = 0.35$, it would entail that $P_{\geq} > P_{\leq}$. In other words, it would be more likely that the former individual is superior and the latter inferior. This does not entail that all the "elements" of the former are superior to the corresponding elements of the latter.

In the genetic algorithm, each individual of the population is encoded with n_p genes, representing the ID codes of the pipe diameters to be installed in the network; in particular, $\sum_{k=1}^{n} n_{p1,k}$ genes (representative of the diameters of the initial pipes to be installed in the various installation sites) take on values within the range 1-n_D; the other $\sum_{k=1}^{n} n_{p2,k}$ genes (representative of the diameters of the pipes installed in parallel to previously existing pipes) take on values within the range 0-n_D; taking into account value 0 in the latter case helps us considering the possibility that, in some sites, the parallel pipe does not have to be laid since the pipe(s) previously laid already meet(s) the demands. Incidentally, the genes inside each individual are arranged in accordance with the phases considered in the construction period. As to the number of decisional variables, it is worth underlining, as had already been stated by Creaco et al. (2013), that even though each optimal solution comprises diameters to be laid in the various sites at the various time steps (i.e., at the beginning of the various phases), in practice engineers can use the methodology herein presented in order to make decisions for the near future, i.e., for the first phase of system growth by having a vision of the long term growth of the system.

At the end of the optimization process, optimal solutions in the space *total present-worth cost* $C - IS_{min}$ are obtained; since the second objective function is a discrete random variable (with lower and upper limit values) instead of a crisp variable, optimal solutions form a *Pareto band*, rather than a Pareto front; inside the band, the generic solution is individualized by a vertical interval, drawn from a given value of the cost and comprising the whole set of IS_{min} values, which the solution itself features in the various demand-growth scenarios considered within the optimization process. If two solutions of the band with two different values of present-worth cost are taken, that with the higher cost will feature a better (average) performance in terms of surplus than the others in the various demand-growth scenarios considered, as will be clear in the numerical example reported below.

To pick a solution of the band, the user may make reference to a certain crisp value for IS_{min} and plot a horizontal line (y-axis: IS_{min}; x-axis: C); then, the user can search for the solution of the band where all or most of the values of the random variable IS_{min} are higher than the assumed threshold.

In the case of no uncertainty in demand growth, the present methodology collapses into the deterministic methodology of Creaco et al. (2013), as the second objective function, the discrete random variable IS_{min}, collapses into a single crisp value of IS_{min}.

Applications

Case Study

The case study considered here makes reference to the network of a town in northern Italy. A skeletonized layout, obtained by Farina et al. (2013) from the original network (Creaco and Franchini 2013) by discarding pipes that only play a distribution function, was taken into account because the problem of phasing of construction mainly concerns transmission mains. This layout is made up of 26 nodes with outflow and 31 sites for pipe laying (Fig. 1). The network is fed by $n_0 = 1$ reservoir (node 26), which presents a value of the head equal to 38 m with respect to all the nodes (whose ground elevation is assumed to be 0 m above sea level). The lengths of the possible pipes (i.e., the first pipe to be laid and the subsequent potential parallel pipes) are reported in Table 1. Pipe Manning

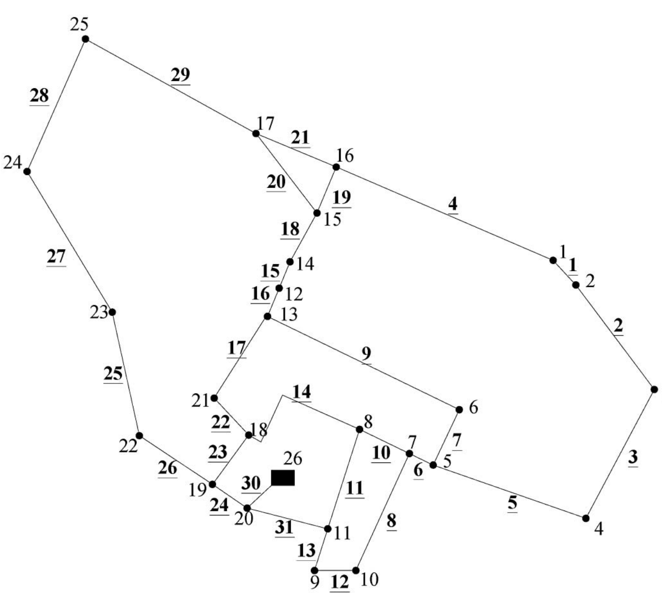

Fig. 1. Case study network; source node with ID 26; the lines indicate the connections, i.e., the sites where the initial pipes and subsequent parallel pipes can be laid; IDs of the connections are underscored and in bold

Table 1. Lengths of the Various Connections, i.e., the Sites Where Pipe Installation Can Take Place

Pipe	ID node 1	ID node 2	L (m)
1	1	2	10
2	2	3	2,874
3	3	4	1,733
4	1	16	2,851
5	4	5	2,648
6	5	7	144
7	5	6	365
8	7	10	817
9	6	13	1,270
10	7	8	333
11	8	11	628
12	9	10	270
13	11	9	241
14	8	18	888
15	12	14	2,056
16	13	12	131
17	21	13	991
18	14	15	7
19	15	16	607
20	15	17	1,670
21	17	16	1,047
22	18	21	132
23	18	19	393
24	19	20	155
25	22	23	2,469
26	22	19	1,594
27	24	23	2,567
28	25	24	2,338
29	17	25	2,453
30	26	20	20
31	20	11	491

roughness coefficients were assumed equal to $0.015\ \mathrm{s/m^{1/3}}$ and variations over time were not considered. The layout in Fig. 1 comprises the whole group of nodes and connections at the end of the $T = 100$-year-long construction period. The various construction phases, instead, are represented in Fig. 2, which shows that a development toward either side of an initial core takes place. As a matter of fact, a number of upgrade phases $n = 4$ $(k = 1,4)$ and thus a time step Δt equal to 25 years were considered.

The choice of $\Delta t = 25$ years represents a (didactic) simplification that is made for explicative purposes, because water distribution networks are systems that are upgraded continuously; as a matter of fact, the phase 0–25 includes all the interventions that are assumed to be necessary between year 0 and year 25; the phase 25–50 includes all the interventions that are assumed to be necessary between year 25 and year 50, and so on. In other words the procedure presented here can also be applied with different and shorter Δt than that used here. The information relative to the initial demand D_0 (L/s) and to the year t_0 of appearance of the various nodes is reported in Table 2. For all nodes and phases, a desired pressured head $h_{des} = 20$ m was considered.

Table 3 shows the unit costs c relative to the pipe diameters that can be installed in the network. These unit costs are multiplied by 1.2 raised to n_{par} in case of network upgrades obtained by laying parallel pipes to pipes previously laid, where n_{par} is the number of parallel pipes already present in the site where the new pipe is positioned; this penalty makes it possible to take account of the fact that the insertion of a parallel pipe is more expensive and complicated than the insertion of the first pipe with the same diameter and cost and complication increases as the number of pipes already laid grows.

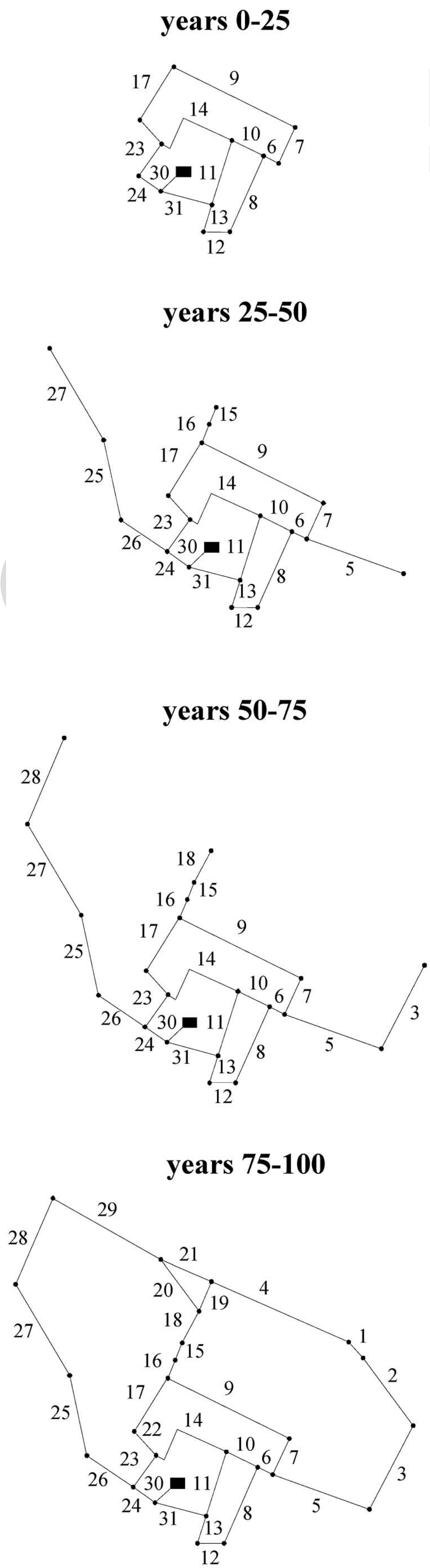

Fig. 2. Expansion pattern of the network; only IDs of the pipes are reported

Table 2. Initial Demands D_0 and Initial Construction Time t_0 for Network Nodes

Node ID	D_0 (L/s)	t_0 (year)
1	0	75
2	0	75
3	0	50
4	0	25
5	6.55	0
6	19.32	0
7	7.91	0
8	12.31	0
9	3.52	0
10	1.99	0
11	5.25	0
12	0	25
13	19.69	0
14	0	25
15	0	50
16	0	75
17	0	75
18	6.51	0
19	0.06	0
20	0.71	0
21	6.00	0
22	0	25
23	0	25
24	0	25
25	0	50

Table 3. Pipe Diameters D and Unit Costs C

D (mm)	Cost (\$/m)
102	8.2
152	15.1
203	23.2
254	32.4
305	42.6
356	53.6
406	65.5
457	78.2
508	91.6
559	105.7
609	120.4
660	135.8
711	151.7

As to the demand-growth model in Eq. (1), the discrete random variable $\bar{A}_k$ is assumed to take on $v = 3$ values (0.02, 0.05, 0.08 L/s/year), representative of a low, average, and high growth rate, respectively. Each of these values is assigned a probability equal to $p = 1/3$. By considering all the possible combinations of these three values in the four construction phases, a total number of $n_s = 3^4 = 81$ scenarios is derived, each of which featuring a probability value $P_m = 1/81 = 1/(3 \times 3 \times 3 \times 3)$. In this set of scenarios, the first three have a constant value of $\bar{A}_k$ ($k = 1{:}4$) equal to 0.02, 0.05, and 0.08 L/s/year, respectively, whereas the others feature a randomly variable value of $\bar{A}_k$, over the construction period. As to the age-related decrease rate r_d, a value equal to 0.0002 L/s/year^2 was considered for all the scenarios.

As to D_{tot} (L/s), the total demand of the network at the end of the construction period (100 years), the smallest value is obtained in the first scenario (constant growth rate equal to 0.02 L/s/year), where demand is equal to 116.2 L/s and remains quite close to the initial value (89.8 L/s) whereas the largest value is obtained in the third scenario (constant growth rate equal to 0.02 L/s/year), where

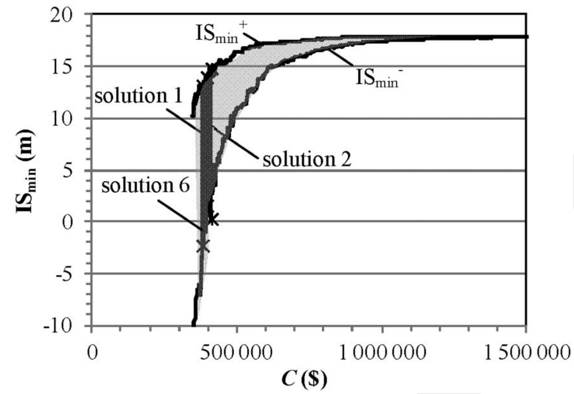

Fig. 3. Probabilistic second objective optimization—Pareto band of optimal solutions in the total present-worth cost—minimum pressure surplus space (see text); Solutions 1, 2, and 6; see the "Results" section

demand is more than duplicated (230.2). All the other scenarios yield intermediate values of total demand at the end of the construction period.

For the application of the genetic algorithm described in the "Methodology" section, a first optimization (hereafter indicated as probabilistic second objective optimization, or PSOO) was performed considering a T = 100-year-long planning horizon and taking into account the entire set of 81 scenarios. For cost conversion to present-worth value at year 0, a discount rate $R = 2\%$ was adopted; this is a low neutral value with respect to the place where the investment is made, which also makes it possible to take account of the presence of inflation. In real cases, the discount rate should be assessed on the basis of the rate of time preference and is generally indicated by the government authorities of the various countries. In the absence of indications, it is possible to make reference to the rate used by the main organization that provides funds for the public bodies (Boardman 2006).

To analyze the effects of the probabilistic approach presented in this paper, a series of deterministic optimizations (hereinafter indicated as deterministic second objective optimizations, or DSOOs) was performed using the methodology of Creaco et al. (2013); DSOOs a, b, and c were performed with reference to each of the first three scenarios, which feature a single and constant value of growth rate coefficient $\bar{A}_k$; an age-related decrease rate r_d equal to 0.0002 L/s/year^2 was always considered in the DSOOs. In these optimizations, the first objective function was the present-worth cost, like in the probabilistic methodology presented in this paper; instead of the discrete random variable IS_{min} (considered in PSOO), the second objective function was the crisp minimum temporal surplus IS_{min} experienced by the network over the 100-year-long construction period in the scenario of the optimization.

In all optimizations, a population of 500 individuals and a total of 1,500 generations were considered; the previous choices are due to the fact that those values of populations and generations represent a good trade-off between accuracy of the results and computational burden.

Results

The results of PSOO are reported as a band in the space present-worth cost-minimum pressure surplus in the graph in Fig. 3; for each solution, i.e., for each value of the expected present-worth cost C (\$), a vertical line helps obtaining the interval which comprises the 81 values of IS_{min} in all the 81 scenarios for that solution. In the

Table 4. Solutions 1–2 Selected in the "Results" Section

Solution	C (\$)	IS_{min} (m) Scenarios																					$P \geq$	$P \leq$
		1	2	**3**	4	5	6	7	8	9	10	**11**	12	13	14	15	16	17	18	19	**20**			
1	395,400	14.0	10.4	**1.6**	13.6	10.7	13.0	11.8	8.5	8.1	8.1	**5.8**	13.4	12.7	9.6	11.7	10.7	7.1	5.7	5.7	**4.1**			
2	411,664	14.8	10.9	**0.3**	14.1	11.9	13.4	12.5	9.3	11.2	9.8	**5.8**	13.9	13.3	10.6	12.1	11.0	7.3	9.5	7.8	**3.3**			
Solution	C (\$)	IS_{min} (m) Scenarios																						
		21	22	23	24	**25**	**26**	27	28	**29**	30	31	32	33	34	35	36	37	38	39	40			
1	395,400	12.4	11.6	8.3	9.9	**9.4**	**5.6**	2.7	2.7	**2.4**	13.5	13.3	10.4	12.7	11.5	8.2	8.1	8.1	5.4	12.9	12.4			
2	411,664	12.8	12.1	8.7	10.7	**9.1**	**5.0**	6.6	5.5	**0.6**	14.5	13.9	11.7	13.0	12.2	9.2	10.9	9.7	5.6	13.5	13.0			
Solution	C (\$)	IS_{min} (m) Scenarios																						
		41	42	43	44	45	**46**	47	48	49	50	**51**	**52**	53	54	**55**	56	57	58	59	60			
1	395,400	9.3	11.4	6.8	5.6	5.6	**3.8**	12.0	11.3	8.0	9.8	**9.1**	**5.2**	2.7	2.7	**2.0**	13.1	12.8	10.1	12.3	11.2			
2	411,664	10.4	11.8	7.2	9.3	7.7	**3.2**	12.4	11.9	8.6	10.3	**9.0**	**4.8**	6.5	5.3	**0.4**	14.1	13.6	11.4	12.7	11.9			
Solution	C (\$)	IS_{min} (m) Scenarios																					$P \geq$	$P \leq$
		61	62	63	64	65	66	67	68	69	70	71	72	**73**	74	75	76	77	78	**79**	80	81		
1	395,400	7.8	8.1	8.1	5.0	12.4	12.1	9.0	11.0	10.0	6.4	5.6	5.6	**3.4**	11.5	11.0	7.6	9.4	8.7	**4.8**	2.7	2.7		
2	411,664	9.1	10.5	9.6	5.5	13.1	12.7	10.2	11.4	10.8	7.0	8.9	7.6	**3.0**	11.9	11.6	8.4	9.9	8.9	**4.7**	6.5	5.2	0.85	0.15

Note: Values of present-worth cost C, of the minimum temporal surplus IS_{min} for the various scenarios 1–81, and of probabilities $P \geq$ and $P \leq$ calculated for Solution 2 with respect to Solution 1. In bold, scenarios where Solution 1 prevails; in regular text, scenarios where Solution 2 prevails.

Table 5. Solution 1 Selected in the "Results" Section

			Phases of construction				
ID pipe	ID node 1	ID node 2	Years 0–25	Years 25–50	Years 50–75	Years 75–100	
1	1	2	0	0	0	152	
2	2	3	0	0	0	102	
3	3	4	0	0	152	0	
4	1	16	0	0	0	152	
5	4	5	0	152	0	0	
6	5	7	254	0	0	254	
7	5	6	203	0	0	0	
8	7	10	102	0	0	0	
9	6	13	102	0	0	0	
10	7	8	305	0	0	0	
11	8	11	356	0	0	0	
12	9	10	102	0	0	0	
13	11	9	152	0	0	0	
14	8	18	102	0	0	0	
15	12	14	0	152	0	254	
16	13	12	0	152	0	406	
17	21	13	254	0	0	203	
18	14	15	0	0	152	0	
19	15	16	0	0	0	203	
20	15	17	0	0	0	102	
21	17	16	0	0	0	102	
22	18	21	305	0	0	203	
23	18	19	305	0	0	254	
24	19	20	254	0	356	0	
25	22	23	0	203	0	0	
26	22	19	0	203	0	0	
27	24	23	0	152	0	0	
28	25	24	0	0	102	0	
29	17	25	0	0	0	152	
30	26	20	406	457	0	0	
31	20	11	406	0	0	0	
Costs ($)			184,146	207,859	55,375	282,543	C_{tot} ($)
Present-worth costs ($)			184,146	126,697	20,573	63,984	395,400
IS (m) Scenario 1			14.4	14.1	14.0	14.3	
IS (m) Scenario 2			13.9	13.0	11.4	10.4	
IS (m) Scenario 3			13.4	11.5	2.7	1.6	

Note: Data for the various temporal phases, diameters (mm) of the pipes laid, costs, and present-worth costs of pipe laying and values of pressure surpluses IS in the first three demand-growth scenarios.

graph, the lower values of IS_{min} of all the solutions are joined together by a piecewise linear curve, which represents the lower edge (IS^{-}_{min}) of the band. Similarly, a piecewise linear curve, which represents the upper edge (IS^{+}_{min}) of the band, joins together the upper values of IS_{min} for all the solutions. The analysis of the results highlights that in the case study herein considered the top curve represents IS_{min} if the lowest growth rate (Scenario 1) is realized while the bottom curve represents IS_{min} if the highest growth rate (Scenario 3) is realized.

The Pareto band comprises solutions for which higher costs yield better performance in terms of IS_{min}. In order to better clarify how the Pareto band has to be read and how the solutions are related to each other, two close solutions were taken out from the Pareto band (Solution 1 and Solution 2 in Fig. 3), featuring $C = \$395{,}400$ and $C = \$411{,}664$, respectively, as values of expected present-worth cost; in particular, Solution 1 is the solution *with lowest cost* which features a positive value of IS_{min} *over all scenarios* (i.e., representative of the minimum cost design, in which pressure surplus is always positive; this solution is hereinafter indicated as *minimum cost solution of the probabilistic approach*) whereas Solution 2 is a PSOO solution with a larger cost, considered herein

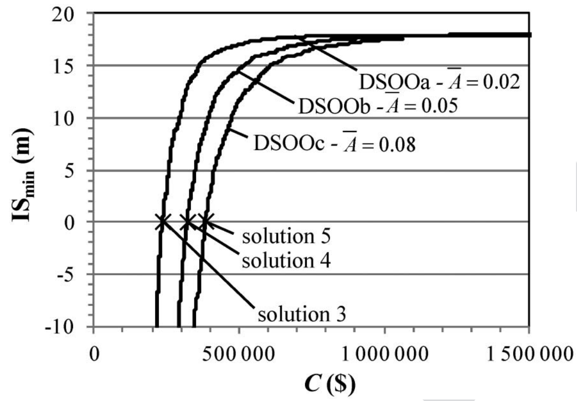

Fig. 4. Deterministic second objective optimizations a, b, and c—Pareto fronts of optimal solutions in the total present-worth cost—pressure surplus space (see text) for the various values of demand-growth coefficient *a*; minimum cost Solutions 3–5; see the "Results" section

only for the sake of comparison with Solution 1. Whereas the graph only highlights the lower IS^{-}_{min} and upper IS^{+}_{min} values of IS_{min} for each solution [equal to 1.6 m (Scenario 3) and 14.0 m (Scenario 1) for Solution 1 and to 0.3 m (Scenario 3) and 14.8 m (Scenario 1) for Solution 2, respectively], Table 4 reports in detail the values taken on by IS_{min} over all the 81 demand-growth scenarios for Solutions 1 and 2; in compliance with the comparison criterion presented in the previous section and taking into account that a probability equal to 1/81 corresponds to each of the 81 values assumed by IS_{min}, the table also reports the probability $P_{\geq}$ for Solution 2 to be larger than or equal to Solution 1 in terms of IS_{min}, as well as the probability $P_{\leq}$ for Solution 2 to be smaller than or equal to Solution 1. The table shows that Solution 2 has a higher value of $P_{\geq}$ (0.85) than $P_{\leq}$ (0.15), and this entails that discrete variable IS^{2}_{min} (made up of 81 values and connected to Solution 2) is larger than discrete variable IS^{1}_{min} (made up of 81 values and connected to Solution 1) and that Solution 2, being more expensive, then performs overall better in terms of IS_{min}.

As an example, diameters of the pipes laid in the four phases of construction encoded in Solution 1 are reported in Table 5, along with the costs and the present-worth costs of the various construction phases and the crisp surpluses IS (m) observed in the first three scenarios (taken as reference scenarios) at the various time steps or phases. In the table, for each pipe, the diameter used is indicated; for given construction step, number 0 indicates that no pipe is laid in that case. The analysis of Table 5 shows that, due to reasons of economy of scale, the first phase (years 0–25) of construction is sized in such a way that the resulting value of IS at the end of the first phase is very high (around 14 m). This excess in the initial phase is due to the installation of pipes with abundant size (see, for instance, Pipes 30 and 31 with 406 mm diameters in the first phase, larger than we would have installed if we only looked at a 25-year planning horizon). This excess is dissipated in the following phases. Under the most severe growth condition (Scenario 3), the lowest value of IS is then reached in the last construction phase (years 75–100).

To compare the results of the probabilistic approach presented in this paper with the deterministic approach proposed by Creaco et al. (2013), the results of deterministic optimizations with deterministic second objective DSOOa, DSOOb, and DSOOc, performed considering each of the first three demand-growth scenarios, were reported as Pareto fronts in the graph in Fig. 4; the solutions are here represented as *dots* rather than intervals since both objective

Table 6. Solution 3 Selected in the "Results" Section

ID pipe	ID node 1	ID node 2	Phases of construction: Years 0–25	Years 25–50	Years 50–75	Years 75–100	
1	1	2	0	0	0	152	
2	2	3	0	0	0	102	
3	3	4	0	0	102	0	
4	1	16	0	0	0	102	
5	4	5	0	102	0	0	
6	5	7	203	0	0	203	
7	5	6	152	0	0	0	
8	7	10	102	0	0	0	
9	6	13	102	0	0	0	
10	7	8	254	0	0	0	
11	8	11	254	0	0	0	
12	9	10	102	0	0	0	
13	11	9	102	0	0	0	
14	8	18	102	0	0	0	
15	12	14	0	102	0	102	
16	13	12	0	152	0	0	
17	21	13	203	0	0	102	
18	14	15	0	0	152	0	
19	15	16	0	0	0	102	
20	15	17	0	0	0	102	
21	17	16	0	0	0	102	
22	18	21	203	0	0	254	
23	18	19	254	0	0	0	
24	19	20	305	0	0	0	
25	22	23	0	102	0	0	
26	22	19	0	102	0	0	
27	24	23	0	102	0	0	
28	25	24	0	0	102	0	
29	17	25	0	0	0	102	
30	26	20	457	0	0	0	
31	20	11	254	0	152	0	
Costs ($)			131,375	94,913	42,375	133,611	C_{tot} ($)
Present-worth costs ($)			131,375	57,852	15,744	30,257	235,228
IS (m) Scenario 1			2.0	0.6	0.9	0.1	
IS (m) Scenario 2			−0.1	−4.6	−92.1	−78.1	
IS (m) Scenario 3			−2.3	−32.1	−271.8	−228.7	

Note: Data for the various temporal phases, diameters (mm) of the pipes laid, costs, and present-worth costs of pipe laying and values of pressure surpluses IS in the first three demand-growth scenarios.

Table 7. Solution 4 Selected in the "Results" Section

ID pipe	ID node 1	ID node 2	Phases of construction: Years 0–25	Years 25–50	Years 50–75	Years 75–100	
1	1	2	0	0	0	152	
2	2	3	0	0	0	102	
3	3	4	0	0	152	0	
4	1	16	0	0	0	102	
5	4	5	0	152	0	0	
6	5	7	254	0	0	0	
7	5	6	152	0	0	0	
8	7	10	102	0	0	0	
9	6	13	102	0	0	0	
10	7	8	254	0	0	203	
11	8	11	305	0	0	0	
12	9	10	102	0	0	0	
13	11	9	102	0	0	0	
14	8	18	102	0	0	0	
15	12	14	0	203	0	0	
16	13	12	0	203	0	0	
17	21	13	254	0	0	0	
18	14	15	0	0	152	0	
19	15	16	0	0	0	152	
20	15	17	0	0	0	102	
21	17	16	0	0	0	102	
22	18	21	203	0	0	254	
23	18	19	254	0	0	305	
24	19	20	254	0	0	356	
25	22	23	0	152	0	0	
26	22	19	0	152	0	0	
27	24	23	0	152	0	0	
28	25	24	0	0	102	0	
29	17	25	0	0	0	152	
30	26	20	254	0	305	0	
31	20	11	305	0	0	0	
Costs ($)			150,766	190,828	46,436	159,986	C_{tot} ($)
Present-worth costs ($)			150,766	116,316	17,252	36,230	320,564
IS (m) Scenario 1			6.6	5.7	5.6	6.9	
IS (m) Scenario 2			5.2	2.3	0.0	0.1	
IS (m) Scenario 3			3.8	−1.6	−23.0	−21.0	

Note: Data for the various temporal phases, diameters (mm) of the pipes laid, costs, and present-worth costs of pipe laying and values of pressure surpluses IS in the first three demand-growth scenarios.

functions are deterministic variables, each of which featuring a single crisp value.

As term of comparison for the minimum cost solution of the PSOO (Solution 1 in Fig. 3), the solution featuring the lowest value of C corresponding to a nonnegative IS_{min} (minimum cost solution) was taken from each of the three Pareto fronts in Fig. 4: these are represented by Solutions 3–5 extracted from the fronts DSOOa, DSOOb, and DSOOc, respectively (see Tables 6–8 for the diameters of the pipes laid in the four phases of construction encoded in Solutions 3–5, along with the costs and the present-worth costs of the various construction phases and the crisp surpluses IS observed in the first three scenarios). As to the total present-worth cost C, the comparison of Solution 1 (which is the least-cost solution within the framework of the probabilistic approach) with Solutions 3–5 (which are the least cost solutions within the framework of the deterministic approach, i.e., without any uncertainty in the water demand) shows that the former has a higher value of C, in particular larger than that of Solution 5 (equal to \$383,136), which belongs to the *deterministic* optimization performed under the most severe demand-growth scenario. However this result is not surprising since the higher cost of Solution 1 is paid back by a higher pressure surplus (1.6 m) at the end of the fourth phase with respect to that reproduced in the case of Solution 5 (0.1 m). A better comparison requires that the present-worth cost C is fixed and equal for both types of solutions. Thus, Solution 6, which features a very close present-worth cost C to the deterministic Solution 5 (actually slightly lower), is taken from the PSOO in Fig. 3. Its characteristics are summarized in Table 9. The values taken on by IS_{min} over all the 81 demand-growth scenarios for Solutions 5 and 6 are reported in Table 10. This table shows that, in light of the comparison criterion presented in the previous section and here applied to relate Solution 6 to Solution 5, Solution 6 has a higher value of $P_{\geq}$ (0.84) than $P_{\leq}$ (0.16) and this entails that discrete variable IS^6_{min} (made up of 81 values and connected to Solution 6) is larger than discrete variable IS^5_{min} (made up of 81 values and connected to Solution 5), though there are three scenarios (3, 29, 55) where $IS^6_{min} < 0$. Since Solution 6 performs overall better in terms of IS_{min}, (deterministic) Solution 5 taken from the DSOO is dominated by the equally expensive (probabilistic) Solution 6 obtained through the PSOO.

All the cases discussed above and summarized in Table 11 entail that taking uncertainty into account tends to lead to solutions where, for fixed present-worth cost, the behavior in terms of

Table 8. Solution 5 Selected in the "Results" Section

ID pipe	ID node 1	ID node 2	Phases of construction: Years 0–25	Years 25–50	Years 50–75	Years 75–100	
1	1	2	0	0	0	152	
2	2	3	0	0	0	102	
3	3	4	0	0	152	0	
4	1	16	0	0	0	152	
5	4	5	0	203	0	0	
6	5	7	254	0	0	0	
7	5	6	203	0	0	0	
8	7	10	102	0	0	0	
9	6	13	102	0	0	0	
10	7	8	305	0	0	0	
11	8	11	305	0	0	0	
12	9	10	102	0	0	0	
13	11	9	152	0	0	0	
14	8	18	102	0	0	0	
15	12	14	0	203	0	0	
16	13	12	0	203	0	0	
17	21	13	254	0	0	254	
18	14	15	0	0	203	0	
19	15	16	0	0	0	203	
20	15	17	0	0	0	102	
21	17	16	0	0	0	152	
22	18	21	305	0	0	0	
23	18	19	203	0	356	0	
24	19	20	305	0	0	356	
25	22	23	0	203	0	0	
26	22	19	0	203	0	0	
27	24	23	0	152	0	0	
28	25	24	0	0	102	0	
29	17	25	0	0	0	152	
30	26	20	406	0	0	0	
31	20	11	305	0	0	203	
Costs ($)			159,955	245,186	70,738	209,538	C_{tot} ($)
Present-worth costs ($)			159,955	149,448	26,281	47,452	383,136
IS (m) Scenario 1			11.2	10.5	11.5	12.5	
IS (m) Scenario 2			10.3	8.0	8.2	8.5	
IS (m) Scenario 3			9.3	5.1	1.3	0.1	

Note: Data for the various temporal phases, diameters (mm) of the pipes laid, costs and present-worth costs of pipe laying and values of pressure surpluses IS in the first three demand-growth scenarios.

Table 9. Solution 6 Selected in the "Results" Section

ID pipe	ID node 1	ID node 2	Phases of construction: Years 0–25	Years 25–50	Years 50–75	Years 75–100	
1	1	2	0	0	0	152	
2	2	3	0	0	0	102	
3	3	4	0	0	152	0	
4	1	16	0	0	0	152	
5	4	5	0	152	0	0	
6	5	7	254	0	0	254	
7	5	6	203	0	0	0	
8	7	10	102	0	0	0	
9	6	13	102	0	0	0	
10	7	8	305	0	0	0	
11	8	11	356	0	0	0	
12	9	10	102	0	0	0	
13	11	9	152	0	0	0	
14	8	18	102	0	0	0	
15	12	14	0	152	0	254	
16	13	12	0	152	0	406	
17	21	13	254	0	0	203	
18	14	15	0	0	305	0	
19	15	16	0	0	0	203	
20	15	17	0	0	0	102	
21	17	16	0	0	0	102	
22	18	21	254	0	0	203	
23	18	19	254	0	0	254	
24	19	20	254	0	356	0	
25	22	23	0	203	0	0	
26	22	19	0	203	0	0	
27	24	23	0	152	0	0	
28	25	24	0	0	102	0	
29	17	25	0	0	0	102	
30	26	20	406	0	660	0	
31	20	11	356	0	0	0	
Costs ($)			172,953	206,025	58,746	265,620	C_{tot} ($)
Present-worth costs ($)			172,953	125,579	21,826	60,152	380,509
IS (m) Scenario 1			13.7	13.3	13.2	13.8	
IS (m) Scenario 2			13.1	11.6	9.2	9.4	
IS (m) Scenario 3			12.6	9.6	2.7	−2.3	

Note: Data for the various temporal phases, diameters (mm) of the pipes laid, costs and present-worth costs of pipe laying and values of pressure surpluses IS in the first three demand-growth scenarios.

pressure surplus is expected to be better than that obtainable through a deterministic approach where the worst combination of the water demand in all the phases is assumed. This may be ascribed to the fact that the probabilistic solution is intended to be flexible to adapt itself to various possible and unknown future conditions of demand growth, thanks to higher surplus values on average.

A further comparison was made concerning the diameters of the pipes laid in the first phase of construction, which represent the real choice at year 0, in Solutions 1, 6 and Solutions 3, 4, 5. Referring to Tables 5–9, with particular emphasis on the column relative to the first phase of construction, the comparison shows that highest cost and surplus values are obtained in Solutions 1 and 6, which also show the largest pipe diameter in almost all locations. The results then point out that, also at year 0 when the choice of the initial pipes to be laid has to be made, designing a network taking account of uncertainty in demand growth is more expensive than without uncertainty.

In the final choice of the solution to adopt, with particular emphasis to the first phase of construction, engineers could orientate themselves to Solution 6 (or more prudently Solution 1), which offers a suitable safety margin for all the possible demand-growth scenarios. As to the successive phases of the construction, when demand growth may be easier to predict, new probabilistic or deterministic optimizations could be performed to plan the necessary upgrades of the construction. The compact reliability index used in this paper, i.e., pressure surplus, may yield only a first attempt indication of the effective construction reliability, since it does not involve simulating pipe failures. Accordingly, some results of the optimizations could be slightly modified following engineering judgment; as examples of these engineering judgment modifications, engineers could prefer to have two pipes coming out of the source (Pipe 30) installed since the first construction phase or to have homogeneous diameters for Pipes 1–5, which belong to the same loop, in a bid to increase network reliability in the case of pipe failure.

A last remark concerns the fact that the applications of this paper were made considering for the diameters of the pipes installed in the various phases only the initial cost, relative to pipe purchase and installation. This assumption seems reasonable since preliminary analyses conducted by using the Shamir and Howard (1979) model and considering suitable values of the model parameters had proven

Table 10. For Solutions 5–6 Selected in the "Results" Section, Values of Present-Worth Cost C, of the Minimum Temporal Surplus IS_{min} for the Various Scenarios 1–81, and of Probabilities $P \geq$ and $P \leq$ Calculated For Solution 6 with Respect to Solution 5

Solution	C ($)	IS_{min} (m) Scenarios																						
		1	2	**3**	4	5	6	7	8	9	10	**11**	12	13	14	15	16	**17**	18	19	**20**			
5	383136	10.5	8.0	**0.1**	10.5	8.9	10.3	10.3	6.7	7.4	7.4	**4.17**	9.1	9.1	7.8	9.1	9.1	**5.4**	4.7	4.7	**2.7**			
6	380509	13.2	9.2	**−2.3**	13.0	10.0	11.4	11.1	7.0	8.2	7.9	**2.74**	12.0	12.0	8.8	9.8	9.8	**5.2**	5.7	5.7	**0.6**			
Solution	C ($)	IS_{min} (m) Scenarios																						
		21	22	23	24	25	**26**	27	28	**29**	30	31	32	33	34	35	36	37	**38**	39	40			
5	383136	7.3	7.3	6.5	7.3	7.3	**3.9**	1.6	1.6	**0.7**	9.6	9.6	8.6	9.4	9.4	6.3	7.3	7.3	**3.8**	8.0	8.0			
6	0	10.6	10.6	7.2	8.0	8.0	**3.1**	2.8	2.8	**−1.9**	12.6	12.5	9.6	10.9	10.7	6.8	8.2	7.7	**2.5**	11.5	11.5			
Solution	C ($)	IS_{min} (m) Scenarios																						
		41	42	43	44	45	**46**	47	48	49	50	51	**52**	53	54	**55**	56	57	58	59	60			
5	383136	7.4	8.0	5.0	4.6	4.6	**2.3**	6.2	6.2	6.1	6.2	6.2	**3.5**	1.5	1.5	**0.4**	8.5	8.5	8.1	8.4	8.4			
6	380509	8.4	9.2	5.0	5.7	5.7	**0.4**	10.1	10.1	6.9	7.4	7.4	**2.9**	2.7	2.7	**−2.1**	12.0	11.9	9.2	10.3	10.3			
Solution	C ($)	IS_{min} (m) Scenarios																					$P \geq$	$P \leq$
		61	62	63	**64**	65	66	67	68	69	70	71	72	**73**	74	75	76	77	78	**79**	80	81		
5	383136	5.9	6.9	6.9	**3.3**	6.9	6.9	6.9	6.9	6.9	4.5	4.5	4.5	**1.8**	5.1	5.1	5.1	5.1	5.1	**3.0**	1.3	1.3		
6	380509	6.6	7.7	7.5	**2.4**	11.0	11.0	7.9	8.6	8.6	4.8	5.6	5.6	**0.2**	9.5	9.5	6.5	6.8	6.8	**2.7**	2.7	2.7	0.84	0.16

Note: In bold, scenarios where Solution 5 prevails; in regular text, scenarios where Solution 6 prevails.

Table 11. For Each Solution Discussed, Type of Optimization, Optimization Scenario, Total Present-Worth Cost C and Values of Minimum Surplus in the Optimization Scenarios

Solution	Type of optimization	Scenario	C (\$)	IS_{min} (m)
1	Probabilistic	[1–81]	395,400	[1.6–14.0]
2	Probabilistic	[1–81]	411,664	[0.3–14.8]
3	Deterministic	1	235,228	0.1
4	Deterministic	2	320,564	0.1
5	Deterministic	3	383,136	0.1
6	Probabilistic	[1–81]	380,509	[−2.3, 13.2]

maintenance costs to be negligible with respect to the initial cost and the optimal replacement year to be always larger than the construction duration $T = 100$ years for the diameters considered in this study (this also complies with the findings of Walski and Pelliccia 1982). Rehabilitation methodologies (such as Farmani et al. 2005; Alvisi and Franchini 2006; Dandy and Engelhardt 2006; Giustolisi et al. 2006; Nafi and Kleiner 2010; Roshani and Filion 2013) could then be superimposed to the network designed with the proposed procedure.

Conclusions

This paper belongs to a research line that deals with the need to view the long-term expansion of a system and application of the phasing of construction for the design of water-distribution networks. Following the paper of Creaco et al. (2013), which showed the merits of this approach with respect to the traditional one, based on a single phase, this work was aimed at setting up a multiobjective probabilistic methodology, which makes it possible to take account of uncertainty in demand growth.

The application of the methodology and the comparison of the results, for preset value of minimum pressure surplus over time, with those of the deterministic methodology developed by Creaco et al. (2013) pointed out that the new methodology yields more expensive solutions, i.e., featuring a larger total present-worth cost. The cost difference is particularly evident in the first phase of construction, which concerns pipes to be laid at year 0 and then represents the real design choice to be made. As a matter of fact, taking uncertainty in demand growth into account leads to the installation of larger pipe diameters (mainly in the first phases), which are necessary to render the construction more flexible to adapt itself to various conditions of demand growth over time. Results seem to be in agreement with the saying *when in doubt, build it stout*, which encourages practitioners to slightly oversize infrastructures when future forcing conditions are not known with certainty. Taking account of uncertainty in phasing of construction then produces the effect of anticipating some expenses over time. This effect goes in the opposite direction from the effect produced by adopting the phasing of construction in the design process *when the demand growth is known* in each node: in this latter case, in fact, costs are deferred toward the end of the construction period (Creaco et al. 2013).

A further analysis, performed with reference to a fixed present-worth cost, showed that designing a network by using the deterministic approach and by referring to the most severe scenario, when the latter can be easily identified, leads to solutions that are dominated by those produced by the probabilistic approach, if viewed in the context of network overall performance (expressed by a compact reliability index) over the various scenarios considered. However, the probabilistic approach has the additional advantage of being applicable when the most severe scenario cannot be identified univocally, like in the case of networks featuring multiple loading conditions.

A future paper will be dedicated to the generalization of the procedure presented in this paper to consider, besides the uncertainty in demand growth, the uncertainty characterizing the expansion of the network layout.

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References

Alperovits, E., and Shamir, U. (1977). "Design of optimal water distribution systems." *Water Resour. Res.*, 13(6), 885–900.
Alvisi, S., Creaco, E., and Franchini, M. (2011). "Segment identification in water distribution systems." *Urban Water J.*, 8(4), 203–217.
Alvisi, S., and Franchini, M. (2006). "Near optimal rehabilitation scheduling of water distribution systems based on multiobjective genetic algorithms." *Civ. Eng. Environ. Syst.*, 23(3), 143–160.
Basupi, I., and Kapelan, Z. (2012). "Flexible water distribution system design under uncertainty." *WDSA 2012: 14th Water Distribution Systems Analysis Conf.*, Engineers Australia, A.C.T., Adelaide, South Australia, Barton, 786–797.
Basupi, I., and Kapelan, Z. (2013). "Flexible water distribution system design under future demand uncertainty." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)WR.1943-5452.0000416.
Beh, E. H. Y., Dandy, G. C., and Maier, H. R. (2011a). "Sequencing of water supply options at the regional scale incorporating sustainability objectives." ~~*Proc. 19th Int. Congress on Modelling and Simulation*, Perth, Australia~~, 4001–4007.
Beh, E. H. Y., Maier, H. R., and Dandy, G. C. (2011b). " Development of a modelling framework for optimal sequencing of water supply options at the regional scale incorporating sustainability and uncertainty." ~~*Proc. 19th Int. Congress on Modelling and Simulation*, Perth, Australia~~, 3825–3832.
Beh, E. H. Y., Maier, H. R., and Dandy, G. C. (2012). "Optimal sequencing of water supply options at the regional scale incorporating sustainability, uncertainty and robustness." ~~*Int. Environmental Modelling and Software Society, 2012 Int. Congress on Environmental Modelling and Software Managing Resources of a Limited Planet, Sixth Biennial Meeting*, R. Seppelt, A. A. Voinov, S. Lange, and D. Bankamp, eds., Leipzig, Germany.~~
Bentley Systems. (2006). *WaterGEMS*, Exton, PA.
Bhave, P. R., and Sonak, V. V. (1992). "A critical study of the linear programming gradient method for optimal design of water supply networks." *Water Resour. Res.*, 28(6), 1577–1584.
Boardman, N. E. (2006). *Cost-benefit analysis: Concepts and practice*, Prentice Hall, Upper Saddle River, NJ.
Ciaponi, C. (2009). "Performance analysis in water supply." *Performance indicators for the planning, design and management of water supply systems*, C. Ciaponi, ed., CSDU, Milano.
Creaco, E., and Franchini, M. (2012). "Fast network multi-objective design algorithm combined with an a-posteriori procedure for reliability evaluation under various operational scenarios." *Urban Water J.*, 9(6), 385–399.
Creaco, E., and Franchini, M. (2013). "A new algorithm for the real time pressure control in water distribution networks." *Water Sci. Technol.: Water Supply*, 13(4), 875–882.

Creaco, E., Franchini, M., and Alvisi, S. (2010). "Optimal placement of isolation valves in water distribution systems based on valve cost and weighted average demand shortfall." Water Resour. Manage., 24(15), 4317–4338.

Creaco, E., Franchini, M., and Walski, T. M. (2013). "Accounting for phasing of construction within the design of water distribution networks." J. Water Resour. Plann. Manage., 10.1061/(ASCE)WR.1943-5452.0000358.

Dandy, G. C., and Engelhardt, M. (2006). "Multi-objective trade-offs between cost and reliability in the replacement of water mains." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2006)132:2(79), 79–88.

Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). "A fast and elitist multiobjective genetic algorithm NSGA-II." IEEE Trans. Evol. Comput., 6(2), 182–197.

Eusuff, M. M., and Lansey, K. E. (2003). "Optimization of water distribution network design using the shuffled frog leaping algorithm." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2003)129:3(210), 210–225.

18 Farina, G., Creaco, E., and Franchini, M. (2013). "Using EPANET for modelling water distribution systems with users along the pipes." Civ. Eng. Environ. Syst.

19 Farmani, R., Walters, G., and Savic, D. (2006). "Evolutionary multi-objective optimization of the design and operation of water distribution network: Total cost versus reliability versus water quality." J. Hydroinf., 8(3), 165–179.

Farmani, R., Walters, G. A., and Savic, D. A. (2005). "Trade-off between total cost and reliability for Anytown water distribution network." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2005)131:3(161), 161–171.

Fujiwara, O., Jenchaimahakoon, B., and Edirisinghe, N. C. P. (1987). "A modified linear programming gradient method for optimal design of looped water-distribution networks." Water Resour. Res., 23(6), 977–982.

Fujiwara, O., and Khang, D. B. (1990). "A two-phase decomposition method for optimal design of looped water-distribution networks." Water Resour. Res., 26(4), 539–549.

Fujiwara, O., and Khang, D. B. (1991). "Correction to 'A two-phase decomposition method for optimal design of looped water-distribution networks' by Okitsugu Fujiwara and Do Ba Khang." Water Resour. Res., 27(5), 985–986.

Gargano, R., and Pianese, D. (2000). "Reliability as tool for hydraulic network planning." J. Hydraul. Eng., 10.1061/(ASCE)0733-9429(2000)126:5(354), 354–364.

20 Gessler, J., and Walski, T. M. (1985). "Water distribution system optimization." US Army Corps of Engineers' Waterways Experimentation Station, Technical Rep. TREL-85-11, Vicksburg.

Giustolisi, O., Laucelli, D., and Savic, D. (2006). "Development of rehabilitation plans for water mains replacement considering risk and cost-benefit assessment." Civ. Eng. Environ. Syst., 23(3), 175–190.

Goulter, I. C., Lussier, B. M., and Morgan, D. R. (1986). "Implications of head loss path choice in the optimization of water-distribution networks." Water Resour. Res., 22(5), 819–822.

21 Haghighi, A., Samani, H. M. V., and Samani, Z. M. V. (2011). "GA-ILP method for optimization of water distribution networks." Water Resour. Manage., 25(7), 1791–1808.

22 Kang, D., and Lansey, K. (2012). "Scenario-based multistage construction of water supply infrastructure." Proc.: World Environmental and Water Resources Congress 2012: Crossing Boundaries, 3265–3274.

Kessler, A., and Shamir, U. (1989). "Analysis of the linear programming gradient method for optimal design of water supply networks." Water Resour. Res., 25(7), 1469–1480.

23 Kessler, A., and Shamir, U. (1991). "Decomposition technique for optimal design of water supply networks." Eng. Optimiz., 17(1–2), 1–19.

Krapivka, A., and Ostfeld, A. (2009). "Coupled genetic algorithm—Linear programming scheme for least-cost pipe sizing of water-distribution systems." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2009)135:4(298), 298–302.

Lansey, K. E., Duan, N., Mays, L. W., and Tung, T. K. (1992). "Optimal maintenance scheduling for water distribution systems." Civ. Eng. Syst., 9(3), 211–226. 24

Loganathan, G. V., Greene, J. J., and Ahn, T. J. (1995). "Design heuristic for globally minimum cost water-distribution systems." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(1995)121:2(182), 182–192.

Maier, H. R., et al. (2003). "Ant colony optimization for design of water-distribution systems." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2003)129:3(200), 200–209.

Mortazavi, M., Kuczera, G., and Cui, L. (2012). "Application of multi objective optimization for managing urban drought security in the presence of population growth." Proc. 10th Int. Conf. on Hydroinformatics 2012, Hamburg, Germany. 25

Nafi, A., and Kleiner, Y. (2010). "Scheduling renewal of water pipes while considering adjacency of infrastructure works and economies of scale." J. Water Resour. Plann. Manage., 10.1061/(ASCE)WR.1943-5452.0000062, 519–530.

Pezzinga, G., and Pititto, G. (2005). "Combined optimization of pipes and control valves in water distribution networks." J. Hydraul. Res., 43(6), 668–677.

Prasad, T. D., Sung-Hoon, H., and Namsik, P. (2003). "Reliability based design of water distribution networks using multiobjective genetic algorithms." J. Civ. Eng., 7(3), 351–361. 26

Quindry, G. E., Brill, E. D., and Liebman, J. C. (1981). "Optimization of looped water-distribution systems." J. Environ. Eng. Div., 665–679.

Quindry, G. E., Brill, E. D., Liebman, J. C., and Robinson, A. R. (1979). "Comment on 'Design of optimal water-distribution systems' by E. Alperovits, and U. Shamir." Water Resour. Res., 15(6), 1651–1654.

Roshani, E., and Filion, Y. (2013). "Event-based approach to optimize the timing of water main rehabilitation with asset management strategies." J. Water Resour. Plann. Manage., 10.1061/(ASCE)WR.1943-5452.0000392, 04014004.

Savic, D. A., and Walters, G. A. (1997). "Genetic algorithms for least-cost design of water distribution networks." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(1997)123:2(67), 67–77.

Shamir, U., and Howard, C. D. D. (1979). "An analytical approach to scheduling pipe replacement." J. AWWA, 1(5), 248–258. 27

Simpson, A. R., Dandy, G. C., and Murphy, L. J. (1994). "Genetic algorithms compared to other techniques for pipe optimization." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(1994)120:4(423), 423–443.

Tanyimboh, T. T., Tabesh, M., and Burrows, R. (2001). "Appraisal of source head methods for calculating reliability of water distribution network." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2001)127:4(206), 206–213.

Todini, E. (2000). "Looped water distribution networks design using a resilience index based heuristic approach." Urban Water J., 2(2), 115–122. 28

Todini, E., and Pilati, S. (1988). "A gradient algorithm for the analysis of pipe networks." Computer application in water supply, Vol. I—System Analysis and Simulation, B. Coulbeck and O. Choun-Hou, eds., Wiley, London, 1–20.

Walski, T. M., and Pelliccia, A. (1982). "Economic analysis of water main breaks." J. AWWA, 74(3), 140–147. 29

Wu, Z. Y., and Simpson, A. R. (2001). "Competent genetic algorithm optimization of water distribution systems." J. Comput. Civ. Eng., 10.1061/(ASCE)0887-3801(2001)15:2(89), 89–101.

Wu, Z. Y., Walski, T., Mankowski, R., Cook, J., Tryby, M., and Herrin, G. (2002). "Optimal capacity of water distribution systems." Proc. 1st Annual Environmental and Water Resources Systems Analysis Symp., Roanoke, VA. 30

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