Analysis of non-iterative methods and proposal of a new one

for pressure-driven snapshot simulations with EPANET

E. Pacchin<sup>(1)</sup>, S. Alvisi<sup>(1)</sup>\*, M. Franchini<sup>(1)</sup>

(1) Dipartimento di Ingegneria, Università degli Studi di Ferrara

Abstract

This paper compares different recently proposed methods for pressure-driven snapshot simulations of water

distribution networks using the EPANET software interface and proposes a new one. The proposed method

is based on the insertion of a sequence of devices consisting of a General Purpose Valve (GPV), a fictitious

junction, a reach with a check valve and a reservoir at each water demand node. The proposed method

differs from other methods previously proposed in the literature – and similarly based on the insertion of

sequences of devices consisting of a valve and a reservoir or emitter – in that it uses a GPV; more

specifically, for this valve the user can fix the relationship between outflow discharge (or the flow

delivered to users) and the available pressure head at the node, thus allowing for a flexible representation of

the relationship between these two parameters, whereas with the other methods this relationship remains

fixed, based on the structure of the sequence of devices used.

Practical applications to three different real-life cases show the unreliability and limits, in terms of

accuracy, of some of the methods previously proposed in the literature. They also show, by contrast, the

validity of the new method, which has proven to be reliable and accurate, as well as flexible, since it

enables any relationship to be defined between the delivered flow and available head.

**keywords**: pressure-driven analysis, Epanet, valve, water distribution system

\*Corresponding author. email: stefano.alvisi@unife.it

#### 1. Introduction

Software programs for simulating the hydraulic behaviour of pipe networks are of great importance for the management, design and monitoring of the operation of water supply and distribution systems. Most of these programs are based on the global gradient algorithm – GGA (Todini and Pilati, 1988; Todini and Rossman, 2013), which is also the solver code of EPANET (Rossman, 2000), a software package that is widely used in a variety of settings, from the professional to the academic. In particular, in hydraulic simulation software based on the GGA proposed by Todini and Pilati (1988), including EPANET, it is assumed that the *delivered flow Q* at each node with an unknown head is fixed and equal to the *required flow*, or water demand,  $Q^{req}$  at the node itself, whilst the flow in the pipes and the head at the nodes with an unknown head is assumed to be unknown. In this system, therefore, the flow delivered at the *i*-th node is assigned and independent of the available head at the same node; this type of operation is generally referred to in the scientific literature as Demand-Driven (DD).

In reality, however, the flow delivered at the nodes of a network is tied to the available pressure head and therefore, in the case of pressure head H lower than a prefixed value  $H^{min}$ , the delivered flow Q will be zero, whereas in the case of values H higher than a prefixed value  $H^{des}$ , the delivered flow Q will be equal to  $Q^{req}$ . In the case of intermediate head values, the delivered flow Q will take on values ranging between 0 and  $Q^{req}$ . This type of operation is referred to in the scientific literature as Pressure-Driven (PD) and the Q-H link characterizing it can be expressed by the following relationship:

$$Q = \begin{cases} 0 & \text{if } H \le H^{min} \\ \alpha Q^{req} & \text{if } H^{min} < H < H^{des} \\ Q^{req} & \text{if } H \ge H^{des} \end{cases}$$
 (1)

where  $\alpha$  is the coefficient enabling the flow rate to be modulated when  $H^{min} < H < H^{des}$ . In the literature one may find various formulations for characterizing the trend in  $\alpha$  as a function of H,  $H^{min}$  and  $H^{des}$ , including, for example, the ones proposed by Wagner et al. (1988), Tucciarelli et al. (1999) and Fujiwara and Ganesharajah (1993), given respectively by:

$$Q = Q^{req} \cdot \left(\frac{H - H^{min}}{H^{des} - H^{min}}\right)^{\gamma} \tag{2}$$

$$Q = Q^{req} \cdot \sin^2\left(\frac{H - H^{min}}{H^{des} - H^{min}}\right) \tag{3}$$

$$Q = Q^{req} \cdot \frac{(H - H^{min})^2 \cdot (3H^{des} - 2H - H^{min})}{(H^{des} - H^{min})^3}$$
(4)

It is important to observe that, from an operational viewpoint, a DD simulator can be effectively used in all applications where the minimum required nodal heads are guaranteed (for example, in the context of a minimum-cost design of a network with a minimum head constraint at the nodes). However, in situations where a pressure deficit can occur – for example when assessing the reliability of a system after a pipe or a pumping station is shut off or taken out of service – it is advisable to use a PD simulator, which will make it possible to identify the nodes where the water demand is not completely (or not at all) met (Creaco et al., 2012; Gomes et al., 2013; Tsakiris, 2014; Ciaponi et al., 2015).

In light of this, over the past decade a variety of approaches have been proposed which focus on the development of simulation models enabling a PD analysis of pipe networks. These approaches can be divided into two main types: the solver algorithm of the simulation model can be modified so as to take directly into account the relationship between delivered flow and available head at the node, or else a

simulation of the PD type can be run using a DD simulation model, such as EPANET, after suitable adaptations have been made.

In the former case, algorithms have been developed which, by modifying the GGA method originally proposed by Todini and Pilati (1988), enable the flow delivered at the nodes to be modified according to the available pressure (for example, the ones proposed by Alvisi and Franchini (2006); Giustolisi et al. (2008)). Clearly, by their very nature, these approaches entail the implementation of new hydraulic simulation software.

In the latter case, by contrast, use is made of an already existing hydraulic simulation software program that operates in the DD mode. With specific reference to EPANET, which, as previously noted, is a DD software program whose use is well established, a number of techniques have recently been presented in the technical-scientific literature. Such techniques make it possible to carry out simulations with this type of software while achieving results that are equivalent to those provided by a PD simulator. They can be essentially divided into three types. With the first type, based on the use of dynamic-link libraries (DLLs), it is possible to carry out PD simulations with EPANET by relying on a suitable programming environment (C, C++, Visual Basic, Matlab), from which the DLL is called up; the second and third types enable PD simulations to be run directly using the executable software, i.e. the EPANET interface.

In greater detail, the first technique consists in modifying the dynamic-link libraries (DLLs) of EPANET by introducing a variant of the "emitter" element originally available in the EPANET software program. The emitter is in fact a device that enables a simulation of the flow delivered as a function of pressure head, according to a power relation:

$$Q = cH_e^{\gamma} \tag{5}$$

where c is a discharge coefficient,  $\gamma$  is the exponent and  $H_e$  is the pressure head at the emitter. From eq.(5) it can be clearly deduced that the flow released from an emitter increases with rising pressure; this behaviour

lends itself well to modelling the modulation of flow rate at a node represented by an emitter where the pressure head is greater than zero but less than  $H^{des}$ , whilst for pressure head values above  $H^{des}$  the discharge should be limited to a maximum value of  $Q^{req}$ . Van Zyl et al. (2003) and Morley and Tricarico (2008) have proposed modified EPANET libraries, respectively called OOTEN and EPANETpdd, in which the operating logic of the emitter element is duly modified in such a way as to limit the delivered flow in the event of pressure heads exceeding  $H^{des}$ , as also suggested by Rossman (2007).

In the case of the second technique, a PD simulation is carried out using the executable EPANET software (i.e. via the interface) along with a manually iterative process (Todini, 2003; Ozger and Mays, 2003; Ang and Jowitt, 2006): this entails running a first DD simulation and then identifying the nodes characterized by a pressure head that is insufficient to meet the water demand; at such nodes the demand is set at zero and each of them is connected to a reservoir (having an elevation equal to  $H^{min}$ ) by means of a pipe with negligible resistance. Once the topological matrix has been thus modified, the simulation is repeated and the configuration of the network is updated on the basis of the following rules: in the reservoir where the entering discharge exceeds demand, the node is brought back to the initial conditions (the reservoir is eliminated and the initial demand fixed); if, on the other hand, the discharge entering the reservoir falls short of demand, the reservoir is maintained and the entering discharge represents the actual flow delivered; finally, if there is outflow from the reservoir (rather than discharge entering it), the reservoir is removed and the demand at the node is set equal to zero. The simulation is repeated and the configuration of the network is iteratively modified on the basis of the previous rules until the results in two successive iterations do not change. This approach can clearly be used by an operator directly using the EPANET interface, but it is necessary to iterate the simulations by manually modifying the network structure each time in order to obtain the correct solution. The process can also be automated, at least theoretically, by leaving the EPANET interface environment and modifying, in a suitable programming environment (e.g. MATLAB<sup>TM</sup>), the input file and then calling up the simulator. To the authors' knowledge, however, there are no examples of this type of application.

With the third technique, the topological structure of the network is modified only once and a given sequence of devices, typically a reservoir or an emitter and a valve, are connected to each demand node. This solution makes it possible to obtain a PD solution of the network by directly using the executable EPANET software, via the interface, with a *single simulation*. Various possible sequences of devices have been recently proposed in the literature (Bertola and Nicolini, 2006; Jinesh Babu and Mohan, 2012; Gorev and Kodzhespirova, 2013; Sayyed et al., 2014).

This study is focused on the latter type of approach and makes specific reference to the case of snapshot simulations. Below we will analyze the different sequences of devices proposed in the scientific literature and formulate an original one based on the use of a valve that is of a different type from the ones already proposed and enables the water demand to be modulated based on variations in the available pressure head and according to different relations, such as, for example, the ones proposed by Wagner et al. (1988) (see eq.(2)), Tucciarelli et al. (1999) (see eq.(3)) and Fujiwara and Ganesharajah (1993) (see eq.(4)). We will then analyze and discuss the accuracy, advantages and disadvantages of each of the methods with reference to three real-life cases and conclude with some final considerations.

### 2. Methods analyzed

The methods analyzed in this study for a PD solution of a pressurized network using the executable EPANET software (via the interface), in the *case of a snapshot simulation*, are the ones proposed by a) Bertola and Nicolini (2006), b) Jinesh Babu and Mohan (2012), c) Gorev and Kodzhespirova (2013), d) Sayyed et al. (2014) and e) a new method proposed here. As previously observed, all of these methods can be used to carry out a PD snapshot simulation, via the EPANET interface, by connecting a specific sequence of devices to a demand node. In general, the devices used are a reservoir, emitter, several types of valves (Flow Control Valve – FCV, Pressure Reducing Valve – PRV) and a pipe with a Check Valve – CV, the latter used to prevent reverse flow. The methods differ in terms of the devices considered to form the sequence to be connected to each node in order to transform the simulation from DD (fixed delivered

flow, irrespective of pressure head) to PD (delivered flow varying as a function of pressure head). Each method is presented in detail below. In order to quantitatively show the modulation of the flow rate obtained with the methods, the description of each is immediately followed by a numerical example based on a very simple scheme consisting of a generic demand node n characterized by a water demand corresponding to  $Q^{req} = 1 \text{ L/s}$ , a pressure head value  $H^{des}$  above which the water demand is fully met, equal to 40 m, and a pressure head value  $H^{min}$  equal to 10 m, below which no flow is delivered. In order to characterize the modulation of the flow rate Q at the node, it is assumed that the pressure head H at the node H ranges between 0 and 50 m. From an operational standpoint, in EPANET, the variation in the pressure head H of the node H0 was obtained by setting the elevation H2 of this node at 0 m, connecting it to a reservoir H3 with a pipe of negligible length and resistance, and setting the reservoir H4 an elevation ranging between 0 m and 50 m (see Figure 1).

### 2.1. Method of Bertola and Nicolini (2006): PRV - E

In the method proposed by Bertola and Nicolini (2006), a sequence of devices composed of a PRV and an emitter (Figure 2a) is added at the demand node n (see Figure 1). The demand at the node n is set equal to zero. With regard to the characteristics of the emitter, an elevation  $z_e = z_n + H^{min}$  is assumed, a value of the exponent  $\gamma=0.5$  and a value of the emitter coefficient c (see eq.(5)), equal to:

$$c = \frac{Q^{req}}{\left(H^{des} - H^{min}\right)^{\gamma}} \tag{6}$$

Thus, without considering the PRV, the pressure head  $H_e$  in the emitter would be equal to:

$$H_e = H + z_n - z_e = H + z_n - (z_n + H^{\min}) = H - H^{\min}$$
 (7)

being H the pressure head in the demand node n,  $z_n$  elevation of node n and  $z_e = z_n + H^{min}$  the elevation of the emitter.

Substituting eq.(6) and eq.(7) in eq.(5) will give us the relation that governs the modulation of the flow rate from the emitter without considering the PRV:

$$Q = \frac{Q^{req}}{\left(H^{des} - H^{min}\right)^{\gamma}} \cdot \left(H - H^{min}\right)^{\gamma} \tag{8}$$

It should be kept in mind that, by virtue of continuity, the flow delivered from the emitter will correspond to the delivered/outgoing flow from the node n.

Finally, considering the PRV, a set value of  $H^{des}$ - $H^{min}$  is assumed, so that in the emitter, downstream of the PRV, the maximum pressure head  $H_e$  will be limited to this value.

Thus, if the pressure head at the node n is  $H leq H^{min}$ , the PRV will not permit reverse flow from the emitter to the node (in EPANET the PRV also works like a check valve - CV) and the outflow from the node n is zero; if the pressure head at the node n is  $H^{min} < H < H^{des}$ , the emitter will release flow according to the relation given in eq.; finally, if the pressure head H at the node H is  $H \geq H^{des}$ , and thus the pressure head H at the emitter is  $H = (H^{des} - H^{min}) - (H^{des} - H^{min}) - (H^{des} - H^{min}) - (H^{des} - H^{min})$ , so that the delivered flow does not exceed demand. With reference to the system in Figure 1, combined with sequence of devices a) in Figure 2, the graph in Figure 3 shows the pattern of  $Q/Q^{req}$  as a function of H with the PRV – E method.

It should be noted, moreover, that eq.(8) represents the second case of eq.(1) if  $\alpha$  is expressed with the expression of Wagner et al. (1988) (see eq.(2)).

# 2.2. Method of Jinesh Babu and Mohan (2012): $FCV - CV_0 - R$

In the method proposed by Jinesh Babu and Mohan (2012), a sequence of devices composed of an FCV, a fictitious junction  $n_f$ , a reach with a CV and a reservoir R are added at the demand node n (Figure 2b). The demand at the node n is set equal to zero, whilst the flow delivered from the node will correspond to the discharge through the above-mentioned sequence of devices in the direction of the reservoir. A set value equal to  $Q^{req}$  is assumed in the FCV to prevent the delivered flow from exceeding demand; the node  $n_f$  and the reservoir are placed at an elevation  $z_R = z_n + H^{min}$ ; the reach with check valve is characterized by negligible head losses and this is emphasized here by using the subscript "0" associated with the check valve symbol, i.e.  $CV_0$ .

In the case of pressure heads lower than  $H^{min}$  at the node n, the  $CV_0$  will not permit the passage of flow from the reservoir to the node n; where  $H \ge H^{min}$ , the discharge would tend to grow in an unlimited manner, but the FCV limits the value to  $Q^{req}$ . How the sequence of devices works is thus described by the following function:

$$Q = \begin{cases} 0 & \text{if } H < H^{min} \\ Q^{req} & \text{if } H \ge H^{min} \end{cases}$$
 (9)

With reference to the system in Figure 1, combined with sequence of devices b) in Figure 2, the graph in Figure 3 shows the pattern of  $Q/Q^{req}$  as a function of H with the FCV –  $CV_0$  – R method. It is worth noting that in this case no modulation of flow rate is observed. The  $Q/Q^{req}$  ratio takes on a value of zero as long as the head H is less than  $H^{min}$ ; in the case of heads H at the node n even just above  $H^{min}$ , and in any case less than  $H^{des}$  (a parameter that is not explicitly considered in this schematic representation), Q would immediately take on very high values (tending to infinity), since the reach located between the node n and the reservoir (that is, the FCV –  $CV_0$  – R sequence of devices) is characterized by negligible/infinitesimal head losses; the presence of the FCV limits the delivered flow to  $Q^{req}$ . Therefore, given the absence of

head losses in the proposed sequence of devices, at  $H^{min}$  Q will pass instantly from 0 to  $Q^{req}$  following the typical pattern of a step function. Therefore, no modulation takes place between 0 and  $Q^{req}$ . It is worth noting, moreover, that in some numerical applications proposed by Jinesh Babu and Mohan (2012), there are values of Q ranging between 0 and  $Q^{req}$  at some nodes when  $H=H^{min}$ . This "apparent" modulation of the flow rate depends solely on the upstream network characteristics, and in particular on the heads at the nodes with an imposed head and the head losses in the pipes upstream of the demand node considered, but not on the head H at the node considered. It should be observed, in fact, also with reference to the diagram in Figure 1, that the head H at the node n is in any case always equal to  $H^{min}$ , since the node is connected, via a reach with negligible resistance, to the terminal reservoir R of the sequence of devices  $FCV - CV_0 - R$  having an imposed head equal to  $H^{min}$ ; however, for particular values of the diameter d, length d and resistance coefficient d of the pipe that connects the reservoir d with head d at the node d and thus to the terminal reservoir d of the sequence of devices d be discharge from d at the node d and thus to the terminal reservoir d of the sequence of devices d be d and d be a figure 2b), could d based on the equation of motion, but independently of the flow rate modulation relation of eq.(9) d take on values ranging between 0 and d between 0 and d for the flow rate modulation relation of eq.(9) d take on values ranging between 0 and d for the flow rate modulation relation of eq.(9) d take on values ranging between 0 and d for the flow rate

A technique substantially identical to the one proposed by Jinesh Babu and Mohan (2012) was presented by Sivakumar and Prasad (2014, 2015); they added a reach with a CV between the demand node *n* and the FCV, a solution that does not introduce any change compared to the method discussed above.

# 2.3. Method of Gorev and Kodzhespirova (2013): $FCV - CV_{ml} - R$

In the method proposed by Gorev and Kodzhespirova (2013) a sequence of devices composed of an FCV, a fictitious junction  $n_f$ , a reach with a CV having significant minor losses (hereinafter CV<sub>ml</sub>, where "ml" stands for "minor loss") and a reservoir R are added at the demand node n (Figure 2c). The demand at the node n is set equal to zero. A set value equal to  $Q^{req}$  is assumed in the FCV to prevent the discharge from exceeding the water demand, the fictitious node  $n_f$  and the reservoir R are set at an elevation  $z_R = z_n + H^{min}$ ;

the reach with  $CV_{ml}$  must be characterized by *negligible* friction losses but must have a minor loss  $\zeta_{CV} \cdot V^2/2g$ , wherein V is the velocity and the loss coefficient  $\zeta_{CV}$  is defined as:

$$\zeta_{CV} = 2g \frac{H^{des} - H^{\min}}{\left(V^{req}\right)^2} = 2g \frac{H^{des} - H^{\min}}{\left(\frac{Q^{req}}{\pi d_{CV}^2 / 4}\right)^2}$$
(10)

Where  $H < H^{min}$ , the reach with  $CV_{ml}$  will not permit flow from the reservoir to the node n; where  $H^{min} \le H < H^{des}$ , the reach with  $CV_{ml}$  will allow modulation of the flow rate (towards the reservoir) thanks to the minor loss; finally, when  $H \ge H^{des}$  the FCV will prevent the delivered flow from exceeding  $Q^{req}$ . In greater detail, in this case the flow rate modulation mechanism depends on the motion equation associated with the sequence of devices itself (reach falling between the node n and the reservoir R) as (see Figure 2c):

$$(H + z_n) - z_R = (H + z_n) - (z_n + H^{\min}) = H - H^{\min} = \zeta_{CV} \frac{Q^2}{2g\left(\frac{\pi d_{CV}^2}{4}\right)^2}$$
(11)

which, when inverted, provides the value of Q:

$$Q = \left(2g \frac{H - H^{\min}}{\zeta_{CV}} \over \left(\pi d_{CV}^2 / 4\right)^2\right)^{0.5}$$
 (12)

By substituting eq.(10) in eq.(12) we obtain:

$$Q = Q^{req} \left( \frac{H - H^{\min}}{H^{des} - H^{\min}} \right)^{0.5}$$
 (13)

With reference to the system in Figure 1, combined with sequence of devices c) in Figure 2, the graph in Figure 3 illustrates the pattern of  $Q/Q^{req}$  as a function of H with the FCV –  $CV_{ml}$  – R method. It should be noted, in particular, that like in the PRV – E method, eq.(13) reproduces the second case of eq. (1) if  $\alpha$  is expressed with the expression of Wagner et al. (1988).

### 2.4. Sayyed et al. (2014) Method: $FCV - CV_0 - E$

In the method proposed by Sayyed et al. (2014), a sequence of devices composed of an FCV, a fictitious junction  $n_f$ , a reach with a CV without minor losses (hereinafter CV<sub>0</sub>) and an emitter and (see Figure 2d) at each demand node n. The demand at the node n is set equal to zero. In the FCV,  $Q^{req}$  is fixed so as to prevent the discharge from exceeding the water demand; the fictitious junction  $n_f$  and the emitter are set at an elevation  $z_e = z_n + H^{min}$ . The emitter delivers flow according to relation (5), where c is defined as in expression (6) and  $\gamma$ , according to Sayyed at al. (2014), must take on a value equal to  $\gamma = 2/3$ . With this configuration, where  $H < H^{min}$ , the reach with CV<sub>0</sub> will not permit flow from the emitter to the node n, whilst where  $H^{min} \le H < H^{des}$ , the emitter will modulate the flow rate according to relation (2) and, finally, when  $H \ge H^{des}$ , the FCV will prevent the delivered flow from exceeding the water demand  $Q^{req}$ . With reference to the system in Figure 1, combined with sequence of devices d) in Figure 2, the graph in Figure 3 shows the pattern of  $Q/Q^{req}$  as a function of H, both for  $\gamma = 2/3$  and  $\gamma = 0.5$ . The case of  $\gamma = 2/3$  is the one originally proposed by Sayyed at al. (2014), whereas with  $\gamma = 0.5$ , the flow rate modulation obtained with this method reproduces the second case of eq.(1) if  $\alpha$  is expressed using the expression of Wagner et al. (1988), as with the PRV – E and FCV – CV<sub>ml</sub> – R methods.

### 2.5. Proposed Method: $GPV_{W/T/F} - CV_0 - R$

The method proposed in this paper entails adding a sequence of devices composed of a General Purpose Valve (GPV), a fictitious junction  $n_f$ , a reach with a CV without minor losses (hereinafter CV<sub>0</sub>) and a reservoir (see Figure 2e) at the demand node n. The demand at the node n is set equal to zero. The fictitious node  $n_f$  and the reservoir are set at an elevation  $z_R = z_n + H^{min}$ ; in the reach with CV<sub>0</sub>, the friction head losses (as well as minor losses) must be negligible. A head loss curve is fixed for the GPV; this curve represents the relation between the discharge Q through the valve and the head loss  $(H - H^{min})$  induced by the valve itself (taking into account that upstream of this valve that is, on the node side, the head is  $z_n + H$  whereas downstream of this valve, that is, on the reservoir side, the head is  $z_R = z_n + H^{min}$ ). The curve must have a strictly monotonically increasing pattern (thus without any parts having a constant value), so that each discharge value will have only one head loss value associated with it and vice versa.

Practically speaking, the head loss curve characterizing the GPV is constructed as a continuous piecewise function:

$$Q = \begin{cases} f(H) & \text{if } H^{\min} \le H \le H^{des} \\ Q^{req} \cdot \left( 1 + \varepsilon \cdot \frac{H - H^{des}}{H^{des}} \right) & \text{if } H > H^{des} \end{cases}$$
(14)

where f(H) can be represented, for example, by the relation of Wagner et al. (1988) (see eq.(2)), Tucciarelli et al. (1999) (see eq.(3)) or Fujiwara and Ganesharajah (1993) (see eq.(4)) (see also Figure 4). If  $H > H^{des}$ , the function characterizing the head loss curve, in accordance with eq.(1), should be constant, i.e. the discharge Q should be equal to the demand  $Q^{req}$  irrespective of the head H; moreover, since the head loss curve must have a strictly monotonically increasing pattern, as previously observed, in order to numerically overcome the problem it is assumed that where  $H > H^{des}$ , the discharge will increase

infinitesimally, or in any case in a negligible manner from an operational standpoint, according to the second relation of eq. (13),  $\varepsilon$  being a small magnitude selected at will (for example in the order of  $10^{-5}$ ). With the proposed sequence of devices, therefore, in the case of  $H < H^{min}$  (remembering that the head H refers to the node n), the reach with  $CV_0$  will not permit flow from the reservoir to the node n and thus the discharge Q will be zero; where  $H^{min} \le H < H^{des}$ , the GPV will modulate the flow rate according to the relation described by the first part of the function characterizing the head loss curve (see eq.(14)); finally, where  $H > H^{des}$ , the delivered flow will remain practically constant and equal to  $Q^{req}$  in accordance with the second part of the piecewise function characterizing the head loss curve. With reference to the system in Figure 1, combined with sequence of devices e) in Figure 2, the graph in Figure 3 shows the pattern of  $Q/Q^{req}$  as a function of H assuming different expressions for the function f(H) of eq.(14), i.e. the relation of Wagner et al. (1988) (GPV<sub>W</sub>), Tucciarelli et al. (1999) (GPV<sub>T</sub>) and Fujiwara and Ganesharajah (1993) (GPV<sub>F</sub>). It is evident that the advantage of this approach compared to the previous ones lies in the possibility of using any relation whatsoever between the delivered flow Q and the value  $(H - H^{min})$ ; this introduces a significant novelty compared to the methods presented in the literature to date.

#### 3. Case studies

Three real-life cases were used to compare the analysed methods. The first case is a looped network fed by two reservoirs set at the same elevation taken from Ozger and Mays (2003) and it was later also adopted in the article of Sayyed et al. (2014) to which the reader can refer for numerical details. The network used in the second case study was taken from an article by Estrada et al. (2009) to which the reader can refer for numerical details; it is a network with a tree-like structure fed by one reservoir and equipped with a pressure sustaining valve (PSV) immediately downstream of the tank/reservoir. The third case consists in the network of the historical centre of a city in northern Italy and the reader can refer to Creaco et al. (2010) for numerical details.

With reference to the methods of Jinesh Babu and Mohan (2012), Gorev and Kodzhespirova (2013), Sayyed et al. (2014) and the proposed method, we assumed a diameter  $d_{CV}$  =0.3 m, length  $L_{CV}$  =1 m and Hazen – Williams coefficient of  $C_{CV}$  =130 for the reach with CV<sub>0</sub> that should have negligible resistance (as suggested by Ang and Jowitt, 2006). This assumption, in all three case studies, implies wholly negligible friction head losses in the reach with CV<sub>0</sub>, in any case always less than 0.05 m/km.

In all three cases, *related to snapshot simulations*, the results obtained with the analysed methods were compared with the values obtained using the pressure-driven algorithm developed by Alvisi and Franchini (2006). In particular, within the framework of this algorithm the relation between discharge Q and pressure head H expressed by eq.(1) was associated with the formulation expressed by (2) (Wagner et al. (1988)) for comparisons with the PRV – E (Bertola and Nicolini, 2006), FCV – CV<sub>0</sub> – R (Jinesh Babu and Mohan, 2012), FCV – CV<sub>L</sub> – R (Gorev and Kodzhespirova,2013), FCV – CV<sub>0</sub> – E (Sayyed et al., 2014) and  $GPV_W$ –  $CV_0$  – R methods and the formulations expressed by eqs.(3) and (4) for the comparisons with the  $GPV_T$  –  $CV_0$  – R and  $GPV_F$  –  $CV_0$  – R methods, respectively, so as to compare each analysed method with the PD algorithm using the same relation between discharge Q and pressure head H.

#### 4. Discussion of the results and conclusions

With reference to the first case study, different snapshot situations obtained were analysed by varying the elevation of the tanks and opening or closing the pipe 3. In this manner, various simulation scenarios were obtained for the network. More specifically, in a first scenario a head of 70.96 m was assumed both for RES<sub>1</sub>, and RES<sub>2</sub> and the pipe 3 was assumed to be open; in this case the network operates in a purely DD mode, in which the nodal heads enable the water demand to be completely met. In a second scenario the head at RES<sub>1</sub> and RES<sub>2</sub> was assumed to be 60.96 m and the pipe 3 was assumed to be closed, resulting in a hybrid DD and PD operation of the system, in which the heads would remain above the threshold  $H^{des}$  only at some of the nodes. Finally, the last scenario was obtained by assuming a head of 35.96 m at RES<sub>1</sub> and RES<sub>2</sub> and the pipe 3 to be closed, resulting in a purely PD operation of the system, in which the pressure

head also falls below  $H^{min}$  at some nodes. Table 1 summarizes the mean, minimum and maximum absolute errors obtained when comparing the results provided by EPANET "structured" according to the different methods previously described with those of the PD algorithm of Alvisi and Franchini (2006), on the basis of the combined data for the three scenarios. Percentage errors are also provided. From an analysis of the errors it appears clear that the FCV – CV<sub>0</sub> – R method (proposed by Jinesh Babu and Mohan, 2012) shows marked differences compared to the other methods. This is due to the inability of the method to modulate flow rates between 0 and  $Q^{req}$  when  $H = H^{min}$  according to a pre-established relation, as they are derived only from the combination of the heads at the surrounding nodes and the flow in the pipes converging at the node concerned. For example, in the third scenario, at the node 8, a head  $H = H^{min}$  is generated, with a delivered flow equal to 65.62 m<sup>3</sup>/h (versus 97.90 m<sup>3</sup>/h produced by the reference PD model with a modulation function according to Wagner et al., 1988), intermediate between 0 and  $Q^{req}$ =327.6 m<sup>3</sup>/h, which does not derive from any particular modulation relation, but only respects the balance at the node, as the discharges entering the node 8 through the pipes 20 and 21 are respectively 70.90 and 62.91 m<sup>3</sup>/h and the outgoing discharge through the pipe 14 at the node is 68.19 m<sup>3</sup>/h. It is also worth noting that the "uncontrolled" value of the flow delivered at the node gives rise to a particular flow circuit in the network pipes, with increasingly less evident effects as the distance from the node considered increases, and this also has a repercussion on the heads of the other nodes. In some cases, this particular flow circuit leads to head values such as to cause conditions of insufficient head, and hence zero delivered flow at nodes where the purely PD model, by contrast, would give a delivery of flow, such as, for example, in the case concerned, at node 4 (delivered flow of  $Q = 133 \text{ m}^3/\text{h}$ , according to the benchmark PD method). In short, this method does not enable the flow rate to be modulated according to a pre-established relation and in this respect it differs distinctly from all the other methods considered here and the purely PD model used as a benchmark.

As regards the remaining methods, they provide very similar results in all of the situations examined and can be considered equivalent. It is worth noting, in particular, that when the  $GPV-CV_0-R$  method is applied using the relations of Tucciarelli et al. (1999)  $(GPV_T-CV_0-R)$  and Fujiwara and Ganesharajah

(1993) (GPV<sub>F</sub> – CV<sub>0</sub> – R), the resulting mean and maximum errors are similar and slightly higher than with the other methods. However, this is not due not so much to the type of sequence of devices and the use of a GPV per se, but rather to the particular functions f(H) used (see eqs.(3) and(4)), which are characterized by a different pattern in the relation between head loss and Q and for heads close to  $H^{min}$  gives rise to errors, during interpolation, that are slightly greater than those that occur when the Wagner relation is used. Nevertheless, they are always very modest, in the order of about 0.06%. This is also confirmed by the fact that with the same sequence of devices  $GPV - CV_0 - R$ , but using the relation of Wagner et al. (1988)  $(GPV_W - CV_0 - R)$  to characterize the "head loss" curve, the resulting errors are equivalent, and indeed slightly smaller than those produced by the other methods relying on an analogous flow rate modulation equation. The efficiency of the analysed methods is also evaluated considering the number of iterations needed to perform the simulations. In Table 1, the average and maximum number of iterations over the whole set of simulation scenarios considered are shown. It can be noticed that in general all the analysed methods require similar average and maximum number of iterations. More in details, it is worth noting that for  $GPV_W - CV_0 - R$  method slightly higher numbers of iteration (both average and maximum) are required, but this is mainly due to those scenarios in which the network operates in a purely DD mode, whereas when the network operates in a hybrid DD and PD condition or in a purely PD condition the number of iterations of this method is equivalent to those of the other methods.

The second case considered is a network featuring a PSV downstream of the tank. Given that this network has already been analysed in the literature in order to assess the conflicts that can arise as a result of the interaction between a PSV and the use of sequences of devices, it is addressed again in this article to verify the compatibility of the methods analysed here with the use of a PSV. It was seen that, when the PRV – E (Bertola and Nicolini, 2006) and FCV –  $CV_0$  – R (Jinesh Babu and Mohan, 2012) methods were used, EPANET could fail to achieve convergence (at the end of the simulation the warning "System Unbalanced" appeared even when 1000 iterations are carried out, versus the 10-15 required by EPANET for the solution of the network modified with the other methods). Indeed, it is worth noting that several tests were performed varying the value of the setting of the PSV valve and the "Unbalanced" condition

occurred only for some particular values, thus showing that the "Unbalanced" condition seems to be due to particular combinations of the PRV – E (Bertola and Nicolini, 2006) and FCV –  $CV_0$  – R (Jinesh Babu and Mohan, 2012) strings and some PSV's setting values.

The remaining methods, on the other hand, showed to be compatible with the use of the PSV for all the PSV setting values considered and EPANET converged to solutions that were very similar to one another and characterized by mean and maximum errors, as compared to the results output by the PD algorithm PD of Alvisi and Franchini (2006), in the order of 0.02 L/s and 0.2 L/s respectively, as shown in Table 2. The number of iterations (see Table 2) shows that for this case study the proposed methods ( $GPV_W - CV_0 - R$ ,  $GPV_T - CV_0 - R$  and  $GPV_F - CV_0 - R$ ) are more efficient than the others.

In the third case studied, the elevation of the tanks was varied from  $H^{des} = 30$  m to  $H^{min} = 20$  m. Figure 5 shows histograms of the absolute error that occurred when the schematized network was solved with the different methods described above as compared to the solution with the PD algorithm of Alvisi and Franchini (2006). Table 3 shows the corresponding mean, minimum and maximum absolute errors. The distributions of the errors that occurred with the different methods and the mean and maximum values generally confirm the results of the previous case studies. In fact, in this case as well it may be observed that the  $FCV - CV_0 - R$  method generally gives larger errors than the other methods (see Figure 5) as a result of the inability of the method to modulate the flow rates between 0 and  $Q^{req}$  according to a preestablished relation; in this case, too, the maximum error, equal to about 13 L/s (see Table 3), occurs at the nodes with head values close to but slightly greater than  $H^{min}$ , for which the FCV – CV<sub>0</sub> – R method gives a delivered flow equal to  $Q^{req}$ , whereas all the other methods give a delivered flow close to 0 and in any case governed by the modulation relation. As far as the other methods are concerned, it can be observed that in this specific case the FCV – CV<sub>ml</sub> - R method (Gorev and Kodzhespirova, 2013) also gives a fairly large error, equal to almost 6 L/s (see Table 3), less than the maximum error of the  $FCV - CV_0 - R$  method but greater than those found with the other methods. This error occurs at a specific node of the network with a head value close to  $H^{min}$ . All the other methods produce mean, minimum and maximum absolute errors that are similar to one another.

Finally, Table 3 shows that in this case study all the methods require on average a rather similar number of iterations.

In conclusion, with respect to snapshot simulations, it can be affirmed that the techniques FCV –  $CV_0$  – E (Sayyed et al., 2014) and  $GPV_{W/T/F}$  –  $CV_0$  – R (proposed method) are capable of correctly reproducing the functioning of a network in the Pressure-Driven mode with a rather similar computational burden, unlike the other techniques, which, when different real-life cases were considered showed a number of limits, specifically: the FCV –  $CV_0$  – R technique (Jinesh Babu and Mohan, 2012) does not enable the flow rate to be modulated between 0 and  $Q^{req}$  according to a pre-established relation, does not provide for a distinction between  $H^{min}$  and  $H^{des}$  and proved unreliable where a PSV was present; the PRV – E technique (Bertola and Nicolini, 2006) showed to be unreliable where a PSV was present; and the FCV –  $CV_{ml}$  - R technique (Gorev and Kodzhespirova, 2013), in a specific case study, gave a large error for a nodal head value close to  $H^{min}$ .

The proposed technique,  $GPV_{W/T/F} - CV_0 - R$ , also has an advantage in that enables any  $Q - (H-H^{min})$  relation to be established between discharge and head loss. Therefore, any relation between delivered flow and available head can be used without necessarily having to use the formulation of Wagner et al. (1988) as in all the other methods analysed.

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# **Tables**

**Table 1.** Case Study I: Mean, minimum and maximum absolute and percentage (within brackets) errors obtained using the various methods versus the PD algorithm of Alvisi and Franchini (2006) and average and maximum number of iterations needed to perform the simulation.

| Method              | Mean error<br>[m³/h] | Minimum error<br>[m³/h] | Maximum error<br>[m³/h] | Iterations<br>(average –<br>maximum) |
|---------------------|----------------------|-------------------------|-------------------------|--------------------------------------|
| PRV – E             | 0.0889 - (0.02)      | 0.00 - (0.00)           | 0.6946 - (0.11)         | 7.50 - 8                             |
| $FCV - CV_0 - R$    | 40.0813 – (10.35)    | 0.00 - (0.00)           | 349.5270 – (54.49)      | 6.00 - 11                            |
| $FCV - CV_{ml} - R$ | 0.0682 - (0.02)      | 0.00 - (0.00)           | 0.4850 - (0.07)         | 5.50 - 8                             |
| $FCV - CV_0 - E$    | 0.0589 - (0.16)      | 0.00 - (0.00)           | 0.4950 - (0.07)         | 5.75 - 9                             |
| $GPV_W - CV_0 - R$  | 0.0584 - (0.02)      | 0.00 - (0.00)           | 0.4250 - (0.06)         | 14.25 - 21                           |
| $GPV_T - CV_0 - R$  | 0.1642 - (0.06)      | 0.00 - (0.00)           | 1.7944 - (0.28)         | 6.50 - 8                             |
| $GPV_F - CV_0 - R$  | 0.1628 - (0.06)      | 0.00 - (0.00)           | 1.6996 - (0.26)         | 6.75 - 9                             |

**Table 2**. Case Study II: Mean, minimum and maximum absolute and percentage (within brackets) errors obtained using the various methods versus the PD algorithm of Alvisi and Franchini (2006) and average and maximum number of iterations needed to perform the simulation.

| Method              | Mean error<br>[L/s] – [%] | Minimum error<br>[L/s] – [%] | Maximum error<br>[L/s] – [%] | Iterations<br>(average –<br>maximum) |
|---------------------|---------------------------|------------------------------|------------------------------|--------------------------------------|
| PRV – E             | Unbalanced                | Unbalanced                   | Unbalanced                   | Unbalanced                           |
| $FCV - CV_0 - R$    | Unbalanced                | Unbalanced                   | Unbalanced                   | Unbalanced                           |
| $FCV - CV_{ml} - R$ | 0.03 - (0.07)             | 0.01 - (0.04)                | 0.12 - (0.11)                | 17.8 - 27                            |
| $FCV - CV_0 - E$    | 0.02 - (0.04)             | 0.00 - (0.00)                | 0.13 - (0.12)                | 22 - 43                              |
| $GPV_W - CV_0 - R$  | 0.03 - (0.08)             | 0.01 - (0.01)                | 0.15 - (0.08)                | 6.5 - 8                              |
| $GPV_T - CV_0 - R$  | 0.045 - (0.15)            | 0.00 - (0.00)                | 0.17 - (0.71)                | 7.7 – 9                              |
| $GPV_F - CV_0 - R$  | 0.028 - (0.08)            | 0.00 - (0.00)                | 0.07 - (0.29)                | 7.5 – 9                              |

**Table 3**. Case Study III: Mean, minimum and maximum absolute and percentage (within brackets) errors obtained using the various methods versus the PD algorithm of Alvisi and Franchini (2006) and average and maximum number of iterations needed to perform the simulation.

| Method              | Mean error<br>[L/s] – [%] | Minimum error<br>[L/s] – [%] | Maximum error<br>[L/s] | Iterations<br>(average –<br>maximum) |
|---------------------|---------------------------|------------------------------|------------------------|--------------------------------------|
| PRV - E             | 0.0221 - (0.31)           | 0.00 - (0.00)                | 0.1602 - (0.64)        | 7.5 - 8                              |
| $FCV - CV_0 - R$    | 1.6658 - (27.59)          | 0.00 - (0.00)                | 13.011 – (52.05)       | 7.42 - 22                            |
| $FCV - CV_{ml} - R$ | 0.1467 - (2.25)           | 0.00 - (0.00)                | 5.9285 - (23.71)       | 6.17 - 7                             |
| $FCV - CV_0 - E$    | 0.0221 - (0.31)           | 0.00 - (0.00)                | 0.1602 - (0.64)        | 5.83 - 6                             |
| $GPV_W - CV_0 - R$  | 0.0230 - (0.33)           | 0.00 - (0.00)                | 0.1702 - (0.68)        | 5.42 - 7                             |
| $GPV_T - CV_0 - R$  | 0.03089 - (0.56)          | 0.00 - (0.00)                | 0.3399 - (6.79)        | 7.33 – 14                            |
| $GPV_F - CV_0 - R$  | 0.0213 - (0.29)           | 0.00 - (0.00)                | 0.1978 - (0.79)        | 7.33 - 15                            |

### **Figures**

**Figure 1.** Layout of the system used to analyse the methods considered.

**Figure 2.** Sequences of devices characterizing the analyzed methods: (a) PRV - E (Bertola and Nicolini, 2006), (b)  $FCV - CV_0 - R$  (Jinesh Babu and Mohan, 2012), (c)  $FCV - CV_L - R$  (Gorev and Kodzhespirova, 2013), (d)  $FCV - CV_0 - E$  (Sayyed et al., 2014), (e)  $GPV_{W/T/F} - CV_0 - R$ .

**Figure 3.** Pattern of  $Q/Q^{req}$  as a function of  $H_{ref}$  for the different methods analysed.

**Figure 4.** Head loss curve for the GPV, obtained using the relations of Wagner et al. (1988), Tucciarelli et al. (1999) and Fujiwara and Ganesharajah (1993).

**Figure 5.** Case Study III: Histogram of the absolute error occurring in comparison with the Pressure-Driven method of Alvisi and Franchini (2006) when discharge was computed with the methods: (a) PRV - E, (b)  $FCV - CV_0 - R$ , (c)  $FCV - CV_{ml} - R$ , (d)  $FCV - CV_0 - E$ , (e)  $GPV_W - CV_0 - R$ , (f)  $GPV_T - CV_0 - R$  and (g)  $GPV_F - CV_0 - R$ .

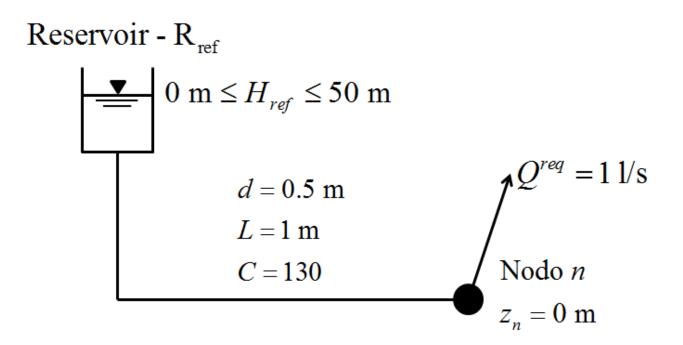
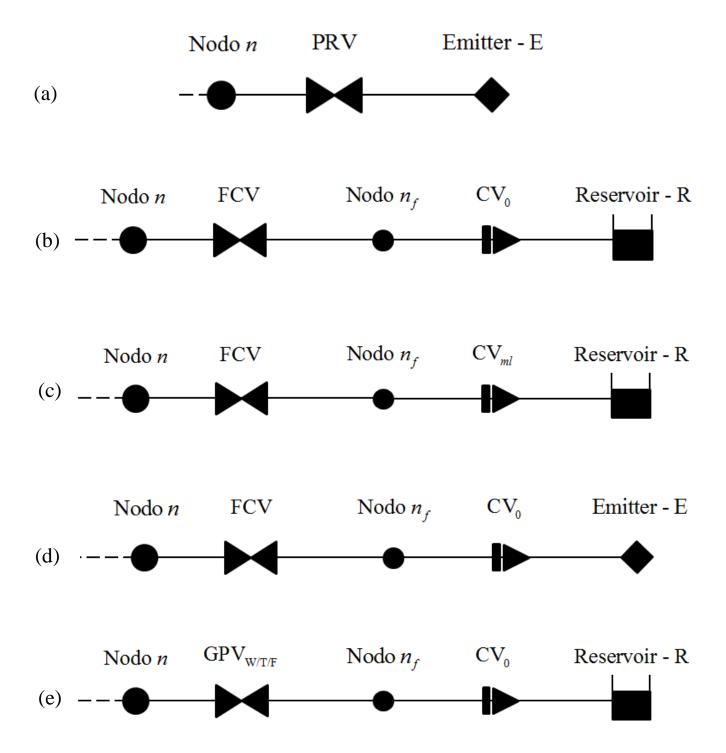
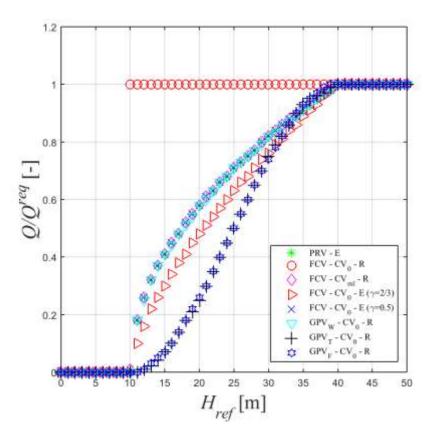


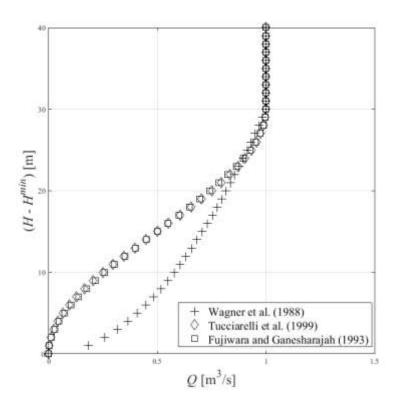
Figure 1. Layout of the system used to analyse the methods considered.



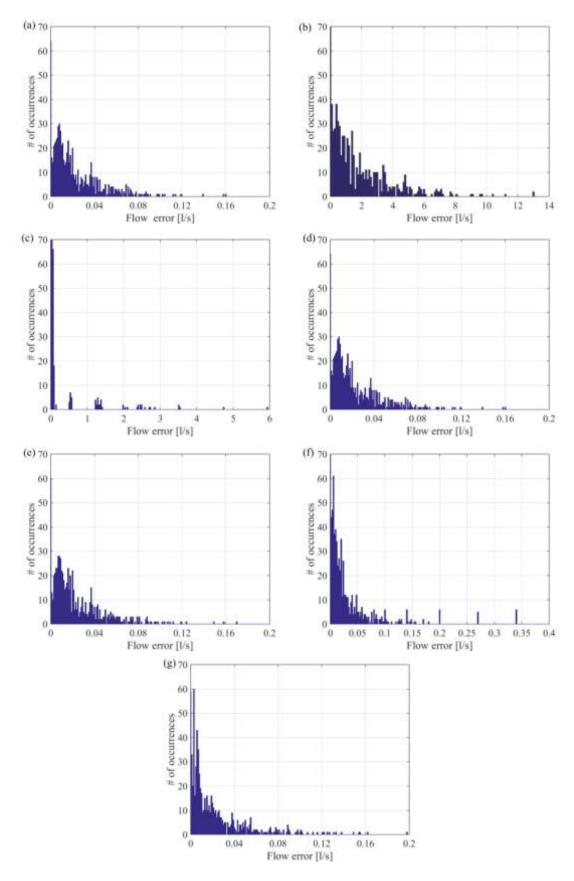
**Figure 2.** Sequences of devices characterizing the analyzed methods: (a) PRV – E (Bertola and Nicolini, 2006), (b) FCV –  $CV_0$  – R (Jinesh Babu and Mohan, 2012), (c) FCV –  $CV_L$  – R (Gorev and Kodzhespirova, 2013), (d) FCV –  $CV_0$  – E (Sayyed et al., 2014), (e)  $GPV_{W/T/F}$  –  $CV_0$  – R.



**Figure 3.** Pattern of  $Q/Q^{req}$  as a function of  $H_{ref}$  for the different methods analysed.



**Figure 4.** Head loss curve for the GPV, obtained using the relations of Wagner et al. (1988), Tucciarelli et al. (1999) and Fujiwara and Ganesharajah (1993).



**Figure 5.** Case Study III: Histogram of the absolute error occurring in comparison with the Pressure-Driven method of Alvisi and Franchini (2006) when discharge was computed with the methods: (a) PRV - E, (b)  $FCV - CV_0 - R$ , (c)  $FCV - CV_{ml} - R$ , (d)  $FCV - CV_0 - E$ , (e)  $GPV_W - CV_0 - R$ , (f)  $GPV_T - CV_0 - R$  and (g)  $GPV_F - CV_0 - R$ .