QoS-aware Admission Control and Resource Allocation for D2D Communications Underlaying Cellular Networks

Sergio Cicalò and Velio Tralli

***Abstract*— Device-to-Device (D2D) communications enable user equipments (UEs) in proximity to exchange information by taking advantage from high data-rate and low energy consumption. When D2D transmissions share the radio resources with the cellular UEs, efficient admission control (AC) and radio resource allocation (RRA) strategies play a key-role to control the co-channel interference and to allow QoS provision to UEs. This paper proposes a novel joint AC and RRA strategy that provides long-term QoS support to cellular and D2D communications. The AC algorithm derives the best set of cellular and D2D links by maximizing the revenues of the service provider under QoS constraints. The RRA algorithm assigns the available channels and transmit powers to admitted users on the short-term, in order to maximize an average weighted sum-rate under the same QoS constraints of the AC. Due to the NP-hard nature of the optimization problem, we propose an AC greedy algorithm that achieves near-optimal results for reasonable numbers of D2D links. Then, we propose a low-complexity RRA algorithm that decouples channel and power allocation. Numerical results show that the proposed joint AC and RRA strategy outperforms existing frameworks by increasing up to 40% the number of satisfied cellular and D2D links and by reducing energy consumption by more than 50%.**

***Index Terms*— D2D communications, radio resource allocation, admission control, cellular networks, QoS, OFDMA.**

I. Introduction

The ever-increasing proliferation of portable devices and the exponential growth of data-rate demanding communications for social applications increase the probability that user equipments (UEs) in proximity wish to communicate with each other [1]. In this scenario, direct device-to-device (D2D) transmission enables UEs to communicate with other proximate UEs by exploiting a four-fold gain: proximity gain, reuse gain, hop gain and pairing gain. It allows to reduce the cell load and the energy consumption, and may benefit from high data-rate and low end-to-end delay. Moreover, D2D transmissions underlaying a licensed cellular network may share the resources with the standard cellular UE (CUs), thereby improving the resource utilization and enhancing the cellular capacity. However, without efficient radio resource allocation (RRA) and admission control (AC) strategies, D2D communications may generate harmful interference to the standard CUs and *vice versa*, thereby compromising the quality of service (QoS) [2]–[23].

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To address this problem, several works in the literature have investigated QoS-aware optimized AC and RRA strategies for D2D communications underlaying cellular network (see also [2], and more recently [3], [4] and references therein). Most of them have mainly addressed RRA [8]–[16], [19]–[21], [23]. Few have jointly considered AC and RRA [3]–[7]. The Authors in [8] investigated the achievable sum-rate of hybrid cellular networks under three different transmission modes, *i.e.*, non-orthogonal sharing mode, orthogonal sharing mode and cellular mode. They showed that the use of non-orthogonal sharing mode provides the best performance. Similarly, the recent work in [9] focused on resource allocation and power control, under heterogeneous QoS requirements of the applications, with the aim to select the best resource sharing mode. However, the analyses in [8] and [9], as well as in [10], are limited to the simplified scenario with one CU and one D2D pair. In [6] and [7] short-term AC and RRA methods for multiple D2D communications underlaying cellular network have been considered. The Authors derived AC and power allocation to maximize the sum-rate of D2D links under instantaneous SNIR constraints for both D2D links and CUs showing that their proposed frameworks were outperforming other methods, in terms of D2D access rate and D2D throughput gain. More recently, [3] and [4] have jointly considered AC, mode selection and RRA to maximize both access rate and network throughput, under minimum signal-to-noise ratio constraint. However, AC and mode selection are performed as short-term processes, and [4] does not consider limits on the available spectrum.

The work in [11] proposed an efficient joint resource assignment and power control strategy to maximize the sum-rate of D2D communications while guaranteeing short-term QoS of CUs. This strategy provides superior performance with respect to similar existing methods. Other QoS-aware RRA strategies have been more recently proposed in [12]–[14]. However, all of them do not consider AC with QoS requirements for D2D links, thereby significantly limiting the D2D performance when the number of D2D pairs and/or the D2D distance increase. Among the recent works addressing link and power allocation with minimum-rate constraint on D2D links, [15] proposed a decentralized algorithm by modeling the system as a Stackelberger game, while [19] proposed a strategy based on optimal power allocation and graph-based subchannel matching. Distributed algorithms for channel and power allocation based on games have been also proposed and investigated in [17] and [18], without considering rate constraints. Moreover, [16] addressed the energy efficiency optimization with fairness and [20], [21] addressed the RRA with partial CSI by maximizing ergodic sum-rate with constraints on outage/success probability for each link. The RRA for multicast D2D communications underlaying the uplink of an LTE network has been addressed in [22]. With an alternative view, [23] proposed to minimize the maximum buffer size of each user by exploiting an LTE network based scheduler.

All the aforementioned works consider short-term QoS-aware strategies, which may fail to provide QoS in the long-term. However, the long-term QoS should be guaranteed for both CUs and D2D links, especially when the D2D links are the result of an off-loading strategy of the service provider [24].

In this paper, we propose a joint AC and RRA strategy for D2D communications underlaying cellular network that preserves the QoS in the long-term. The optimization problem for the AC jointly considers CUs and D2D pairs in order to derive the links admitted to the system that maximize the total revenue at the service provider under QoS and resource constraints. As a consequence of the NP-hard nature of this optimization problem, we also propose a greedy algorithm based on clustering and iterative linear programming (CILP) methods.

Moreover, we provide novel methods to optimize the short-term scheduling and power-allocation, by looking for the solutions of the weighted ergodic sum-rate maximization problem under average rate-constraints for both CUs and D2Ds in OFDMA networks. Due to the discrete nature of the allocation variables and the non-linear relationship between powers and rates, both objective and constraints of the problem reveal to be non-convex. Since finding the optimal solution of such NP-hard problem is computationally prohibitive, we derive a low-complexity sub-optimal algorithm which decouples power allocation and scheduling, and iteratively finds a provably-convergent sub-optimal solution. We also propose a novel and efficient method to evaluate the initialization point of the RRA algorithm, which greatly improves the final performance.

To the best of our knowledge, this is the first time that the AC and the RRA strategies, which include both scheduling and power allocation, have been investigated for long-term QoS support in D2D communications underlaying cellular networks.

Numerical results will show that the proposed low-complexity joint AC and RRA strategy is near-optimal for a reasonable number of D2D links. Furthermore, the proposed strategy outperforms existing frameworks in terms of QoS support and energy efficiency by increasing up to 40% the number of satisfied CUs and D2D links and by reducing energy consumption of more than 50%.

The next section presents the system model. The AC model and optimization are illustrated in Section III, whereas Section IV presents the proposed RRA framework. In section V we provide extensive numerical results and, finally, we draw our conclusions in Section VI.

Notation: Vectors and sets are denoted by bold and calligraphic fonts, respectively. The notation $\hat{x}, \tilde{x}, x^*$ denotes intermediate, approximated, and optimal solution of x. $\boldsymbol{x}^{\mathrm{T}}$ indicates

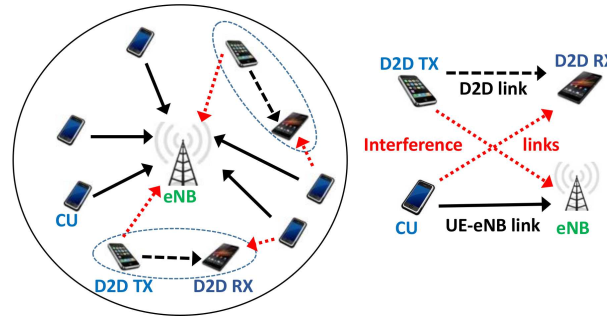


Fig. 1. System model.

transpose of the vector $\boldsymbol{x}$ and $|\mathcal{X}|$ indicates the cardinality of set $\mathcal{X}$. $\mathbb{E}[\cdot]$ denotes the statistical expectation. We use notation $[x]^+ = \max(x, 0)$. Given the vectors $\boldsymbol{x} = [x_1, \ldots, x_N]$, $\boldsymbol{x}' = [x'_1, \ldots, x'_N]$, we use the following element-wise inequality:

$$\boldsymbol{x} \succeq \boldsymbol{x}' \Leftrightarrow x_n \geq x'_n, \forall n = 1, \ldots, N$$

The most used symbols of this paper are summarized in Table I.

II. System Model

We consider a time-slotted uplink OFDMA cellular system. The total available bandwidth B is divided into S orthogonal subchannels indexed by the set $\mathcal{S} = \{1, \ldots, S\}$. Each subchannel has bandwidth ΔB (*e.g.*, 180 kHz in LTE). The time-frequency unit composed of one subchannel and one time-slot is defined as the resource block (RB), which is the elementary resource unit for RRA.

Multiple cellular UEs (CUs), indexed by the set $\mathcal{K}$ with $|\mathcal{K}| = K$, are uniformly distributed in a cell and wish to establish a long-term communication with the base station (BS). The CUs coexist with a set $\mathcal{D}$, with $|\mathcal{D}| = D$, of D2D pairs which establish a direct communication without going through the BS. Each D2D pair is assumed to be distributed in a cluster of radius ρ, and clusters are randomly located within the cell area, according to the clustered distributed model [7]. For each D2D pair $d \in \mathcal{D}$ there is a transmitter (TX) and a receiver (RX). The total number of links is then $L = K + D$ and we use $\mathcal{L} = \mathcal{K} \cup \mathcal{D}$ to denote the total link set.

Each link $l \in \mathcal{L}$ has its own QoS requirement, defined in terms of minimum average bit-rate q_l, with a correspondent revenue w_l for the service provider. In this work we consider a scenario where the cellular links have the highest priority and the network is overloaded, i.e., almost all the network resources have to be assigned to the admitted cellular links. In this scenario the D2D pairs can only be served in sharing mode with the CUs and some links could not be supported by the network due to the lack of resources to meet QoS requirements. Therefore, we assume that a RB can be allocated to either a CU in orthogonal mode or to a couple CU and D2D pair in sharing mode, thus with a possible cross-interference generation as depicted in the system model of Fig. 1. Although in principle a RB might be allocated to more than one D2D links in sharing mode, this option is often inconvenient due

Symbol	Definition
$k, \mathcal{K}$	index and set of CU links
$d, \mathcal{D}$	index and set of D2D links
$l, \mathcal{L}$	index and set of total links
$s, \mathcal{S}$	index and set of RBs
w_l, q_l	revenue (weight) and minimum rate of link l
p_l, P_l	allocated power and power budget of link l
$G_{u,i}$	long-term gain between TX of link u and RX of link i
$\Gamma_u^{(i)}$	long-term SNIR of link u interfered by link i
$c_u^{(i)}$	long-term rate per RB of link u interfered by link i
A', A''_u	Parameters of long-term rate model
α_k	portion of RBs allocated to CU k
$\beta_d^{(k)}$	portion of RBs allocated to D2D link d in sharing mode with CU k
x_k, z_d	binary AC variables of CU k and D2D pair d
$U(\cdot), \Psi(\cdot)$	Utility and cost function of AC
$g_{u,i,s}$	short-term gain between TX of link u and RX of link i on RB s
$\gamma_{u,s}^{(i)}$	SNIR of link u when interfered by link i on RB s
$e_u^{(i)}$	short-term rate per RB of link u interfered by link i
r_l	short-term ergodic rate of link l
Λ', Λ''	parameters of short-term rate model
$y_{k,s}^{(d)}$	allocation variables of CU k and D2D pair d on RB s

TABLE I

SUMMARY OF MOST USED SYMBOLS.

to the increased level of cross-interference. Hence, we do not consider this option in our model.

The aim of our work is to jointly derive efficient AC and RRA strategies that allow to maximize the number of both CU and D2D admitted links while satisfying their QoS requirements. Since we consider long-term QoS, we first analyze the AC problem in the long-term in the following section and then investigate a proper short-term RRA strategy that meets rate requirements in Section IV.

III. ADMISSION CONTROL

The objective of the AC is to select a suitable set of of CUs and D2D users that can be supported by the cellular network in order to maximize the total revenue under QoS, *i.e.*, long-term data-rate, and resource constraints.

One of the main issues is to characterize the resource requirement of each UE, *i.e.*, the expected number of RBs required to achieve the prescribed QoS, considering that the UEs experience heterogeneous short-term and long-term channel conditions. In fact, in OFDMA multi-user scenario, when CUs and D2D users are allowed to share the same RB, the long-term achievable data-rate is dependent on the statistical distribution and on the correlation properties of the short-term channel gain samples of both direct and interfering links, and on the underlaying short-term scheduler employed. This makes the resource requirement characterization hard to be derived.

To overcome this issue, state-of-the-art cellular AC, *e.g.*, [25], [26], as well as load balancing strategies [27], are generally built by considering an estimation of the achievable UE data-rate based on the long-term channel conditions, *i.e.*, obtained by averaging out the short-term dynamics of the channel. Here, we consider a similar approach based on the average channel conditions of direct and interfering links, by also introducing a suitable model to account for the multi-user diversity gain achievable through the underlaying RRA.

A. Rate Model for AC

Let $G_{u,j}$ be the *long-term* power gain between the transmitter of the link u and the receiver of link j, with $u, j \in \mathcal{L}$. We denote the average SNR of link u in orthogonal mode with $\Gamma_u^{(0)}$, and the average SINR of link u when it shares the channel with the link $i \in \mathcal{L}$ as $\Gamma_u^{(i)}$. In compact notation, they are given by

$$\Gamma_u^{(i)} = \begin{cases} \frac{G_{u,u} P_u}{\sigma^2} & \text{if } i = 0 \\ \frac{G_{u,u} P_u}{\sigma^2 + G_{i,u} P_i} & \text{otherwise} \end{cases} \tag{1}$$

where P_u is the power budget of UE u, and σ^2 is noise power. Accordingly, we model the average rate achievable over one RB by the link u, when it shares the channel with link i, as

$$c_u^{(i)} = A' \Delta B \log_2(1 + A''_u \Gamma_u^{(i)}) \tag{2}$$

where A' and A''_u are parameters of the model. More specifically, A''_u is used to account for the multi-user diversity gain of the user u, achieved through the actual RRA algorithm, which depends on the number of CUs and D2D pairs considered. In [28] the Authors have shown that the achievable multi-user diversity gain in OFDMA systems, when all the UEs have the same average SNR and share the channel without interference, is upper-bounded by the logarithm of the number of UEs sharing the orthogonal RBs in the system. Motivated by this result, we extend the use of (2) to also include the effects of interference by defining $A''_u = \varsigma \ln(K)$, if $u \in \mathcal{K}$ and $A''_u = \varsigma \ln(D)$ if $u \in \mathcal{D}$, where ς takes into account the impact of the possible interference on the multi-user diversity gain. The particular adaptive modulation and coding (AMC) adopted at the physical layer influences both the parameters A', A''_u. Note that the model in (2) is a rough, but simple, approximation of the ergodic achievable rate. The setup of the parameters A', ς and the accuracy of the model will be discussed in the first part of Section V.

Each subchannel may be shared by a couple of UEs (one CU and one D2D user), or may be alternatively allocated in orthogonal mode (one CU). We denote with α_k the fraction of RBs to be allocated to CU k, and with $\beta_d^{(k)}$ the fraction of RBs that a D2D link d shares with CU k. Here, the fraction of RBs is the average number of RBs over a sufficiently long time interval, divided by the length in time-slots of this interval. The following constraint must hold:

$$\sum_{d \in \mathcal{D}} \beta_d^{(k)} \leq \alpha_k, \quad \forall k \in \mathcal{K} \tag{3}$$

The average rate achievable by the CU k when it shares the RBs with a set of D2D links having $\beta_d^{(k)} > 0$ can be then estimated as

$$R_k(\alpha_k, \boldsymbol{\beta}) = \left(\alpha_k - \sum_{d \in \mathcal{D}} \beta_d^{(k)} \right) c_k^{(0)} + \sum_{d \in \mathcal{D}} \beta_d^{(k)} c_k^{(d)} \tag{4}$$

where $\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_D]$, with $\boldsymbol{\beta}_d = [\beta_d^{(1)}, \ldots, \beta_d^{(K)}]$, and $c_k^{(d)}$ is the average rate of link k when it shares one RB with link i. The first and second additive terms account for the rates achievable in orthogonal mode and in sharing mode, respectively. Given the values of $\boldsymbol{\beta}_d$, the rate achievable by the D2D link $d \in \mathcal{D}$ can be written as

$$R_d(\boldsymbol{\beta}_d) = \sum_{k \in \mathcal{K}} \beta_d^{(k)} c_d^{(k)}. \tag{5}$$

According to the proposed long-term rate model, in the following, we evaluate the minimum fraction of RBs required by both CUs and D2D pairs to achieve their minimum rates, and accordingly we formulate the optimization problem for the AC.

B. Optimization Problem for AC

We use the binary variables x_k and z_d to indicate whether (1) or not (0) the CU k and the D2D pair d are admitted in the system, respectively. The aggregate utility of the service provider is here evaluated as the sum of the revenues from each admitted UE, *i.e.*,

$$U(\boldsymbol{x}, \boldsymbol{z}) = \sum_{k \in \mathcal{K}} x_k w_k + \sum_{d \in \mathcal{D}} z_d w_d \tag{6}$$

where $\boldsymbol{x} = [x_1, \ldots, x_K]$ and $\boldsymbol{z} = [z_1, \ldots, z_D]$.

Since the aggregate utility defined in (6) only depends on the number of admitted UEs through the binary variables $\boldsymbol{x}, \boldsymbol{z}$, its maximum value is attained by searching for the optimum values of α_k and $\boldsymbol{\beta}_d$ satisfying $R_k(\alpha_k, \boldsymbol{\beta}) = q_k$ and $R_d(\boldsymbol{\beta}_d) = q_d$, by taking into account that α_k and $\beta_d^{(k)}$ are implicitly related to the allocation variables x_k and z_d, *i.e.*, $\sum_k \beta_d^{(k)} > 0$ if and only if $z_d = 1$, and $\sum_d \beta_d^{(k)} > 0$ if and only if $x_k = 1$.

The minimum cost for the admission of the CU k, *i.e.*, the fraction of the RBs $\alpha_k^{\min}$ required to achieve minimum rate q_k, thus $\alpha_k \geq \alpha_k^{\min}$, can be then derived by setting $R_k = q_k$ in (4), *i.e.*,

$$\alpha_k^{\min} = \frac{q_k}{c_k^{(0)}} + \sum_{d \in \mathcal{D}} \beta_d^{(k)} \left(1 - \frac{c_k^{(d)}}{c_k^{(0)}}\right) \tag{7}$$

which is function of the variables $\beta_d^{(k)}$ only. Note that the second additive term can be seen as the resource increment required by CU k to support resource sharing with a set of D2D links in order to still achieve its minimum rate. When $R_k = q_k$, the resource sharing constraint in (3) can be rewritten, after some simple algebra manipulation, as follows

$$\sum_{d \in \mathcal{D}} \beta_d^{(k)} \leq \alpha_k^{\min} \Rightarrow \sum_{d \in \mathcal{D}} \beta_d^{(k)} c_k^{(d)} \leq q_k, \forall k \in \mathcal{K}. \tag{8}$$

By exploiting eq. (7) we can evaluate the total amount of the RBs per time-slot required to support the set of CUs (given by $\boldsymbol{x}$) admitted at their minimum rate, when they share the resources with the D2D pairs (according to $\boldsymbol{\beta}$), as

$$\Psi(\boldsymbol{x}, \boldsymbol{\beta}) = \sum_{k \in \mathcal{K}} x_k \left[\frac{q_k}{c_k^{(0)}} + \sum_{d \in \mathcal{D}} \beta_d^{(k)} \left(1 - \frac{c_k^{(d)}}{c_k^{(0)}}\right)\right] \tag{9}$$

Within this framework, the optimization problem for the AC can be then stated as follows

$$\max_{\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\beta} \succeq \boldsymbol{0}} U(\boldsymbol{x}, \boldsymbol{z}) \tag{10a}$$

$$\text{s.t. } \Psi(\boldsymbol{x}, \boldsymbol{\beta}) \leq S \tag{10b}$$

$$\sum_{d \in \mathcal{D}} \beta_d^{(k)} c_k^{(d)} \leq x_k q_k, \qquad \forall k \in \mathcal{K} \tag{10c}$$

$$\sum_{k \in \mathcal{K}} \beta_d^{(k)} c_d^{(k)} = z_d q_d, \qquad \forall d \in \mathcal{D} \tag{10d}$$

$$x_k \in \{0, 1\}, \qquad \forall k \in \mathcal{K} \tag{10e}$$

$$z_d \in \{0, 1\}, \qquad \forall d \in \mathcal{D} \tag{10f}$$

Eq. (10b) is the wireless resource constraint, whereas (10c) describes the resource sharing constraints as in (8). Eq. (10d) defines the minimum rate constraints for the D2D links. Note that the constraints (10c) and (10d) also imply that $\beta_d^{(k)} = 0$, if at least one of the binary variables x_k, z_d is set to zero.

The problem in (10) is a mixed integer linear problem (MILP) and represents a variant of the well-known knapsack problem, which is NP-complete. Optimal solution can be attained via, *e.g.*, Branch & Bound (B&B) search and linear programming (LP) relaxation, whose complexity is exponential in the worst-case. As an example, in the test case with $S = 15$ RBs, $K = 40$ CUs, $D = 20$ D2D pairs and D2D cluster radius $\rho = 250$ m , the Gurobi solver [29] achieves in several cases an optimal solution with 10^6 node explorations of the B&B algorithm. Although the AC algorithm may work off-line and run on a time scale of several seconds, practical low-complexity algorithms with worst-case polynomial complexity should be considered. In the next section, we propose a low-complexity algorithm based on clustering of UEs and iterative LP (CILP), which is shown to be near-optimal for reasonable values of number of the D2D pairs and the radius of D2D cluster.

C. CILP Greedy Algorithm

In order to reduce the complexity of the optimal solution of the AC problem in (10), we propose a low-complexity greedy algorithm. The key-idea is to build clusters of UEs achieving the maximum value of a suitably defined objective function and to select them for admission in the cellular system until constraint (10b) holds. This objective function includes both the utility, *i.e.*, the total revenue of a UEs cluster, and the total cost, *i.e.*, the amount of RBs required to achieve the required QoS for each UE in the cluster.

A cluster $\mathcal{C} = \mathcal{K}' \cup \mathcal{D}'$ is defined as a subset of CUs and D2D pairs that includes at least one CU. For each choice of $\boldsymbol{\beta}$, its cost is given by $\psi_{\mathcal{C}}(\boldsymbol{\beta}) = \Psi(\boldsymbol{x}', \boldsymbol{\beta})$, while its objective function is defined as

$$U_{\mathcal{C}}(\boldsymbol{\beta}) = U(\boldsymbol{x}', \boldsymbol{z}') - F\Psi(\boldsymbol{x}', \boldsymbol{\beta}) \tag{11a}$$

$$= \sum_{k \in \mathcal{K}'} x'_k \left[u_k - F \sum_{d \in \mathcal{D}'} \beta_d^{(k)} \left(1 - \frac{c_k^{(d)}}{c_k^{(0)}}\right)\right] + \sum_{d \in \mathcal{D}'} w_d z'_d \tag{11b}$$

where $u_k = w_k - F q_k / c_k^{(0)}$ is the individual objective of the CU k. Moreover, $x'_k = 1$, $z'_d = 1$, if $k \in \mathcal{K}'$, $d \in \mathcal{D}'$, respectively, and $x'_k = 0$, $z'_d = 0$, $\beta_d^{(k)} = 0$, otherwise. The parameter F is a positive scaling factor.

A cluster $\mathcal{C}$ is said to be optimized if $\boldsymbol{\beta} = \boldsymbol{\beta}_{\mathcal{C}}^*$ such that the objective function achieves its maximum value $U_{\mathcal{C}}(\boldsymbol{\beta}_{\mathcal{C}}^*)$, given the constraints (10d) and (10c). The optimum $\boldsymbol{\beta}_{\mathcal{C}}^*$ is the solution of the following problem

$$\min_{\boldsymbol{\beta} \succeq \boldsymbol{0}} \sum_{k \in \mathcal{K}'} \sum_{d \in \mathcal{D}'} \beta_d^{(k)} \left(1 - \frac{c_k^{(d)}}{c_k^{(0)}}\right) \tag{12a}$$

$$\text{s.t. } \sum_{d \in \mathcal{D}'} \beta_d^{(k)} c_k^{(d)} \leq q_k, \qquad \forall k \in \mathcal{K}' \tag{12b}$$

$$\sum_{k\in\mathcal{K}'} \beta_d^{(k)} c_d^{(k)} \geq q_d, \qquad \forall d \in \mathcal{D}' \tag{12c}$$

if and only if (12) is feasible. This is a simple LP problem which can be optimally solved with polynomial complexity, *e.g.*, by using the interior-point or primal/dual-simplex methods. The resulting algorithm is also used to check the feasibility of the problem. If the problem is unfeasible, cluster $\mathcal{C}$ is said to be unfeasible. If the problem is feasible, it may happen that one CU $k \in \mathcal{K}'$ exists such that $\beta_d^{(k)} = 0, \forall d \in \mathcal{D}'$. This means that such CU k does not support resource sharing with the D2D links in $\mathcal{C}$. We finally define the minimal form $\bar{\mathcal{C}}$ of an optimized cluster $\mathcal{C}$ as the subset of the cluster which includes only the CUs that support resource sharing with D2D pairs in $\mathcal{C}$, i.e. $\bar{\mathcal{C}} = \mathcal{D}' \cup \{k \in \mathcal{K}' : \sum_{d\in\mathcal{D}'} \beta_d^{(k)} > 0\}$.

The basic steps of the CILP algorithm are illustrated at the end of the paragraph, after having defined $\mathcal{A}$ as the set of admitted links, $\Delta\psi_{\mathcal{C},\mathcal{A}} = \psi_{\mathcal{C}}(\boldsymbol{\beta}^*_{\mathcal{C}}) - \psi_{\mathcal{C}\cap\mathcal{A}}(\boldsymbol{\beta}^*_{\mathcal{A}})$ as the *marginal* cost of the cluster $\mathcal{C}$ with respect to $\mathcal{A}$, $\Delta U_{\mathcal{C},\mathcal{A}} = U_{\mathcal{C}}(\boldsymbol{\beta}^*_{\mathcal{C}}) - U_{\mathcal{C}\cap\mathcal{A}}(\boldsymbol{\beta}^*_{\mathcal{A}})$ as the *marginal* objective of the cluster $\mathcal{C}$ with respect to $\mathcal{A}$. The details can be found in Algorithm 1.

Step 1: Initialize $\mathcal{A}$ to $\{0\}$.

Step 2: Sort the CUs in the set $\mathcal{K}$ in decreasing order according to their individual objectives u_k. For each $k \in \mathcal{K}$ build the cluster $\mathcal{C}_k^{\text{o}}$ by using $\mathcal{K}' = \{k' = 1, \ldots, k\}$ and $\mathcal{D}' = \{0\}$.If its cost is larger than S set it as not-admissible.

Step 3: For each D2D pair d, build the cluster $\mathcal{C}_d^{\text{h}}$ by using $\mathcal{K}' = \mathcal{K}$ and $\mathcal{D}' = \{d\}$. If the optimized cluster of $\mathcal{C}_d^{\text{h}}$ exists, derive the minimal form $\bar{\mathcal{C}}_d^{\text{h}}$ of $\mathcal{C}_d^{\text{h}}$. If its cost is larger than S set it as not-admissible.

Step 4: For each admissible cluster $\bar{\mathcal{C}}_d^{\text{h}}$ build a new cluster $\bar{\mathcal{C}}_d^{\text{h}} \cup \mathcal{A}$: if its cost is larger than S, then mark the cluster $\bar{\mathcal{C}}_d^{\text{h}}$ as not-admissible. Compute the *marginal* cost and the *marginal* objective of all the admissible clusters $\mathcal{C}_k^{\text{o}}$ and $\bar{\mathcal{C}}_d^{\text{h}}$ with respect to $\mathcal{A}$.

Step 5: Find the cluster $\bar{\mathcal{C}}_{d^*}^{\text{h}}$ with minimum *marginal* cost among the admissible clusters $\bar{\mathcal{C}}_d^{\text{h}}$. If some admissible clusters $\mathcal{C}_k^{\text{o}}$ exist that have both a larger *marginal* objective and a smaller *marginal* cost than those of $\mathcal{C}_{d^*}^{\text{h}}$, then select the cluster $\mathcal{C}_{k^*}^{\text{o}}$ with minimum k, include it in $\mathcal{A}$ and set the clusters $\mathcal{C}_k^{\text{o}}$, $k \leq k^*$ as not-admissible. Otherwise, if $\bar{\mathcal{C}}_{d^*}^{\text{h}} \cup \mathcal{A}$ is feasible, include cluster $\bar{\mathcal{C}}_{d^*}^{\text{h}}$ in $\mathcal{A}$ and set it as not-admissible.

Step 6: Go to *step 4* if at least one admissible cluster $\bar{\mathcal{C}}_d^{\text{h}}$ non yet included in $\mathcal{A}$ still exists.

Step 7: Iteratively consider the single (ordered) not yet admitted CUs $k' \notin \mathcal{A}$. If the cluster $\{k'\} \cup \mathcal{A}$ is not unfeasible and its cost, when optimized, does not exceed S, then include k' in $\mathcal{A}$.

At the end of the algorithm, the resulting set of admitted links is denoted with $\mathcal{L}^*$ and the variables x_k, z_d are set to one for each CU k and D2D pair d belonging to $\mathcal{L}^*$.

1) Complexity Analysis: In the worst-case the CILP Algorithm solves $2D$ LP problems: the first D problems involve K variables and $K+1$ constraints (*step 4*), whereas the latter D LP problems (*step* 5) involve iK variables and $K+i$ constraints, with $i = 1, \ldots, D$. By neglecting the fixed number of products and the low order terms, the complexity is $\mathcal{O}(DKI_{\text{mjr}})$, where I_{mjr} is the number of iterations required to solve each LP problem. By considering the problem (12) in canonical form with matrix notation, it is straightforward to show that the $(D' + K') \times (D'K')$ matrix of the constraints results to be sparse. Hence, each LP optimal solution is achieved with few iterations in all the investigated cases.

Algorithm 1 CILP Algorithm

Define $\boldsymbol{h}^{(d)} = [h_1^{(d)}, \ldots, h_K(d)]$; $h_k(d) = 1$ if $\beta_d^{(k)} > 0$; 0 otherwise; $\Phi(x) = 1$, if $x > 0$; $\Phi(x) = 0$, otherwise
Initialize $\boldsymbol{x}^* = \boldsymbol{0}$, $\boldsymbol{z}^* = \boldsymbol{0}$, $\boldsymbol{z}' = \boldsymbol{0}$, $\mathcal{A} = \{0\}$, $\mathcal{D}_{\text{temp}} = \{0\}$
Sort the set $\mathcal{K}$ according to $w_k - Fq_k/c_k^{(0)}$
for all $k \in \mathcal{K}$: **do**
  Build $\mathcal{C}_k^{\text{o}}$ with $\mathcal{K}' = \{k' = 1, \ldots, k\}$, $\mathcal{D}' = \{0\}$.
end for
for all $d \in \mathcal{D}$ **do**
  Build $\mathcal{C}_d^{\text{h}}$ with $\mathcal{K}' = \mathcal{K}$, $\mathcal{D}' = \{d\}$
  if Problem feasible & $\Psi(\boldsymbol{h}^{(d)}, \boldsymbol{\beta}) \leq S$ **then**
    $\mathcal{D}_{\text{temp}} \leftarrow \mathcal{D}_{\text{temp}} \cup \{d\}$; $z'_d = 1$
    $\psi_d = \Psi(\boldsymbol{h}^{(d)}, \boldsymbol{\beta})$
    $\Delta U_d = U(\boldsymbol{h}^{(d)}, \boldsymbol{z}') - F\psi_d$
  end if
end for
repeat
  $d^* = \text{argmin}_{d\in\mathcal{D}'} \ \psi_d$
  $A = 0$; $B = 0$; $x''_k = 0, \ \forall k \in \mathcal{K}$
  for all $k \in \mathcal{K} : x_k^* = 0$ **do**
    $A \leftarrow A + q_k/c_k^{(0)}$; $B \leftarrow B + w_k - Fq_k/c_k^{(0)}$;
    $x''_k = 1$;
    if $A < \psi_{d^*}$ & $B \geq \Delta U_{d^*}$ **then**
      $x_k^* = x''_k, \ \forall k \in \mathcal{K}$;
      break
    else
      Solve (12) with $\mathcal{A} \cup \{d^*\}$, to get $\boldsymbol{\beta}$,
      $x'_k = x_k^* \vee \Phi(\sum_d \beta_d^{(k)}), \ \forall k \in \mathcal{K}$
      if Problem Feasible & $\Psi(\boldsymbol{x}', \boldsymbol{\beta}) \leq S$ **then**
        $\mathcal{D}^* \leftarrow \mathcal{D}^* \cup \{d^*\}$;
        $\boldsymbol{x}^* = \boldsymbol{x}'$; $\boldsymbol{\beta}^* = \boldsymbol{\beta}$; $z_d^* = 1$; .
      end if
      $\mathcal{D}_{\text{temp}} \leftarrow \mathcal{D}_{\text{temp}} \setminus \{d^*\}$
    end if
  end for
  for all $d \in \mathcal{D}_{\text{temp}}$ **do**
    for all $k \in \mathcal{K} : x_k^* = h_k^{(d)}$ **do**
      $\Delta U_d \leftarrow (\Delta U_d - w_k) - F(\psi_d - q_k/c_k^{(0)})$
      $\psi_d \leftarrow \psi_d - q_k/c_k^{(0)}$
    end for
    if $\Psi(\boldsymbol{x}^*, \boldsymbol{\beta}^*) + \psi_d > S$ **then**
      $\mathcal{D}_{\text{temp}} \leftarrow \mathcal{D}_{\text{temp}} \setminus \{d\}$.
    end if
  end for
until $\mathcal{D}_{\text{temp}} \neq \{0\}$
for all $k \in \mathcal{K} : x_k^* = 0$ **do**
  if $\Psi(\boldsymbol{x}^*, \boldsymbol{\beta}^*) + q_k/c_k^{(0)} \leq S$ **then**
    $x_k^* = 1$
  end if
end for

IV. Radio Resource Allocation and Scheduling

Given the set of admitted links $\mathcal{L}^*$, *i.e.*, the union of the sets of admitted CUs $\mathcal{K}^*$ and D2D pairs $\mathcal{D}^*$, resulting from the AC algorithms proposed in the previous section, the aim of the RRA strategy is now to allocate each RB to a CU in orthogonal mode or to a couple CU - D2D pair in sharing mode, and the UE's power budget to the different subchannels, in the *short-term* time scale, i.e., slot by slot. We use time index n to indicate time-slot sequence. The objective considered here is to maximize the weighted sum of short-term averaged (*i.e.*, ergodic) rates under rate constraints.

Let $y_{k,s}^{(d)}[n] \in \{0,1\}$ be the binary variable that indicates whether or not the subchannel s at time slot n is allocated to CU $k \in \mathcal{K}^*$ and the D2D link $d \in \mathcal{D}_0^* = \mathcal{D}^* \cup \{0\}$, where $d = 0$ indicates orthogonal subchannel assignment to CU k. We denote with $g_{u,i,s}[n]$ the *short-term* random power gain, including fading and path-loss components, between the TX of the link $u \in \mathcal{L}^*$ and the RX of link $i \in \mathcal{L}^*$, on subchannel s and time-slot n. Similarly to eq. (1), we define the *short-term* SINR of link u when it shares the channel with link i, given $u, i \in \mathcal{L}^* \cup \{0\}$, as follows

$$\gamma_{u,s}^{(i)}(p_{u,s}, p_{i,s})[n] = \begin{cases} 0 & \text{if } u = 0 \\ \frac{g_{u,u,s}[n]p_{u,s}[n]}{\sigma^2} & \text{if } i = 0,\ u \neq 0 \\ \frac{g_{u,u,s}[n]p_{u,s}[n]}{\sigma^2 + g_{i,u,s}[n]p_{i,s}[n]} & \text{otherwise} \end{cases} \tag{13}$$

where $p_{u,s}[n]$, $u \neq 0$, denotes the power allocated to the TX of link u on subchannel s and time slot n. The instantaneous (short-term) achievable rate is modeled as

$$e_{u,s}^{(i)}(p_{u,s}, p_{i,s})[n] = \Lambda' \Delta B \log_2(1 + \gamma_{u,s}^{(i)}(p_{u,s}, p_{i,s})[n]/\Lambda'') \tag{14}$$

where Λ', Λ'' are two parameters, namely the rate adjustment and the SNR-gap, respectively, depending on the specific AMC scheme adopted. Note that the parameter Λ'' is different from Λ''_u used in (2).

To summarize, given the set $\boldsymbol{g} = \{g_{u,i,s}[n], u, i \in \mathcal{L}^*, s \in \mathcal{S}\}$ of the $S[2KD+K+D]$ realizations of the random channel gains, the RRA algorithm at the BS determines the sets of allocation variables $\boldsymbol{y} = \{y_{k,s}^{(d)}[n], k \in \mathcal{K}^*, d \in \mathcal{D}_0^*, s \in \mathcal{S}, \forall n\}$ and powers $\boldsymbol{p} = \{p_{u,s}[n], u \in \mathcal{L}^*, s \in \mathcal{S}, \forall n\}$ as functions of the channel realizations $\boldsymbol{g}$.

The ergodic rate achievable by the link $l \in \mathcal{L}^*$ is then defined by $r_l(\boldsymbol{y}, \boldsymbol{p}) = \mathbb{E}[E_l(\boldsymbol{y}, \boldsymbol{p})[n]]$, where $E_l(\boldsymbol{y}, \boldsymbol{p})[n]$ is the instantaneous rate given by

$$E_l(\boldsymbol{y}, \boldsymbol{p})[n] = \begin{cases} \sum\limits_{s \in \mathcal{S}} \sum\limits_{d \in \mathcal{D}_0^*} y_{l,s}^{(d)}[n] e_{l,s}^{(d)}(p_{l,s}, p_{d,s})[n] & \text{if } l \in \mathcal{K}^* \\ \sum\limits_{s \in \mathcal{S}} \sum\limits_{k \in \mathcal{K}^*} y_{k,s}^{(l)}[n] e_{l,s}^{(k)}(p_{l,s}, p_{k,s})[n] & \text{if } l \in \mathcal{D}^* \end{cases} \tag{15}$$

The ergodic weighted sum-rate maximization problem under rate constraints is stated as follows

$$\max_{\boldsymbol{y}, \boldsymbol{p}} \left[\sum_{k \in \mathcal{K}^*} w_k r_k(\boldsymbol{y}, \boldsymbol{p}) + \sum_{d \in \mathcal{D}^*} w_d r_d(\boldsymbol{y}, \boldsymbol{p}) \right] \tag{16a}$$

$$\text{s.t. } r_l(\boldsymbol{y}, \boldsymbol{p}) \geq q_l, \qquad \forall l \in \mathcal{L}^* \tag{16b}$$

$$\sum_{k \in \mathcal{K}^*} \sum_{d \in \mathcal{D}_0^*} y_{k,s}^{(d)}[n] \leq 1,\ \forall s \in \mathcal{S} \tag{16c}$$

$$y_{k,s}^{(d)}[n] \in \{0,1\}, \qquad \forall k \in \mathcal{K}^*, \forall d \in \mathcal{D}_0^*, \forall s \in \mathcal{S} \tag{16d}$$

$$\sum_{s \in \mathcal{S}} p_{l,s}[n] \leq P_l, \qquad \forall l \in \mathcal{L}^* \tag{16e}$$

$$p_{l,s}[n] \geq 0, \qquad \forall l \in \mathcal{L}^*, \forall s \in \mathcal{S} \tag{16f}$$

where (16b) is the minimum average rate constraint for all the admitted links. Constraints (16c) and (16d) indicate exclusive RB allocation to a single CU or to a couple of CU and D2D links, whereas constraint (16e) limits the power allocated to each link to the maximum budget of the related transmitter. Finally, (16f) dictates a non-negative power allocation.

The feasibility of problem (16) is not guaranteed if the set $\mathcal{L}^*$ of admitted links is too large with respect to the available resources, the channel conditions and the QoS constraints. This event might happen when the rate model in (2) overestimates the long-term rate achievable by each link over one RB. The behavior of the proposed algorithms in this case will be discussed in the next subsection.

The ergodic RRA and scheduling problem in (16) is a combinatorial non-convex problem due to the discrete nature of the variables in $\boldsymbol{y}$ and the non-linear expression of the power-dependent SINR in eq. (13) which appears in both objective (16a) and constraints (16b). Finding optimal solution of such NP-hard problem is computationally prohibitive. In order to derive a low-complexity optimized RRA we propose a framework which decouples the power allocation and the scheduling problems, and iteratively find a provably-convergent sub-optimal solution. The algorithm is inspired by the methods proposed in [30] and extended in [31] to solve the RRA problem in the downlink of multi-cell OFDMA systems. However, several modification are introduced here to adapt the methods to our problem. Moreover, compared to [31], we introduce a novel and efficient method to evaluate the RRA initialization, which greatly impacts the final performance. In the next subsections, we provide efficient solutions for the decoupled scheduling and power-allocation sub-problems. We first illustrate the solution to the RRA problem when the set of powers is fixed to constant values (no power allocation), then we extend it to include power allocation.

A. Scheduling with Fixed Power

In this subsection we consider the scheduling sub-problem obtained when the values of $\boldsymbol{p}$ are fixed to given feasible values $\hat{\boldsymbol{p}}$, *i.e.*, the problem described by the equations (16a)-(16d) with $\boldsymbol{p} = \hat{\boldsymbol{p}}$. Due to the binary variables in $\boldsymbol{y}$ this subproblem is still a combinatorial non-convex optimization problem. Nevertheless, motivated by the "zero duality gap" result of ergodic RRA in OFDMA systems [32], we solve the problem through Lagrangian dual decomposition by applying a continuos relaxation of the variables in $\boldsymbol{y}$.

By defining the relaxed allocation variables as $\tilde{\boldsymbol{y}} = \{\tilde{y}_{k,s}^{(d)}[n], k \in \mathcal{K}^*, d \in \mathcal{D}_0^*, s \in \mathcal{S}, \forall n\}$, with $0 \leq \tilde{y}_{k,s}^{(d)}[n] \leq 1$, the relaxed version of problem (16a)-(16d) can be then written as:

$$\max_{\tilde{\boldsymbol{y}}} \left[\sum_{k \in \mathcal{K}^*} w_k r_k(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) + \sum_{d \in \mathcal{D}^*} w_d r_d(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) \right] \tag{17a}$$

$$\text{s.t. } r_l(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) \geq q_l, \qquad \forall l \in \mathcal{L}^* \tag{17b}$$

$$\sum_{k \in \mathcal{K}^*} \sum_{d \in \mathcal{D}_0^*} \tilde{y}_{k,s}^{(d)}[n] \leq 1, \quad \forall s \in \mathcal{S} \tag{17c}$$

$$0 \leq \tilde{y}_{k,s}^{(d)}[n] \leq 1 \quad \forall k \in \mathcal{K}^*, \forall d \in \mathcal{D}_0^*, \forall s \in \mathcal{S} \tag{17d}$$

which is a convex optimization problem [32].

Let $L(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}; \boldsymbol{\mu})$ be the Lagrangian function, and $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_{L^*}]$ be the dual vector related to the constraint (17b). The dual problem becomes

$$\min_{\boldsymbol{\mu}} \Theta(\boldsymbol{\mu}) \quad \text{s.t. } \boldsymbol{\mu} \succeq \mathbf{0} \tag{18}$$

where

$$\Theta(\boldsymbol{\mu}) = \max_{\tilde{\boldsymbol{y}}} L(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}; \boldsymbol{\mu}) \tag{19a}$$

$$= \max_{\tilde{\boldsymbol{y}}} \left[\sum_{k \in \mathcal{K}^*} \pi_k r_k(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) + \sum_{d \in \mathcal{D}^*} \pi_d r_d(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) \right] \tag{19b}$$

is the dual objective, under the constraints in (17c), (17d), and $\pi_l = (w_l + \mu_l)$, $\forall l \in \mathcal{L}^*$. For any feasible solution of $\boldsymbol{\mu}^*$, the solution of the dual objective is the solution, time slot by time slot, of the instantaneous weighted sum-rate (WSR) maximization:

$$\max_{\tilde{\boldsymbol{y}}} g(\tilde{\boldsymbol{y}}, \hat{\boldsymbol{p}}) \tag{20a}$$

$$\text{s.t.} \sum_{k \in \mathcal{K}^*} \sum_{d \in \mathcal{D}_0^*} \tilde{y}_{k,s}^{(d)} \leq 1, \ \forall s \in \mathcal{S} \tag{20b}$$

$$0 \leq \tilde{y}_{k,s}^{(d)} \leq 1 \qquad \forall k \in \mathcal{K}^*, \forall d \in \mathcal{D}_0^*, \forall s \in \mathcal{S} \tag{20c}$$

where the time n is omitted for the sake of simplicity and

$$g(\tilde{\boldsymbol{y}}, \boldsymbol{p}) \tag{21a}$$

$$= \sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}^*} \sum_{d \in \mathcal{D}_0^*} \tilde{y}_{k,s}^{(d)} \left[\pi_k e_{k,s}^{(d)}(p_{k,s}, p_{d,s}) + \pi_d e_{d,s}^{(k)}(p_{d,s}, p_{k,s}) \right] \tag{21b}$$

with π_0 defined as 0.

Problem (20) corresponds to the OFDMA single-cell WSR maximization problem, with $(D+1)K$ users, investigated in [32]. Hence, the almost-sure optimal scheduling solution is given by a "*winner-takes-all*" strategy [32], where here the "*winner*" may be either a CU or a couple of CU and D2D. At each time slot n, the subchannel s is assigned to a CU k_s^* and a D2D pair $d_s^* > 0$, or exclusively to a CU k_s^* if $d_s^* = 0$, according to the following rule:

$$(k_s^*, d_s^*) = \underset{k \in \mathcal{K}^*, d \in \mathcal{D}_0^*}{\operatorname{argmax}} \left[\pi_k e_{k,s}^{(d)}(\hat{p}_{k,s}, \hat{p}_{d,s}) + \pi_d e_{d,s}^{(k)}(\hat{p}_{d,s}, \hat{p}_{k,s}) \right] \tag{22}$$

Since the dual problem in (18) is in general not tractable analytically, an iterative sub-gradient method as in [33] can be used to solve it. However, in realistic applications, the adaptive implementation is suggested, where the dual variables are updated at each time slot as

$$\mu_l[n+1] = \left[\mu_l[n] - \delta \big(E_l(\boldsymbol{y}^*, \hat{\boldsymbol{p}})[n] - q_l \big) \right]^+ \tag{23}$$

where $E_l(\boldsymbol{y}^*, \hat{\boldsymbol{p}})[n]$ is the instantaneous rate evaluated as in (15) with $\boldsymbol{y}^* = \{y_{k,s}^{*(d)}[n], k \in \mathcal{K}^*, d \in \mathcal{D}_0^*, s \in \mathcal{S}, \forall n\}$ such that $y_{k,s}^{*(d)}[n] = 1$ if $k = k_s^*[n]$, $d = d_s^*[n]$, and $y_{k,s}^{*(d)}[n] = 0$ otherwise, and δ is a constant step-size selected to ensure convergence [34].

It should be remarked that when the set of admitted users is too wide with respect to available system capacity, thus making the problem in (17) infeasible, the adaptive algorithm in (23) does not converge. However, in this case, the convergence to a result that does not satisfy the QoS constraints in (16b) for some links can be forced by simply setting an upper limit to all the dual variables in μ. This event should be easily detected and this information may be suitably exploited to update the set of admitted links.

B. Joint Scheduling and Power Allocation

Here we come back to the combinatorial non-convex problem in (16) and try to build an algorithmic solution by extending the methods used to derive optimal scheduling with fixed power (illustrated in Sect. IV-A) in order to include power allocation. This solution will be in general suboptimal.

As for scheduling with fixed power, we apply Lagrangian dual decomposition and continuous relaxation of $\boldsymbol{y}$ into $\tilde{\boldsymbol{y}}$. The new problem can be formulated as in (17) by leaving both $\boldsymbol{p}$ and $\tilde{\boldsymbol{y}}$ as optimization variables and by also including the constraints (16e) and (16f). The dual problem becomes as in (18) and its solution can be still obtained through stochastic algorithm in (23). Within this framework, the solution of the dual objective is, slot by slot, given by

$$\max_{\tilde{\boldsymbol{y}}, \boldsymbol{p}} g(\tilde{\boldsymbol{y}}, \boldsymbol{p}) \tag{24a}$$

$$\text{s.t.} \sum_{k \in \mathcal{K}^*} \sum_{d \in \mathcal{D}_0^*} \tilde{y}_{k,s}^{(d)} \leq 1, \ \forall s \in \mathcal{S} \tag{24b}$$

$$0 \leq \tilde{y}_{k,s}^{(d)} \leq 1 \qquad \forall k \in \mathcal{K}^*, \forall d \in \mathcal{D}_0^*, \forall s \in \mathcal{S} \tag{24c}$$

$$\sum_{s \in \mathcal{S}} p_{l,s} \leq P_l, \qquad \forall l \in \mathcal{L}^* \tag{24d}$$

$$p_{l,s} \geq 0, \qquad \forall l \in \mathcal{L}^*, \forall s \in \mathcal{S} \tag{24e}$$

where time n is omitted for the sake of simplicity.

The problem (24) replaces problem (20) in this new case. It is not convex and has a structure similar to that of the optimization problem for coordinated multicell downlink scenario in [31] where the total power constraint is per base station instead of per link. A suboptimal solution can be obtained with an algorithm that iteratively plays between two suitably defined decoupled subproblems. The algorithm updates the solutions of a first subproblem, *i.e.*, the scheduling subproblem, by using the solutions of a second subproblem, *i.e.*, the power allocation subproblem, and viceversa. The scheduling subproblem in our case is the one obtained from (24) by fixing the power values to given feasible fixed values $\hat{\boldsymbol{p}}$, which is exactly the problem in (20), with solution in (22). In the same way, the power allocation sub-problem is the one obtained from (24) by fixing the scheduling variables to any given feasible set $\tilde{\boldsymbol{y}} = \hat{\boldsymbol{y}}$, whose elements must be in $\{0, 1\}$ as stated in (22), *i.e.*,

$$\max_{\boldsymbol{p} \succeq \mathbf{0}} g(\hat{\boldsymbol{y}}, \boldsymbol{p}) \tag{25a}$$

$$\text{s.t.} \sum_{s \in \mathcal{S}} p_{l,s} \leq P_l, \qquad \forall l \in \mathcal{L}^* \tag{25b}$$

The sub-problem (25) is still non-convex. A similar problem was investigated in [30], [35]. By following the methods proposed in [30], the solution of (25) can be approximated by exploiting a tight lower-bound of the logarithmic function in the objective (25a), which allows to build a successive convex approximation procedure, also known as SCALE. Here, we briefly retrace the key steps of the procedure in [30], with a formulation coherent to our proposed model.

For any $\xi \geq 0$, $\bar{\xi} \geq 0$, the following inequality holds

$$\log_2(1+\xi) \geq a \log_2(\xi) + b \tag{26}$$

$$a = \frac{\bar{\xi}}{1+\bar{\xi}}, \quad b = \log_2(1+\bar{\xi}) - a \log_2(\bar{\xi}). \tag{27}$$

By exploiting it and the transformation $\boldsymbol{p} = e^{\boldsymbol{p}'}$, we can write

$$g(\hat{\boldsymbol{y}}, \boldsymbol{p}) \geq \bar{g}(\hat{\boldsymbol{y}}, \boldsymbol{p}') \tag{28a}$$

$$= \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}^*} \sum_{d\in\mathcal{D}_0^*} \hat{y}_{k,s}^{(d)} \left[\pi_k \bar{e}_{k,s}^{(d)}(p'_{k,s}, p'_{d,s}) + \pi_d \bar{e}_{d,s}^{(k)}(p'_{d,s}, p'_{k,s})\right] \tag{28b}$$

where

$$\bar{e}_{u,s}^{(i)}(p'_{u,s}, p'_{i,s}) = \Delta B \Lambda' a_{u,s}^{(i)} \log_2(\gamma_{u,s}^{(i)}(e^{p'_{u,s}}, e^{p'_{i,s}})/\Lambda'') + b_{u,s}^{(i)} \tag{29}$$

and $a_{u,s}^{(i)}$ and $b_{u,s}^{(i)}$ are parameters evaluated as in (27) by using the values of SINR $\bar{\xi}_{u,s}^{(i)}$ resulting from the preceding iteration. These values can be evaluated through the set of powers $\hat{\boldsymbol{p}}$ resulting from the preceding iteration, also used in (22) to update $\boldsymbol{y}$, as

$$\bar{\xi}_{u,s}^{(i)} = \gamma_{u,s}^{(i)}(\hat{p}_{u,s}, \hat{p}_{i,s}), \ \ \forall u, i \in \mathcal{L}^*, s \in \mathcal{S} \tag{30}$$

Thus, the new transformed power allocation problem becomes

$$\max_{\boldsymbol{p}'} \bar{g}(\hat{\boldsymbol{y}}, \boldsymbol{p}') \tag{31a}$$

$$\text{s.t.} \sum_{s\in\mathcal{S}} e^{p'_{l,s}} \leq P_l, \quad \forall l \in \mathcal{L}^* \tag{31b}$$

Note that the problem (31) is a standard concave maximization. In Appendix I we provide a low-complexity method to optimally solve it by using the Lagrangian dual decomposition method. To summarize, if we denote with $\boldsymbol{p}'^*$ the solution of problem (31), the algorithm that solves problem (24) iterates the following steps:

- given the set $\hat{\boldsymbol{y}}$, find $\boldsymbol{p}'^*$ solution of (31)
- update $\hat{\boldsymbol{p}} = e^{\boldsymbol{p}'^*}$
- given the set $\hat{\boldsymbol{p}}$, find $\boldsymbol{y}^*$ as in (22)
- update $\hat{\boldsymbol{y}} = \boldsymbol{y}^*$
- update $\xi_{u,s}^{(i)}$ as in (30), update $a_{u,s}^{(i)}$ and $b_{u,s}^{(i)}$ as in (27).

We finally observe that the solution of the ergodic WSR optimization in (16) requires two layers of iterations. In the outer layer, at each step, the problem (24) is solved and the dual variables $\boldsymbol{\mu}$ are updated as in (23). This can be used as adaptive loop along time, also able to track long-term channel variations. In the inner layer, the two problems (20), (31) are solved and both scheduling and power allocation variables are updated until convergence. We name the algorithm ED2D-SCALE. The convergence of the ED2D-SCALE algorithm can be easily proved as extension of [31]. The next issue is how to derive an initial starting point for the iterative procedure in the inner layer, which is the subject of the next subsection.

C. Improved RRA initialization: SAA-SLM

As pointed out in [31], the iterative algorithm provides a suboptimal solution, e.g. a local maximum, of problem (24), whose values depend on the initialization used. It is shown there that an enhanced power initialization strategy based on per-tone binary power selection and uniform power allocation among active tones allows to significantly improve the performance of the SCALE algorithm. However, due to the different nature of the downlink considered in [31] with respect to the uplink where the power budget is per user and each subchannel can only be shared by one CU and one D2D pair, this technique leads to a poor performance. Note that, since user-based binary power selection in the uplink is already inside power allocation mechanism, the initialization method of [31] in the uplink is just uniform power allocation among tones. Therefore, we propose here a novel initialization method which exhibits better performance as shown in Sect. V. It is based on two steps, *i.e.*, an initial subchannel amount assignment (SAA) and a subsequent subchannel link matching (SLM) with uniform power allocation.

1) Subchannel amount assignment (SAA): To derive an efficient low-complexity initialization procedure we first simplify the general problem in (24) by assuming that: (i) direct and interfering channel gains are constant over the set of subchannels at their average values given by $\tilde{g}_{u,i} = \frac{1}{S}\sum_{s\in\mathcal{S}} g_{u,i,s}, \forall u, i \in \mathcal{L}^*$; (ii) a constant power budget is first assigned to each set of RBs shared by a CU and a D2D pair, which is denoted by $\tilde{P}_k$ for CU $k \in \mathcal{K}^*$ and by $\tilde{P}_d$ for D2D pair $d \in \mathcal{D}^*$, then the assigned power budget is uniformly distributed among the RBs in the set (the same power budget $\tilde{P}_k$ is also assigned to each set of RBs used by CU k in orthogonal mode). According to these approximations, it is straightforward to show that the RRA problem in (24) collapses to the problem of finding the optimal number of RBs allocated to each link with uniform power distribution among them.

In fact, let $N_k^{(d)}$ be the fractional number $0 \leq N_k^{(d)} \leq S$ of RBs allocated to CU k and shared with the D2D pair d, where, once again, $d = 0$ indicates orthogonal CU allocation. By following the approximation, we can rewrite the achievable instantaneous rate $\rho_k^{(d)}(N_k^{(d)})$ of link $k \in \mathcal{K}^*$, when it shares $N_k^{(d)}$ RBs with link $d \in \mathcal{D}^*$, and the achievable instantaneous rate $\rho_d^{(k)}(N_k^{(d)})$ of link $d \in \mathcal{D}^*$, when it shares $N_k^{(d)}$ RBs with link $k \in \mathcal{K}^*$, as follows:

$$\rho_k^{(d)}(N_k^{(d)}) = \begin{cases} N_k^{(d)} f\left(\frac{\tilde{g}_{k,k}\tilde{P}_k}{\sigma^2 N_k^{(d)}}\right), & d = 0 \\ N_k^{(d)} f\left(\frac{\frac{\tilde{g}_{k,k}\tilde{P}_k}{N_k^{(d)}}}{\sigma^2 + \frac{\tilde{g}_{d,k}\tilde{P}_d}{N_k^{(d)}}}\right), & d \in \mathcal{D}^* \end{cases} \tag{32a}$$

$$\rho_d^{(k)}(N_k^{(d)}) = N_k^{(d)} f\left(\frac{\frac{\tilde{g}_{d,d}\tilde{P}_d}{N_k^{(d)}}}{\sigma^2 + \frac{\tilde{g}_{k,d}\tilde{P}_k}{N_k^{(d)}}}\right) \tag{32b}$$

where $\boldsymbol{N} = \{N_k^{(d)}, \ k \in \mathcal{K}^*, \ d \in \mathcal{D}_0^*\}$ and $f(x) = \Lambda'\Delta B \log_2(1 + x/\Lambda'')$ is defined in (14). The approximated

RRA problem can be then rewritten as follows:

$$\max_{\boldsymbol{N}} \sum_{k\in\mathcal{K}^*}\sum_{d\in\mathcal{D}_0^*}\left[\pi_k\rho_k^{(d)}(N_k^{(d)})+\pi_d\rho_d^{(k)}(N_k^{(d)})\right] \tag{33a}$$

$$s.t. \sum_{k\in\mathcal{K}^*}\sum_{d\in\mathcal{D}_0^*} N_k^{(d)} \leq S \tag{33b}$$

As shown in [11] and [36], where a problem with a similar structure has been analyzed, (33) is a standard convex optimization problem. Optimal solutions with complexity $O(KD)$ can be easily found via dual relaxation method. Since the resulting subchannel allocation $\hat{N}_k^{(d)}$ is fractional, a suitable rounding method[1] is applied to have integer values for $\hat{N}_k^{(d)}$ that satisfy $\sum_{k\in\mathcal{K}^*}\sum_{d\in\mathcal{D}_0^*}\hat{N}_k^{(d)} \leq S$. It should be finally noted that the result of this SAA procedure depends on the arbitrary choice of the powers $\tilde{P}_k$ and $\tilde{P}_d$. Although it is natural to consider values of the two powers that satisfy the total power constraint, *i.e.*, $\tilde{P}_k(D+1) \leq P_k$ and $\tilde{P}_d K \leq P_d$, other values can be used in the approximated framework. In our results we simply select $\tilde{P}_k = P_k$ and $\tilde{P}_d = P_d$, because in many cases each CU (or D2D pair) shares the resources with only one D2D pair (or CU).

2) Subchannel link matching (SLM) with uniform power allocation: By exploiting the estimate of the number of RBs allocated to each link, we can now initialize the power set $\hat{\boldsymbol{p}}$ with uniform values over the RBs allocated to the same link as follows: $\hat{p}_{k,s} = P_k/(\sum_{d\in\mathcal{D}_0^*}\hat{N}_k^{(d)})$ and $\hat{p}_{d,s} = P_d/(\sum_{k\in\mathcal{K}^*}\hat{N}_k^{(d)})$ where the power values[2] $\hat{p}_{k,s}$ and $\hat{p}_{d,s}$ are allocated to CU k and D2D pair d, respectively, according to the integer solution of problem (33). As a consequence, the initial values of the set $\hat{\boldsymbol{y}}$ can be found as the solution of a subchannel link assignment (SLM) problem stated as:

$$\max_{\boldsymbol{y}} g(\boldsymbol{y},\hat{\boldsymbol{p}}) \tag{34a}$$

$$s.t. \sum_{s\in\mathcal{S}} y_{k,s}^{(d)} = \hat{N}_k^{(d)}, \qquad \forall k\in\mathcal{K}^*, \forall d\in\mathcal{D}_0^* \tag{34b}$$

$$\sum_{k\in\mathcal{K}^*}\sum_{d\in\mathcal{D}_0^*} y_{k,s}^{(d)} = 1, \quad \forall s\in\mathcal{S} \tag{34c}$$

The problem in (34) is a classical matching problem, which can be efficiently solved through Hungarian method with complexity $\mathcal{O}(S^3)$. The resulting solutions $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{y}}$ of the SLM method can be then used as starting values for the ED2D-SCALE algorithm at each time epoch n when the adaptive algorithm updates $\boldsymbol{y}$, $\boldsymbol{p}$, $\boldsymbol{\mu}$. To further reduce the algorithmic complexity, the initialization might be done at time $n=0$ only, and the initial values of the inner layer algorithms for $n>0$ might be replaced by the results $\boldsymbol{y}$ and $\boldsymbol{p}$ of the same inner layer algorithm at time $n-1$. However, we have checked from simulation results that this algorithmic simplification leads to a small, but not negligible, performance loss.

[1] In our numerical evaluation, we evaluate an integer solution by first finding the $m = S - \sum_{k\in\mathcal{K}^*}\sum_{d\in\mathcal{D}_0^*}\lfloor\hat{N}_k^{(d)}\rfloor$ variables having the largest value of $\hat{N}_u^{(i)} - \lfloor\hat{N}_u^{(i)}\rfloor$. For these variables we set $N_u^{(i)} \leftarrow \lfloor\hat{N}_u^{(i)}\rfloor + 1$, while for all the others we set $N_u^{(i)} \leftarrow \lfloor\hat{N}_u^{(i)}\rfloor$.

[2] If the denominator is zero, *i.e.*, no RB is allocated by SAA to link k or d, the minimum power P_k/S or P_d/S will be assigned to this link for initialization, to prevent initial link exclusion.

The ED2D-SCALE algorithm is finally outlined in Algorithm 1. Here, the initialization of $\boldsymbol{\mu}$ can be simply done by setting $\mu_l = 1\ \forall l\in\mathcal{L}$.

Algorithm 2 ED2D-SCALE algorithm with SAA-SLM

```
Initialize I_max and μ, and set n = 0
repeat
    Solve the SAA problem (33) to get N̂_k^(d), ∀k ∈ K*, ∀d ∈ D*
    Set n*_k = Σ_{d∈D*_0} N̂_k^(d), ∀k ∈ K*, n*_d = Σ_{k∈K*} N̂_d^(k), ∀d ∈ D*
    Set p̂_{l,s}[0] = P_l/n*_l, if n*_l > 0; p̂_{l,s}[0] = P_l/S otherwise
    Solve the SLM problem (34) to get ŷ[0]
    Set i = 0
    repeat
        Solve problem (31) to get p'[i+1]
        Set p[i+1] = e^{p'[i+1]}
        Solve problem (17) to get ŷ[i+1]
        Update a, b according to (27) and (30)
        i ← i + 1
    until convergence or i = I_max
    Update μ[n+1] as in (23)
    n ← n + 1
until L* changes
```

V. Numerical Results

We consider the uplink of a LTE-like OFDMA cellular system[3] with $B = 3$ MHz, resulting in $S = 15$ RBs, and carrier frequency equal to 2 GHz. The most important system model parameters are listed in Table II. Monte Carlo simulations with duration of 15 s are carried out by fixing the number of CUs to $K = 40$ and by varying either the number of D2D links D or the radius of the D2D clustered model ρ. The weights $w_k, k\in\mathcal{K}$, of the CUs are uniformly distributed in [0,1], whereas the weights $w_d, d\in\mathcal{D}$, of D2D users are uniformly distributed between zero and the minimum weight of the CUs, in order to prioritize CUs. The required average rates $q_l, l\in\mathcal{L}$, for both CUs and D2D users are set to 512 kbps. The UE-BS path-loss follows 3GPP case-1 model defined in [38], whereas the path-loss of the D2D links follows the outdoor-to-outdoor communication model in [39]. The parameters $\Lambda' = 0.945$ and $\Lambda'' = 2.061$ in the model in (14) are derived through curve-fitting over the actual LTE discrete rate-to-SINR function [40]. In the proposed CILP algorithm the parameter F has to be suitably selected in order to balance the two values of utility and cost functions in eq. (11). After having experimentally checked that the algorithm works well when the maximum cost (weighted by F) is not greater than the utility values, we set $F = 0.05$ for our results, as in our setup the maximum cost is $S = 15$ and the average utility per user is around 0.5.

We first investigate how to select the value of the parameters Λ' and Λ_l'' (which depends on ς) in the model (2), used to estimate the rate per RB for each link. The performance of the joint AC and RRA has been investigated by varying ς for different numbers of CUs and D2D links. We have found that, the maximum value of ς allowing to match the minimum long-term rate requirement for the 99th percentile of the admitted

[3] In LTE systems SC-FDMA is used for the uplink. In this case, some additional specific allocation constraints significantly increase the complexity of the RRA problem. This issue is addressed in [37].

System model	
Cell layout	Single circular cell
Cell range	500 m
Number of CUs	40
CU distribution	Uniform
Minimum CU distance	50 m
CU path loss [dB]	$128.1 + 37.6 \log_{10}(d[km])$ [38]
Penetration loss [dB]	15
Number of D2D links	[5, 10, 20, 30, 40]
D2D users distribution	Clustered model [7]
Radius of D2D cluster [m]	[50, 100, 200, 250, 300, 400]
D2D path loss [dB]	$157.5 + 43.7 \log_{10}(d[km])$ [39]
Channel model	ITU A extended pedestrian
System bandwidth	3 MHz
Subchannel bandwidth	180 kHz
Number of available RBs	15
Time-slot duration	1 ms
UE's power budget	24 dBm
Noise power density	$2 \cdot 10^{-20}$ W/Hz
Minimum rate	512 kbps
Simulations drops	200

TABLE II
SIMULATION PARAMETERS

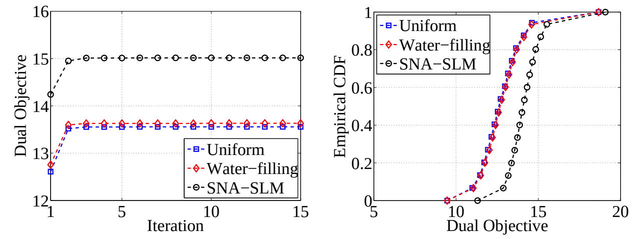
UEs is 0.8. By increasing ς beyond this value, the model in (2) overestimates the achievable rate and the number of admitted UEs increases, thus increasing the probability of infeasible RRA (no solution for problem (16)) using the proposed sub-optimal algorithm. Hence, we use $\varsigma = 0.8$ for the evaluation of the next results. Regarding parameter A' of (2), it is reasonable to set it equal to parameter Λ' of (14).

As a second step we evaluate the convergence properties and the performance of the proposed ED2D-SCALE algorithm for a feasible set of admitted users in the considered scenario according to different RRA initializations, *i.e.*,

- uniform power allocation among subchannels, *i.e.*, $p_{l,s} = P_l/S$, $\forall l \in \mathcal{L}$, and channel allocation as in (22), which is the result of the initialization proposed in [31] when applied to the uplink
- water-filling power allocation, *i.e.*, $p_{l,s} = [A'\pi_l \Delta B/(\ln(2)\lambda_l) - A''/g_{l,l}]^+$, $\forall l \in \mathcal{L}$, where λ_l is derived through the power budget constraint equation (16e), and channel allocation as in (22)
- the SAA-SLM method proposed in Section IV-C.

Fig. 2a shows a representative example of the evolution of the dual objective in (24a) over the inner iterations of the proposed ED2D-SCALE algorithm and the empirical CDF of the dual objective at convergence with D = 20 D2D pairs and cluster radius ρ = 250 m (a). We can note that the ED2D-SCALE algorithm quickly converges in few iteration for all the RRA initialization considered. Moreover, as statistically verified in Fig. 2b, the proposed SAA-SLM initialization method significantly increases the ED2D-SCALE performance of 10 % on average and up to 35 % with respect to the other two benchmark cases.

Now, we compare the system that jointly applies the proposed CILP AC and ED2D-SCALE algorithms with the following benchmark cases: (i) the system with the optimal AC obtained by solving (10) using Gurobi solver [29], jointly



(a) (b)

Fig. 2. Example of the dual objective (19) evolution over the inner iterations of the proposed RRA algorithm (a) and empirical CDF of the final dual objective (b), according to three different power and scheduling initialization strategies with $D = 20$ and $\rho = 250$ m.

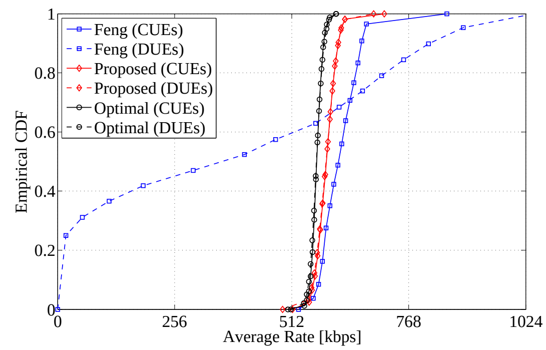


Fig. 3. Empirical CDF of the UE average rates when $D = 20$ and ρ =100 m.

working with ED2D-SCALE algorithm, and (ii) the system with AC and RRA proposed by Feng *et al.* in [7]. In [7] it was assumed that the subchannel allocation to CUs was already been done, thus allowing the D2D links to share a RB if the instantaneous QoS requirements for both the CU and the D2D user were satisfied. Here, to obtain a meaningful comparison, the strategy in [7] is implemented by first performing AC for CUs using the solution of problem (10) without D2D links. In this scenario, problem (10) collapses in the well-known knapsack problem, *i.e.*,

$$\max_{\boldsymbol{x}} \sum_{k \in \mathcal{K}} x_k w_k \qquad \text{s.t.} \sum_{k \in \mathcal{K}} x_k q_k / c_k^{(0)} \leq S(1 - \delta_{\text{D2D}}) \quad (35)$$

where, for the sake of a fair comparison, we introduce in each simulation drop a resource gap δ_{D2D} to the total RBs that accounts for the resources required to accommodate D2D links. The admitted CUs are then scheduled according to the optimal solution of the RRA without D2D links (see [32]).

The performance in terms of average rate achieved by the admitted UEs is investigated in Fig. 3 for the three strategies. Here, the empirical CDF of the rates of both CUs and D2D users is reported for a test-case with $D = 20$ and $\rho = 250$ m. The resource gap δ_{D2D} is set to 0.2. The step-size δ in the ED2D-SCALE algorithm is set to 10^{-4}. Although the AC and RRA strategies in [7] allow short-term QoS support, the resulting long-term performance is highly unfair. In fact, the 40 % of the D2D links, which correspond to the short-distance D2D links, achieve a long-term rate larger than the double of the rate requirement, whereas another 40 % of D2D links are not able to achieve a long-term QoS support. The

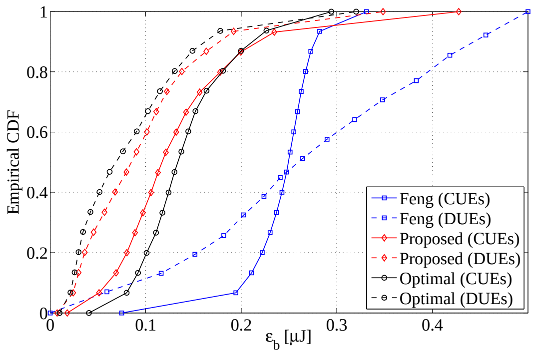
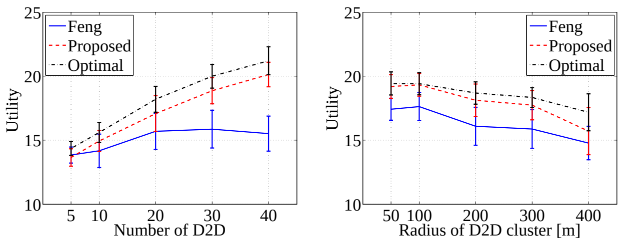
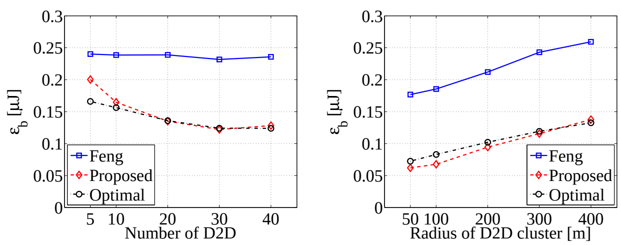


Fig. 4. Empirical CDF of the energy consumption per information bit when $D = 20$ and ρ =100 m.



(a) (b)

Fig. 5. Average utility (6) resulting from the joint use of CILP-AC and ED2D-SCALE strategies, by varying the number D of D2D links with cluster radius $\rho = 250$ m (a), and by varying the cluster radius ρ with $D = 20$ (b). The bars represent the standard deviation of the utilities.

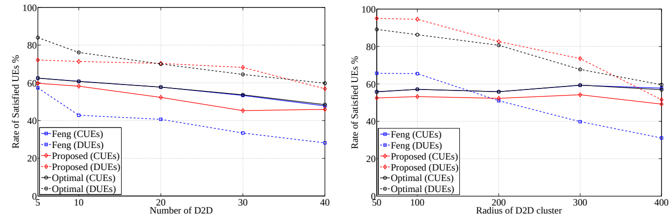


(a) (b)

Fig. 6. Energy consumption per information bit, averaged over UEs set, resulting from the joint use of CILP-AC and ED2D-SCALE strategies, by varying the number D of D2D links with cluster radius $\rho = 250$ m (a), and by varying the cluster radius ρ with $D = 20$ (b).

proposed and the optimal AC jointly used with ED2D-SCALE algorithm performs similarly and fairly with respect to both D2D users and CUs by providing an average rate larger than the requirement of all admitted UEs.

It is important to note that this improvement is not at the expense of an increase of the energy spent by the UE's. In Fig. 4 the empirical CDF of the average transmission energy consumption per information bit, *i.e.*, $\epsilon_{b,l} = \mathbb{E}[\sum_s p_{l,s}]/r_l$ for the generic link $l \in \mathcal{L}^*$, is reported. Although the energy efficiency is not the major focus of our paper, we are able to quantify how the the proposed long-term methods significantly decrease the energy spent by both CUE and D2D TXs. With respect to the benchmark in [7], our proposed strategies save more than one half of energy. This is due to the fact that our strategy better exploits the time, frequency, multiuser, as well as link, diversity of the underlay system.



(a) (b)

Fig. 7. Rate of satisfied UEs evaluated by varying the number D of D2D links with $\rho = 300$ m (a) and by varying the cluster radius ρ with $D = 20$ (b).

Fig. 5 shows, for each strategy, the average utility, *i.e.*, the sum of the utility revenues in (10a) for the links having the average rate guaranteed, along with its standard deviation, according to different numbers of D2D links D with cluster radius $\rho = 250$ m (Fig. 5a), and different cluster radii with $D = 20$ (Fig. 5b). The utility is normalized to have its maximum value equal to the number L of links. The proposed CILP AC achieves near-optimal performance for a reasonable number of D2D connections and D2D maximum distance. As the cluster radius or the number of D2D links increases, the utility gap increases. However, even in the worst-cases when (i) the number of D2D links is much larger than the available RBs and equal to the number of CUs (*i.e.*, $D = 40$) or (ii) the D2D users are almost uniformly distributed in the cell, the average gap is limited to 10%, with a standard deviation not larger than 5 %, resulting in a good trade-off between optimality and complexity. The strategy presented in [7] shows poor performance when compared to the proposed AC and RRA when the number of D2D pairs is large for all the considered cluster radius. The aggregate revenue gap at the service provider with respect to the CILP AC is up to 50% for large numbers of D2D pairs.

The improvement in terms of energy efficiency is also verified at different values of number of D2D users or cluster radii, as shown in Fig. 6. Compared to the benchmark, when the number of D2D pairs increases, our proposed AC and RRA select the best D2D links in terms of both utility and cost by fully exploiting the multi-link diversity. This leads to an increase of the average energy efficiency. Naturally, when the D2D cluster radius increases, the energy efficiency also increases, due to the larger path-loss in all the considered strategies. However, the slope of the energy consumption curve for the proposed strategies is smaller than that of the benchmark.

The performance degradation in terms of QoS experienced by D2D users for the short-term AC and RRA strategies proposed in [7] is highlighted in Fig. 7 for different numbers of D2D pairs and different values of D2D cluster radius. Here, the performance metric is the percentage of satisfied CUs and D2D pairs, *i.e.*, the number of CUs (D2D pairs) achieving the required minimum rate with respect to the total number of CUs (D2D pairs) in the system. Compared to the benchmark, our framework increases the percentage of admitted D2D links up to 40% (30% on average).

VI. Conclusions

In this paper, we have proposed a joint AC and RRA strategy for D2D communications underlaying cellular network that supports QoS, in terms of minimum rate, in the long-term. The proposed AC is built on a simple model for the estimation of the achievable long-term data-rate as function of the average channel condition of each direct and interfering link. After showing that the optimal AC is a NP-complete problem, we have derived a near-optimal CILP algorithm with polynomial complexity. The low-complexity RRA algorithm for the admitted UEs has been derived by maximizing the average weighted sum-rate under average rate constraints for both CUs and D2D users. Joint subchannel and power allocation has been addressed through an iterative algorithm named ED2D-SCALE. Numerical results, obtained through extensive Monte Carlo simulations, have shown that the proposed CILP AC strategy is near-optimal for reasonable values of distance and number of D2D links. Furthermore, the joint use of the proposed CILP AC and RRA strategies outperforms existing frameworks by increasing the number of satisfied D2D links up to 50 % and by reducing the energy expense by more than 50%.

Acknowledgment

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Appendix I
Derivation of problem (31) solutions

Since the problem in (31) is convex, solving the first-order optimality conditions is sufficient to obtain a global optimal solution. Given a set $\hat{y}$ of allocation variables, for each subchannel s let us define $k_s = l$ if a user index $l \in \mathcal{K}^*$ exists such that $\sum_d \hat{y}_{l,s}^{(d)} = 1$, and $d_s = l$ if user index $l \in \mathcal{D}^*$ exists such that $\sum_k \hat{y}_{k,s}^{(l)} = 1$. Let us now consider the following Lagrangian function associated to problem (31)

$$\tilde{L}(\boldsymbol{p}', \boldsymbol{\lambda}) = \bar{g}(\hat{\boldsymbol{y}}, \boldsymbol{p}') - \sum_{l \in \mathcal{L}^*} \lambda_l \left(\sum_{s \in \mathcal{S}} e^{p'_{l,s}} - P_l \right) \tag{36}$$

where $\lambda_l \geq 0$ is the dual variable for the power budget constraint of each link $l \in \mathcal{L}^*$. For any feasible set of dual variables $\boldsymbol{\lambda} = \{\lambda_l, l \in \mathcal{L}^*\}$ the dual objective $\max_{\boldsymbol{p}'} \tilde{L}(\boldsymbol{p}', \boldsymbol{\lambda})$ is obtained through the solution of at most $2S$ equations, two for each subchannel $s \in \mathcal{S}$ having one CU, *i.e.*, $k_s \neq 0$, and one D2D pair, *i.e.*, $d_s \neq 0$, active, as follows:

$$p_{k_s,s}^2 \lambda_{k_s} g'_{k_s,d_s,s} + p_{k_s,s} \left[\lambda_{k_s} + g'_{k_s,d_s,s} (\alpha_{d_s}^{(k_s)} - \alpha_{k_s}^{(d_s)}) \right] - \alpha_{k_s}^{(d_s)} = 0 \tag{37}$$

$$p_{d_s,s}^2 \lambda_{d_s} g'_{d_s,k_s,s} + p_{d_s,s} \left[\lambda_{d_s} + g'_{d_s,k_s,s} (\alpha_{k_s}^{(d_s)} - \alpha_{d_s}^{(k_s)}) \right] - \alpha_{d_s}^{(k_s)} = 0 \tag{38}$$

which are obtained from $\partial \tilde{L}(\boldsymbol{p}', \boldsymbol{\lambda}) / \partial p'_{l,s} = 0$ with the positions: $p_{l,s} = e^{p'_{l,s}}$, $g'_{u,i,s} = g_{u,i,s}/\sigma^2$, $\alpha_{u,s}^{(i)} = \Delta B \Lambda' \pi_u \log_2(e) a_{u,s}^{(i)}$. As example, the solution of equation (37), using $k = k_s$, $d = d_s$ and $\beta_{k,d,s} = \alpha_{d,s}^{(k)} - \alpha_{k,s}^{(d)}$, is given by

$$p_{k,s} = \begin{cases} \frac{\sqrt{\left(\frac{\lambda_k}{g'_{k,d,s}} + \beta_{k,d,s}\right)^2 + 4\alpha_{k,s}^{(d)} \frac{\lambda_k}{g'_{k,d,s}}} - \beta_{k,d,s} - \frac{\lambda_k}{g'_{k,d,s}}}{2\lambda_k}, & \lambda_k > 0 \\ \frac{\alpha_{k,s}^{(d)}}{g'_{k,d,s} \beta_{k,d,s}}, & \lambda_k = 0 \end{cases} \tag{39}$$

with the constraint $p_{k,s} \in [0, P_k]$. If, for a subchannel s, $d_s = 0$, only one equation for $p_{k_s,s}$, *i.e.*, $p_{k_s,s} \lambda_{k_s} - \alpha_{k_s}^{(0)} = 0$ has to be considered.

By observing that each power value $p_{l,s}$ in (37) and (38) depends on λ_l only, *i.e.*, $p_{l,s} = f(\lambda_l)$, the set of dual variables λ_l, $l \in \mathcal{L}^*$, can be found from the set of constraint equations $\sum_{s \in \mathcal{S}} p_{l,s} = P_l$, for each $l \in \mathcal{L}^*$. The solutions of these equations can be independently derived through line-search, *e.g.*, through bi-section method.

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