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Variance of fatigue damage in stationary random loadings: comparison between time- and frequency-domain results

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Abstract

In this paper, the variance of fatigue damage under stationary random loadings is investigated by means of different case studies (linear oscillator system, random process with ideal unimodal or bimodal power spectral density). Monte Carlo method is used to evaluate the variance of fatigue damage in time-domain and frequency-domain approach. The variance from simulations is compared with analytical methods (e.g. Mark and Crandall's method, Bendat's method and Low's method). Furthermore, for each case study, the paper investigates the relationship between the variance of fatigue damage and the bandwidth parameters. The purpose is to provide a simple closed-form solution for the case of ideal unimodal power spectral density. The closed-form solution has been compared with Monte Carlo simulation in time-domain. The results show good agreement not only for narrow-band but also for wide-band process.

Keywords: Random loadings; Time-domain approach; Frequency-domain approach; Fatigue damage; Variance of fatigue damage

1. Introduction

Failures of engineering components or structures may occur as an effect of applied uniaxial random loadings. In a frequency-domain approach, a random process is characterized by a Power Spectral Density (PSD). Analytical expressions are available for estimating the expected damage directly from PSD (Benasciutti and Tovo (2005)). Alternatively, the time-domain approach is based on well-established procedures (rainflow counting method and

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Palmgren-Miner rule) and does not require too complex theories; however, it usually needs long time-history records to achieve good confidence in estimating the damage.

In the random process theory, a time-history of length T can be thought as being one particular realization of an infinite ensemble. Its fatigue damage is then one possible value out of an infinite set. The damage can thus be modelled as a random variable following a damage probability distribution. The variance around the expected fatigue damage is an essential property of the damage probability distribution. Some authors (e.g. Mark and Crandall, Bendat, Low, Shinozuka) have proposed various analytical solutions to assess the variance of fatigue damage for stationary random loadings (Mark (1961), Low (2012)).

The aim of this paper is to investigate the variance of fatigue damage under stationary random loadings pertaining to narrow-band and wide-band Gaussian process. Monte Carlo method is performed to assess the variance of fatigue damage in the time- and frequency-domain approach. By means of different case studies including linear oscillator system, ideal unimodal and ideal bimodal process, the variance from simulations is compared with analytical methods. Furthermore, for each case study, the paper investigates the relationship between the variance of fatigue damage and several bandwidth parameters (e.g. δ, α_1, α_2). The purpose is to provide a simple closed-form solution for the case of ideal unimodal process. Although the existing Low's solution applies to a narrow-band process with any PSD shape, the simple solution proposed here is shown to work not only for narrow-band but also for wide-band ideal unimodal process.

2. Properties of random processes

A zero-mean stationary Gaussian random process, $X(t)$, is characterized in frequency-domain by a one-sided Power Spectral Density (PSD), $S_{XX}(f)$, with spectral moments (Lutes and Sarkani (2004)):

$$\lambda_m = \int_0^\infty f^m S_{XX}(f)df, \quad m = 0,1,2 \ldots \tag{1}$$

where λ_0 is the variance, $\sigma_X{}^2 = Var(X(t))$. The zero mean upcrossing rate, $\nu_0^+ = \sqrt{\lambda_2/\lambda_0}$ and the rate of peaks, $\nu_p = \sqrt{\lambda_4/\lambda_2}$, are computed by combining several spectral moments. The same is true for bandwidth parameters:

$$\alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0\lambda_2}}, \qquad \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0\lambda_4}}, \qquad \delta = \sqrt{1 - \frac{\lambda_1{}^2}{\lambda_0\lambda_2}} \tag{2}$$

The α_1 and α_2 parameters tend to unity for a narrow-band random process. Conversely, for a wide-band process, they tend to zero. The δ parameter (Vanmarcke (1972)) follows an opposite trend: it is close to zero for narrow-band and close to unity for wide-band random process.

3. Fatigue damage evaluation

In the Palmgren-Miner rule, the fatigue damage of a time-history of length T is computed by summing up the damage of every half-cycle with stress amplitude s_i:

$$D(T) = \sum_{i=1}^{n(T)} d_i = \sum_{i=1}^{n(T)} \frac{s_i{}^k}{2A} \tag{3}$$

where $n(T)$ is the number of all counted half-cycles, A is the fatigue strength coefficient and k is the inverse slope of the S–N curve $s^k N = A$. The damage $D(T)$ is a random variable as it strictly depends on the particular time-history considered. For the random process, $X(t)$, the expected damage is determined by:

$$E[D(T)] = E\left[\sum_{i=1}^{n(T)} d_i\right] = E[n(T)]E[d] \tag{4}$$

In stationary processes, $n(T) = 2\nu_a T$, where ν_a is the rate of counted cycles. The number of cycles on a counting method (e.g. rainflow) equals the number of peaks, i.e. $\nu_a = \nu_p$ (Benasciutti and Tovo (2005)).

For narrow-band processes, the stress amplitude distribution coincides with the distribution of a local peak (Rayleigh distribution). The expected damage results in a closed-form equation (Benasciutti and Tovo (2005)):

$$E[D(T)]_{NB} = \frac{\nu_0^+ T}{A}\left(\sqrt{2\lambda_0}\right)^k \Gamma\left(1 + \frac{k}{2}\right) \tag{5}$$

This expression is exact for a strictly narrow-band process, i.e. $\nu_0^+ \cong \nu_p$. When Eq. (5) is also applied for estimating the damage of wide-band processes, it is named "narrow-band approximation" (Benasciutti and Tovo (2005)).

Tovo and Benasciutti developed a more general, though approximate, method (TB method) to estimate the expected damage for both narrow-band and wide-band processes. The method is based on a weighted linear combination of two bound damage values and turns out in a correction of the narrow-band expression (Benasciutti and Tovo (2005)).

$$E[D(T)]_{TB} = [b + (1-b){\alpha_2}^{k-1}]\, E[D(T)]_{NB} = \lambda_{TB}\, E[D(T)]_{NB} \tag{6}$$

where b is a weighting coefficient and λ_{TB} is a bandwidth correction factor.

The expected damage value in (6) refers to the whole ensemble of time-histories and is usually computed from a PSD that is supposed to be known exactly. In practical applications, the damage $D(T)$ is calculated from a particular time-history $x_i(t)$ and it must then be regarded as a random variable. Even if a PSD were estimated from that particular time-history, the expected damage $E[D(T)]$ would be a random variable too, as the estimated spectrum would clearly differ from the exact one (which is actually unknown) and it would slightly change if another time-history be considered. The variance of fatigue damage then becomes a fundamental measure to quantify how fatigue damage (i.e. random variable) deviates from its expected value.

4. Variance of fatigue damage

The variance of fatigue damage $D(T)$ is formulated as (Mark (1961)):

$${\sigma_D}^2 = Var\left[\sum_{i=1}^{n(T)} d_i\right] = E\left[\sum_{i=1}^{n(T)}\sum_{j=1}^{n(T)} d_i, d_j\right] - E\left[\sum_{i=1}^{n(T)} d_i\right]^2 \tag{7}$$

Assuming a deterministic number of half-cycles, n, the variance of fatigue damage is given by:

$${\sigma_D}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} E[d_i, d_j] - \left[\sum_{i=1}^{n} E[d_i]\right]^2 \tag{8}$$

The variance of half-cycle damage is $Var[d_i] = E[{d_i}^2] - E[d_i]^2$ and its covariance $Cov[d_i, d_j] = E[d_i, d_j] - E[d_i]E[d_j]$. The variance of fatigue damage then becomes:

$${\sigma_D}^2 = \sum_{i=1}^{n} Var[d_i] + \sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{j=1}^{n} Cov[d_i, d_j] \tag{9}$$

Since the process is stationary, $E[d_0] = E[d_1] = \cdots = E[d_{n-1}]$, the variance of fatigue damage results into:

$${\sigma_D}^2 = nVar[d_0] + 2\sum_{l=1}^{n-1}(n-l)Cov[d_0, d_l] \tag{10}$$

where $l = j - i$. The quantity $Cov[d_0, d_l]$ is the autocovariance function for the half-cycles damage, $G_d(l)$. The autocorrelation function is likewise obtained, $E[d_0, d_l] = R_d(l)$. Furthermore, the variance of fatigue damage can also be thought as the summation over the elements of $n \times n$ matrix, minus $n^2 E[d_0]^2$, see Fig. 1.

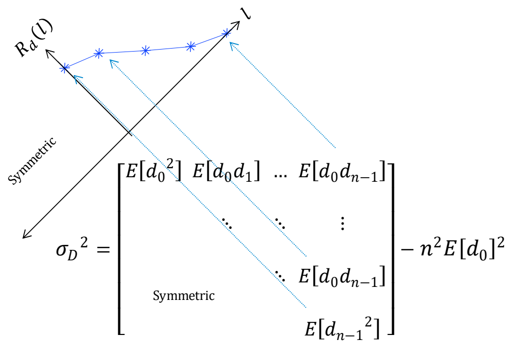

Fig. 1. The variance of fatigue damage and autocorrelation function for the half-cycle damage.

The sum of all elements in the main diagonal represents the term $nE\left[d_0{}^2\right] = R_d(0)$ and the sum of off-diagonal terms corresponds to $E[d_0, d_l] = R_d(l)$. The expectation of the product, $E[d_0, d_l]$, is a function of the joint probability density function (JPDF), $f_{d_0,d_l}(u, v; \tau)$:

$$E[d_0, d_l] = \iint_{-\infty}^{\infty} u^k v^k \, f_{d_0,d_l}(u, v; \tau)\, du\, dv \tag{11}$$

where τ is the time lag from one peak to the next valley. The previous double integral of the JPDF is the key element to compute the variance of fatigue damage. It is, however, rather challenging to solve the integral in closed form, unless some simplifying assumptions is introduced, as done in Mark and Crandall's approach.

4.1. Mark and Crandall's method

Mark and Crandall proposed the first analytical method to estimate the variance of fatigue damage. The method assumes that stress histories are proportional to the response of a linear oscillator system. It is only valid for narrow-band and zero-mean stationary Gaussian random process, and restricted to odd integer values of k (Mark (1961)). The equation for the variance of fatigue damage is:

$$\sigma_D{}^2 = \frac{f(k)\nu_0^+ T}{\zeta}\left(\sqrt{2\lambda_0}\right)^{2k} \Gamma^2\left(1 + \frac{k}{2}\right), \quad \text{for } \zeta \leq 0.05 \text{ and } \zeta\nu_0^+ T \gg 1 \tag{12}$$

where $f(k)$ is a function for odd k and ζ is the damping coefficient of linear oscillator system. The variance of fatigue damage is often normalized to the expected damage $\mu_D = E[D(T)]$ through the coefficient of variation, C_D:

$$C_D = \frac{\sigma_D}{\mu_D} = \sqrt{\frac{f(k)}{\zeta\nu_0^+ T}} \quad \text{for } \zeta \leq 0.05 \text{ and } \zeta\nu_0^+ T \gg 1 \tag{13}$$

The work of Shinozuka (see section two of Desmond (1987)) is similar to Mark and Crandall's method, but it also applies to even integer values of k, which complements Mark and Crandall's method.

4.2. Bendat's method

Bendat developed a more general expression for the variance of fatigue damage. The method has the same assumptions of Mark and Crandall's approach, the main difference being that Bendat's method agrees to odd and even values of k (Bendat (1964)). Bendat's method is based on approximating the autocorrelation coefficient for the half-cycle damage as $\rho_{d_0 d_l} = e^{2l\pi\zeta}$, which turns out in the following closed-form expression:

$$\sigma_D{}^2 = \frac{\nu_0^+ T}{\zeta}\left(\sqrt{2\lambda_0}\right)^{2k}\left(\Gamma(1+k) - \Gamma^2\left(1+\frac{k}{2}\right)\right) \tag{14}$$

The corresponding closed-form expression of the coefficient of variation is:

$$C_D = \sqrt{\frac{1}{2\pi\zeta\nu_0^+ T}\left(\frac{\Gamma(1+k)}{\Gamma^2\left(1+\frac{k}{2}\right)} - 1\right)} \tag{15}$$

4.3. Low's method

The Low's method is applicable to narrow-band process with any spectral density shape, i.e. not exclusive to linear oscillator system (Low (2012)). This is a great advantage compared with the previous methods, which are restricted to the linear oscillator system. The Low's method considers two half-cycle damage, d_0 and d_l, separated by a time lag, $\tau = l/(2\nu_0^+)$. For a narrow-band process, the JPDF of peaks and valleys has been derived by Rice (Rice (1944)).

$$f_{d_0 d_l}(u, v; \tau) = \frac{uv}{1-\rho_{XX}^2(\tau)} I_0\left(\frac{uv\rho_{XX}(\tau)}{1-\rho_{XX}^2(\tau)}\right) e^{\frac{u^2+v^2}{-2\left(1-\rho_{XX}^2(\tau)\right)}} \tag{16}$$

where $I_0(-)$ is the modified Bessel function of the first kind with order zero, and $\rho_{XX}(\tau)$ is the autocorrelation coefficient of the random process. The marginal distribution is Rayleigh. By invoking Eq. (11), the variance is:

$$\sigma_D{}^2 = nVar[d_0] + 2Var[d_0]\sum_{l=1}^{n-1}(n-l)\rho_{d_0 d_l} \tag{17}$$

The Low's method proposes that $\rho_{d_0 d_l}$ be approximated by a quadratic function of $\rho_{XX}^2(\tau)$, i.e. $\rho_{d_0 d_l} = \alpha_k \rho_{XX}^2(\tau) + \beta_k \rho_{XX}^4(\tau)$, where α_k and β_k are best-fitting coefficients. The coefficient of variation is:

$$C_D = \sqrt{\frac{n + 2\sum_{l=1}^{n-1}(n-l)\rho_{d_0 d_l}}{n^2}\left(\frac{\Gamma(1+k)}{\Gamma^2\left(1+\frac{k}{2}\right)} - 1\right)} \tag{18}$$

A few years later, Low extended his method to a multimodal process including two or more narrowband components (more than one frequency mode). The method assumes that all frequency modes do not overlap. The expected damage of each frequency mode is uncorrelated. Closed-form expressions are available for two special cases, i.e. the components are all linear oscillator responses or their spectral density is rectangular (Low (2014a)). The whole analysis of the multimodal Low's method leading to the variance of fatigue damage is too long to be reported here; for details see Low (2014a). The general expression of the variance for any number of frequency modes is:

$$\sigma_D{}^2 = \sum_{i=1}^{M} \sigma_{D_i}{}^2 \tag{19}$$

where M is the number of frequency modes. Then the coefficient of variation is computed as:

$$C_D = \sum_{i=1}^{M} \left(\frac{\mu_{D_i}}{\mu_D}\right) C_{D_i} \tag{20}$$

5. Numerical case studies

Numerical simulations are performed to compare analytical estimations with Monte Carlo results in time-domain and frequency-domain. Simulations consider several power spectra (e.g. linear oscillator response, ideal unimodal and ideal bimodal), for which random processes having different bandwidth parameters can easily be generated. The shape of such power spectra $S_{XX}(f)$ is known (they are a sort of "reference" PSD).

A sample of N random time-histories $x_i(t)$, $i = 1,2,3, \dots N$, with fixed time length T is simulated from each spectrum $S_{XX}(f)$. For each time-history $x_i(t)$, fatigue damage is calculated both in time-domain and in frequency-domain. The time-domain damage, $D_{TD,i}(T)$, is computed using the rainflow counting algorithm and the Palmgren-Miner rule. The frequency-domain damage, $D_{FD,i}(T)$, is computed from the power spectrum, $\widehat{S_{XX}}(f)$, that is estimated back from each simulated signal, $x_i(t)$. Frequency-domain damage is estimated by the TB method. In all case studies, the variance of random process is normalized to unity, ${\sigma_X}^2 = 1$, the fatigue strength coefficient is assumed unity, $A = 1$, and the inverse slope is taken as $k = 3$ (it typically ranges from 2 to 8).

A total of N damage values, $D_{TD,i}(T)$ and $D_{FD,i}(T)$, $i = 1,2,3, \dots N$, is calculated for each reference PSD. The sample mean damage $\overline{D} = N^{-1} \sum_{i=1}^{N} D_i$, the sample variance $\sigma_D^2 = (N-1)^{-1} \sum_{i=1}^{N} (D_i - \overline{D})^2$ and the coefficient of variation $C_D = [(N-1)^{-1} \sum_{i=1}^{N} (D_i - \overline{D})^2]^{1/2} \, N \left(\sum_{i=1}^{N} D_i\right)^{-1}$ are determined accordingly. As for $N \to \infty$ the damage values approximately follow a Gaussian distribution (central limit theorem), large sample size is employed in Monte Carlo simulations, $N = 2 \times 10^5$.

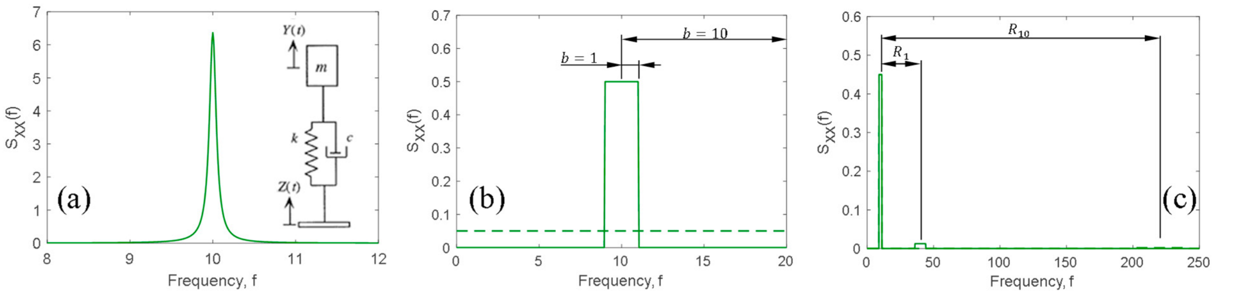

Fig. 2. Power spectral densities considered: (a) Linear oscillator system, (b) Ideal unimodal process, (c) Ideal bimodal process.

5.1. Linear oscillator system

This system is here chosen as it allows Crandal and Mark, Bendat and Low methods to be applied. A linear oscillator system is often used as a simple mechanical model for vibrating structures (e.g. building-foundation system, cantilever beam, a car-quarter model). In this case study, the oscillator system has a natural frequency f_n =10 Hz and a relative damping ratio ζ = 0.005. The system is subjected to a white noise random base acceleration $Z(t)$. Analytical expressions are derived for the mass absolute displacement, $Y(t)$, and the relative displacement, $X(t) = Y(t) - Z(t)$, see Fig. 2a. The response PSD is centered around the natural frequency and it is narrow-band, with bandwidth parameters $\alpha_1 = 0.9979$, $\alpha_2 = 0.994$ and $\delta = 0.065$.

Figure 3a displays the mean and standard deviation of fatigue damage (normalized to the expected damage of the reference PSD) for both time-domain and frequency-domain, as a function of the number of counted cycles. The greater is the number of cycles, the lower is the dispersion around the mean of the distribution of damage. The standard deviation in frequency-domain seems to be slightly lower (about 1%) than that in time-domain, regardless of the number of cycles. Also, the mean damage in frequency-domain seems indeed closer to the expected fatigue damage.

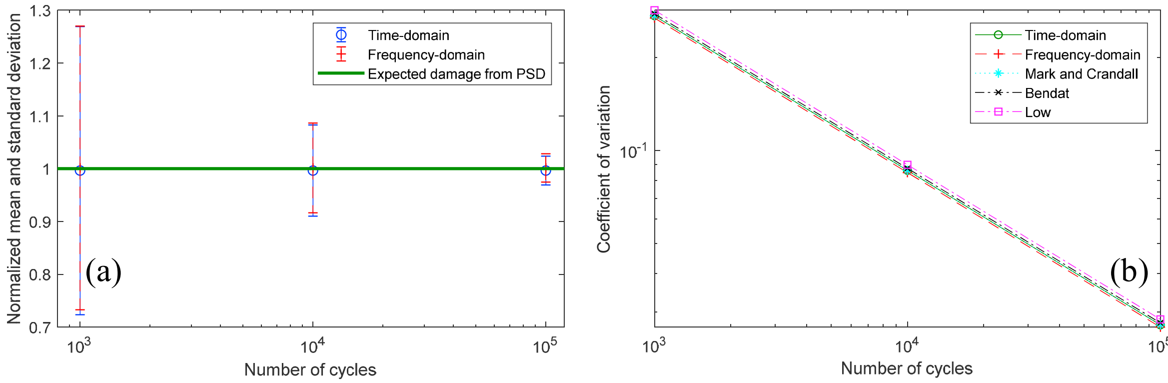

Fig. 3. Linear oscillator system (a) normalized mean and standard deviation and (b) coefficient of variation versus the number of counted cycles.

For the comparison between simulations and all analytical methods, the coefficient of variation from linear oscillator response shows a good agreement over all number of cycles, see Fig. 3b. The straight lines on a log-log scale has somehow to be expected, as the coefficient of variation from any PSD is inversely proportional to the number of cycles, $\sqrt{N_p}$ (Low (2012)). The general equation is $C_D = cst \cdot N_p^{-1/2}$ where cst is a constant of proportionality, which is expected to vary according to k and PSD shape (obtained, for example, by different values of ζ).

5.2. Ideal unimodal process

This case study refers to an idealized rectangular PSD, see Fig. 2b. The mid-frequency is fixed at 10 Hz. The effect of bandwidth (from narrow-band to wide-band process) is incorporated in the half spectral width b, which takes integer values in the interval $b = 1 \div 10$ Hz. By varying b, the bandwidth parameters move from $\alpha_1 = 0.9983$, $\alpha_2 = 0.993$ and $\delta = 0.058$ (narrow-band) to $\alpha_1 = 0.866$, $\alpha_2 = 0.745$ and $\delta = 0.5$ (limiting wide-band). The coefficient of variation from Monte Carlo simulations is compared with Low's method over such a range of bandwidth parameters. A fitting closed-form solution, described later on, is also presented.

Fig. 4 displays a typical trend of the coefficient of variation over the number of cycles, for two limiting cases $b = 1$ Hz and $b = 10$ Hz (the curves for other intermediate cases are not shown to avoid clutter).

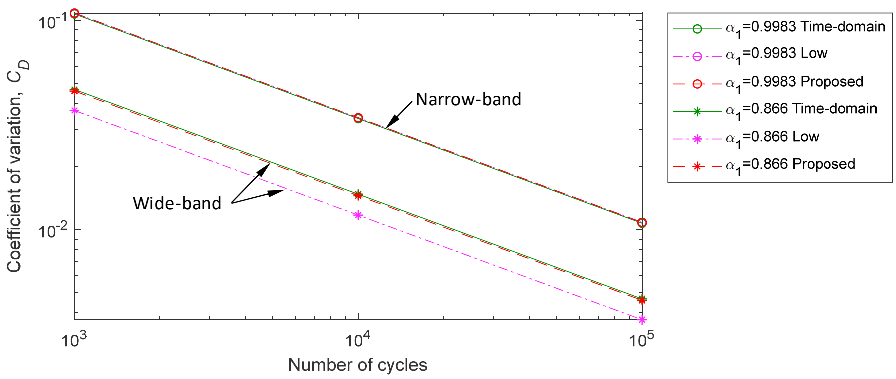

Fig. 4. Coefficient of variation for the ideal unimodal process.

For narrow-band process ($\alpha_1 = 0.9983$, $\alpha_2 = 0.993$), the Low's method overlaps results of time-domain simulations. Once the ideal unimodal tends to the limiting wide-band case ($\alpha_1 = 0.866$, $\alpha_2 = 0.745$), the coefficient of variation decreases. Despite Low's method only applies to the narrow-band case, in the wide-band case it provides estimations for which the disagreement with time-domain results is not excessive (about 25%).

It is also of interest to investigate the relationship (see Fig. 5) that links the coefficient of variation C_D directly to the bandwidth parameters α_1, α_2 and δ. Such parameters are indeed function of spectral moments, which in turn depend on the PSD shape. Not only are bandwidth parameters used to classify a PSD type from narrow-band to wide-band, but they also allow the expected damage to be estimated directly from a PSD (for example, the correction factor λ_{TB} in TB method is a function of α_1 and α_2, see Benasciutti and Tovo (2005)).

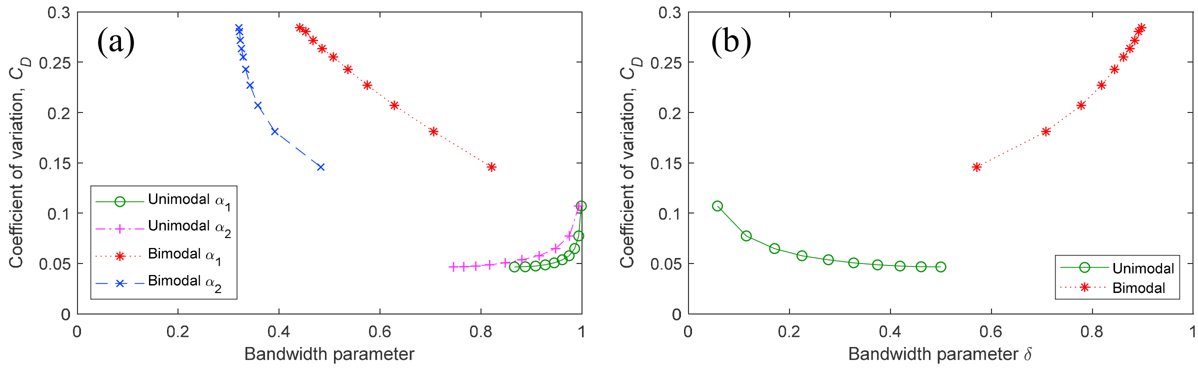

Fig. 5. Coefficient of variation versus (a) α_1 and α_2 and (b) δ for the ideal unimodal and bimodal process.

Figure 5 shows that the greater are α_1 and α_2 (from wide-band to narrow-band process), the greater is the coefficient of variation. An opposite trend is observed for parameter δ. Note that the tendency observed in Fig. 5 characterizes also other unimodal PSD (e.g. linear oscillator system, JONSWAP and Pierson-Moskowitz spectra).

The curves in Fig. 5 are not only increasing or descending, but more importantly, they are also smooth. This attribute is advantageous to transform them into an analytical fitting expression. It is useful to assume that the coefficient of variation is a function of α_1, k and N_p. The bandwidth parameter, α_1, changes from 0.866 to 1. The idea is to find the constant of proportionality cst, which is only a function of α_1 and k. The function $C_D = f(\alpha_1, k, N_p)$ for k and N_p fixed is a monotonic function and must satisfy the constraints $f(0.866, k, N_p) = C_D^{cst}$ and $f(1, k, N_p) = \infty$ where C_D^{cst} is a asymptotic constant value. Among the mathematical expressions that satisfy such a constraint, a rationale polynomial seems to be a simple and enough accurate choice:

$$C_D = \frac{c_1\, e^{k^{c_2}}}{(1 - \alpha_1{}^{c_3})^{c_4}} N_p{}^{-1/2} \tag{21}$$

where c_1, c_2, c_3 and c_4 are unknown fitting coefficients, which are determined by minimizing the root-mean-square (RMS) error between the coefficient of variation from time-domain simulations, $C_{D,i}^{sim}$, and from the proposed fitting expression, $C_{D,i}^{est}$:

$$\varepsilon_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\log_{10}\left(\frac{C_{D,i}^{est}}{C_{D,i}^{sim}}\right)\right]^2} \tag{22}$$

where the sum spans over the whole set of N ideal unimodal power spectra considered in simulations. Replacing the c_1, c_2, c_3 and c_4 values, the final expression is:

$$C_D = \frac{0.241\, e^{k^{0.583}}}{(1 - \alpha_1{}^{19.338})^{0.253}} N_p{}^{-0.5}, \qquad \text{for } 2 \le k \le 8 \text{ and } 0.866 \le \alpha_1 \le 1 \tag{23}$$

The equation gives $C_D = C_D^{cst}$ when $\alpha_1 \to 0.866$ and $C_D = \infty$ when $\alpha_1 \to 1$. It also has the desirable property to depend on four spectral moments λ_0, λ_1, λ_2 and λ_4, similarly to the TB method. Another desirable property is that for large time length T, the equation provides a coefficient of variation approaching zero. The equation thus satisfies the limiting conditions and becomes applicable for any unimodal power spectrum. Besides simplicity and elegance, the equation does not need a computer program.

Fig. 4 confirms a good agreement between the proposed formula (23) and numerical simulations in time-domain. The proposed formula may assess the coefficient of variation not only for narrow-band but also for wide-band ideal unimodal process.

5.3. Ideal bimodal process

This case study refers to a bimodal PSD in which the low- and high-frequency components are ideal rectangular blocks (narrow-band), see Fig. 1c. The half spectral bandwidth of the low-frequency block is equal to the ideal unimodal case, $b = 1$ Hz. The frequency ratio between the two blocks takes the values $R_i = f_{c,2}/f_{c,1} = 2(1 + i)$, where $i = 1 \div 10$ (integer values), while the area ratio (variance ratio) is fixed to $B = \lambda_{0,2}/\lambda_{0,1} = 1/9$. This type of power spectrum behaves as a wide-band PSD.

The coefficient of variation from Monte Carlo simulation in time-domain is compared with multimodal Low's method. The Jiao and Moan (1990) method is used for estimating the expected fatigue damage, $E[D] = E[D]_L + E[D]_H$, where $E[D]_L$ and $E[D]_H$ refer to the low- and high-frequency components, respectively. Fig. 6 depicts the trend of the coefficient of variation in two limiting cases $R_1 = 4$ and $R_{10} = 22$.

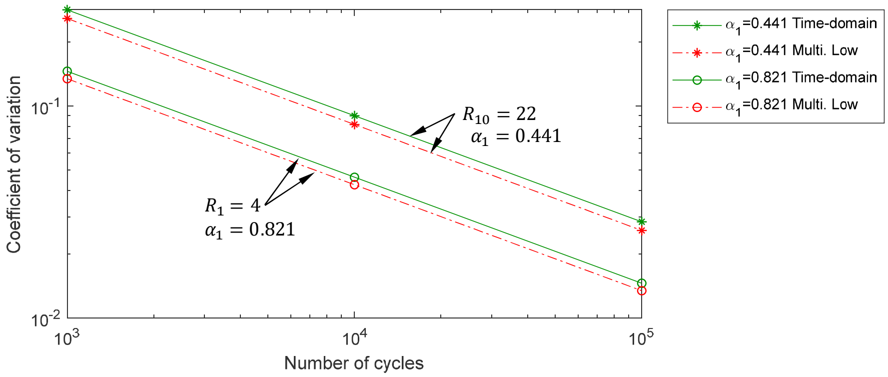

Fig. 6. Coefficient of variation for ideal bimodal process.

Compared to the ideal unimodal process, an opposite behavior is now observed: the more wide-band is the PSD (with $\alpha_1 \to 0.441$), the greater is the coefficient of variation, see Fig. 6. Slight disparities (about 10%) between simulation in time-domain and multimodal Low's method are observed for both small and high-frequency ratios, $R_1 = 4$ and $R_{10} = 22$. The disparity at the lowest R has somehow to be expected, as the damage estimated by Jiao-Moan method is not accurate for a small frequency ratio (Low (2014a)). The difference at the highest R is, however, surprising since, at such a high-frequency ratio, the multimodal Low's method, combined with Jiao-Moan, should work pretty well.

What could improve the results would be to apply the Low's method for estimating the fatigue damage (Low (2010)), in connection with the multimodal Low's method for the coefficient of variation. The Low (2010) approach is, however, intricate and difficult to implement. In fact, Low proposed a surrogate model (Low (2014b)) that approximates the exact expected fatigue damage from bimodal process. The Low's surrogate model does not provide explicitly the low- and high-frequency damages, $E[D]_L$ and $E[D]_H$, which thus makes impossible to apply the multimodal Low's method. A simple and accurate model for both low- and high-frequency damages is required to apply the multimodal Low's method without too much effort.

The coefficient of variation from time-domain simulations versus the bandwidth parameters, α_1, α_2 and δ again follows smooth curves, see Fig. 5. An opposite trend compared to ideal unimodal process is yet observed, i.e. descending for the coefficient of variation versus α_1 and α_2, and ascending versus δ. Although the curves for bimodal spectra are smooth, it is difficult to fit them by an analytical expression, the function $C_D = f(-)$ would depend on R, B, k and N_p. As this relationship is somehow complicated, the attempt to find a simple fitting expression would result in an elaborated formula, and it is not investigated further.

6. Conclusions

The variance and the coefficient of variation of fatigue damage have been investigated through time-domain and frequency-domain simulations, and compared with analytical methods (Mark and Crandall, Bendat, Low). Several power spectra (i.e. linear oscillator system, ideal unimodal and ideal bimodal) have been considered, from which a large sample of time-histories have been simulated and the fatigue damage computed. For all cases examined, the normalized standard deviation of fatigue damage reduces as the number of counted cycles (i.e. the time-history length) increases. This result suggests that, in engineering applications, the longest possible time-history record be used. Simulations and analytical methods were also shown to provide quite similar values of the coefficient of variation. For the ideal unimodal case, the coefficient of variation reduces from narrow-band to wide-band process. The fact that the coefficient of variation versus the bandwidth parameters is a smooth function allowed an analytical fitting expression to be obtained. The proposed expression is very precise when compared against Monte Carlo simulation in time-domain. For the ideal bimodal process, the coefficient of variation followed a trend opposite to the ideal unimodal case. Although this work gave a contribution in the study of the variance of fatigue damage for several types of random processes, further studies may be needed to investigate in more detail the relationship between the coefficient of variation and bandwidth parameters, in particular with the aim to find a relationship of general validity, i.e. applicable to any PSD.

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