

# Application of a physics-based model to predict the performance curves of pumps as turbines

Cite as: AIP Conference Proceedings **2191**, 020106 (2019); <https://doi.org/10.1063/1.5138839>  
Published Online: 17 December 2019

Lucrezia Manservigi, Mauro Venturini, and Enzo Losi



View Online



Export Citation

## Lock-in Amplifiers up to 600 MHz



Zurich  
Instruments



# Application of a Physics-Based Model to Predict the Performance Curves of Pumps as Turbines

Lucrezia Manservigi, Mauro Venturini\*, Enzo Losi

*Dipartimento di Ingegneria, Università degli Studi di Ferrara, Ferrara, Italy*

*\*Corresponding author: mauro.venturini@unife.it*

**Abstract.** This paper presents the application of a physics-based simulation model, aimed at predicting the performance curves of pumps as turbines (PATs) based on the performance curves of the respective pump. The simulation model implements the equations for estimating head, power and efficiency for both direct and reverse operation. Model tuning on a given machine is performed by using loss coefficients and specific parameters identified by means of an optimization procedure, which simultaneously optimizes both the pump and PAT operation. The simulation model is calibrated in this paper on data taken from the literature, reporting both pump and PAT performance curves for head and efficiency over the entire range of operation. The performance data refer to twelve different centrifugal pumps, running in both pump and PAT mode. The accuracy of the predictions of the physics-based simulation model is quantitatively assessed against both pump and PAT performance curves and best efficiency point. Prediction consistency from a physical point of view is also evaluated.

## INTRODUCTION

Micro and small hydropower plants are one of the most important renewable sources in developing countries, in rural and remote areas, in small localities and also in OECD countries. In these contexts, there is the need of a cost-effective and reliable solution for the hydraulic turbine, also capable of handling low and variable power. Since some years, such a solution has been identified with pumps as turbines (PATs), i.e. pumps used in turbine mode by reversing the flow direction and using the electric motor as a generator [1]. One of the strong points of PATs, compared to conventional hydraulic turbines, is their cost effectiveness; in fact, their cost per nominal power can be up to 15 times lower [2].

The scarce information about PAT behavior represents the main drawback for the application of this new technology; in fact, pump manufacturers rarely provide their performance curves. Thus, many scholars have addressed the challenge of predicting the best efficiency point (BEP) and also the complete characteristic curves of a PAT. For example, Derakhshan and Nourbakhsh [3] derived some relationships to predict the BEP by using a theoretical analysis, coupled with a three-dimensional computational fluid dynamic (CFD) model [4]. Renzi and Rossi [5] developed a generalized theoretical methodology making use of non-dimensional parameters, to predict the flow rate, head and efficiency of PATs at BEP. The same authors employed artificial neural networks to determinate both PAT performance curves and BEP [6]. Stefanizzi et al. [7] reported the experimental characterization of a single stage centrifugal pump, in both direct and reverse mode and also proposed a new prediction model. Rossi et al. [8] proposed a model for estimating PAT performance in off-design operating conditions. This model was validated by means of CFD analyses [9].

However, a methodology for estimating PAT performances over the complete operating range has not yet consolidated in the literature. In fact, experimental characterization is required case by case, but published laboratory tests that focus on this topic are scarce. In addition, the models derived from experimental tests and available in the

literature usually refer to a specific type of pump and no methodology suitable to a wide range of specific speeds has yet been developed [10].

For these reasons, Venturini et al. developed in [11] a physics-based model aimed at estimating PAT performance curves over the entire range of operation, given the pump performances. The physics-based model, based on the theoretical approach presented by Gulich in [12], includes both corrective and geometrical parameters which are identified by means of an optimization procedure.

In this paper, the physics-based model developed in [11] is applied by considering two novelties. First, the tuning of the physics-based model is performed by simultaneously optimizing the behavior in both pump and PAT mode. On the contrary, pump and PAT performance was simulated separately in [11]. Second, the physics-based model is tuned in this paper by means of a limited number of experimental data, while model tuning in [11] was performed on 1,000 data obtained by interpolating experimental data. The physics-based model is applied to predict the characteristic curves of twelve centrifugal pumps/PATs reported in [13], characterized by specific nondimensional speed values in the range of 0.16 – 1.15. The reliability of the simulation model is tested against the respective pump and PAT performance curves and also at the best efficiency point of each PAT.

## SIMULATION MODEL

### Pump model

The relations implemented in the simulation model of a pump are taken from the basic theory of pumps, in the form reported by Gulich in [12]. The theoretical head of a pump can be calculated by means of the Euler's equation presented in Eq. (1):

$$H_{th,P} = \frac{1}{g}(u_2 c_{2u} - u_1 c_{1u}) \quad (1)$$

The actual head  $H_P$  can be estimated from theoretical head  $H_{th,P}$  by subtracting all hydraulic losses between the suction and discharge nozzles, i.e. the hydraulic losses in the inlet casing ( $Z_E$ ), the impellers ( $Z_{La}$ ), diffusers ( $Z_{Le}$ ), volutes ( $Z_{sp}$ ) and the outlet casing ( $Z_A$ ), according to Eq. (2):

$$H_P = H_{th,P} - Z_E - Z_{La} - Z_{Le} - Z_{sp} - Z_A \quad (2)$$

Thus, the pump's hydraulic efficiency can be calculated as in Eq. (3):

$$\eta_{h,P} = \frac{H_P}{H_{th,P}} \quad (3)$$

The useful power is defined in Eq. (4), where  $Q$  is the useful flow rate:

$$P_{u,P} = \rho g H_P Q \quad (4)$$

The volumetric efficiency, defined in Eq. (5), accounts for the leakage through the annular seal at the impeller inlet  $Q_{sp}$ , the leakage  $Q_E$  through the device for axial thrust balancing and additional fluid  $Q_h$  circulated within the pump (e.g. branched off for auxiliary purposes such as feeding a hydrostatic bearing, flushing, sealing or cooling):

$$\eta_{v,P} = \frac{Q}{Q + Q_{sp} + Q_E + Q_h} \quad (5)$$

Therefore, it is possible to estimate the power  $P_P$  required at the coupling by accounting for all pump losses. Power losses include mechanical power losses  $P_m$ , power losses due to fluid recirculation  $P_{Rec}$ , disk friction losses  $P_{RR}$ , throttling losses  $P_{s3}$  and friction losses  $P_{er}$  created by the components of axial thrust balance devices. Therefore, the power  $P_P$  can be expressed as in Eq. (6):

$$P_P = \sum_1^{N_{st}} \frac{\rho g H_P Q}{\eta_v \eta_h} + P_m + P_{Rec} + \sum_1^{N_{st}} P_{RR} + \sum_1^{N_{st}} P_{s3} + P_{er} \quad (6)$$

Finally, the overall pump efficiency at coupling  $\eta_P$  can be estimated according to Eq. (7):

$$\eta_P = \frac{P_{u,P}}{P_P} = \frac{\eta_V \eta_h \rho g H_P Q}{\rho g H_P Q + \eta_V \eta_h (P_m + P_{Rec} + \sum_1^{N_{st}} P_{RR} + \sum_1^{N_{st}} P_{S3} + P_{er})} \quad (7)$$

## PAT model

This section presents the modeling approach used to simulate the behavior of a pump running in turbine mode, when the liquid transfers power to the rotor. The discharge nozzle of the pump is an inlet nozzle to the turbine, while the pump suction nozzle becomes the turbine outlet nozzle. As made for modeling pump operation, the equations implemented in the simulation model of a PAT are written in the form reported by Gulich in [12].

The theoretical head of a PAT can be expressed as in Eq. (8):

$$H_{th,T} = \frac{1}{g} (u_2 c_{2u} - u_1 c_{1u}) = \frac{1}{g} (u_2 c_{2m} \cot \alpha_2 - u_1^2 - u_1 c_{1m} \cot \beta_1) \quad (8)$$

The inflow angle  $\alpha_2$  to the runner can be determined from the guide wheel or volute geometry. The flow angle  $\beta_1$  of the fluid exiting the runner, which can be estimated from the throat area, differs from the blade angle  $\beta_{1B}$  because a vane-congruent flow cannot be expected in turbine operation.

According to Eq. (8), the theoretical head of a PAT increases linearly with the flow rate. However, the theoretical head  $H_{th,T}$  transferred from the fluid to the runner is smaller than the actual head  $H_T$  between inlet and exhaust nozzles because of hydraulic losses, as shown in Eq. (9):

$$H_T = H_{th,T} + Z_E + Z_{La} + Z_{Le} + Z_{sp} + Z_A \quad (9)$$

The following relation is used for estimating PAT hydraulic efficiency:

$$\eta_{h,T} = \frac{H_{th,T}}{H_T} \quad (10)$$

Since the power  $P_T$  available at the coupling of the turbine is affected by power losses, the power balance of a PAT can be expressed according to Eq. (11):

$$P_T = \rho g H_T \eta_{h,T} (Q - Q_{sp} - Q_E) - P_m - \sum_1^{N_{st}} P_{RR} - \sum_1^{N_{st}} P_{S3} - P_{er} \quad (11)$$

where flow rates and powers have the same meaning as in the pump model.

PAT overall efficiency  $\eta_T$  at the coupling is given by Eq. (12):

$$\eta_T = \frac{P_T}{\rho g H_T Q} \quad (12)$$

## Model parameters

As extensively discussed in [11], the physics-based model includes twenty-four parameters, which are reported in Table A1.

Fourteen parameters are specific to the pump, by including (i) nondimensional ratios of geometric characteristics, (ii) flow and geometrical angles and (iii) hydraulic and power losses. These parameters may be known from pump geometry or can be estimated through an optimization procedure, as made in this paper due to a lack of detailed geometrical data.

The remaining ten parameters are used to replicate PAT behavior. These parameters (i) allow the identification of the BEP of the PAT in terms of flow rate and head with respect to the corresponding values of the pump (which are known from pump performance curves) and (ii) estimate hydraulic and power losses.

All these parameters are estimated by the simulation model in order to tune it on a given pump/PAT, according to the tuning procedure described in the following.

## Tuning procedure

To identify the optimal value of the parameters listed in Table A1, an optimization procedure was adopted, making use of the tool *Optimtool* [14], which is available in Matlab®, and the solver called *fmincon*, which is also available in Matlab®. This solver can search the minimum value of a scalar Objective Function (OF).

It is worth noting that, in this paper, the tuning procedure differs from the one outlined in [11] from two points of view. In fact, in this paper, the physics-based model is challenged to reproduce pump/PAT behavior by taking into account only the experimental data reported by [13]. Thus, for all machines, the prediction error is calculated by means of Eq. (13), which compares experimental data, i.e.  $(Y_{ki})_e$ , to the corresponding simulated data, i.e.  $(Y_{ki})_s$ .

$$RMSE_{Y_k} = \sqrt{\frac{1}{N_e} \sum_{i=1}^{N_e} \left( \frac{(Y_{ki})_e - (Y_{ki})_s}{(Y_{ki})_e} \right)^2} \quad Y = \psi, \pi, \eta; \quad k = 1, \dots, N_{P,T} \quad (13)$$

On the contrary, all model parameters were tuned in [11] by sampling 1,000 data on the second-order curves which interpolated the considered experimental data.

In addition, the optimization of the twenty-four parameters is performed by means of a new objective function (see Eq. (14)), which includes the Root Mean Square Relative Error (RMSE) referred to nondimensional head, power and efficiency, of both pump and PAT modes. Thus, pump and PAT behavior are simultaneously optimized. Instead, in [11], pump and PAT parameters were optimized separately.

$$OF = (RMSE_{\psi_k} + RMSE_{\pi_k} + RMSE_{\eta_k})_P + (RMSE_{\psi_k} + RMSE_{\pi_k} + RMSE_{\eta_k})_T \quad k = 1, \dots, N_{P,T} \quad (14)$$

The simulation model requires (i) pump geometrical information (i.e. impeller outlet diameter  $d_2$  and number of blades), (ii) pump and PAT operating point (rotational speed and flow rate and head at pump best efficiency point) and (iii) the complete pump and PAT performance curves which have to be reproduced by the simulation model.

According to the optimized parameters, both the simulated pump and PATs curves and their prediction errors with respect to the experimental data are provided in the results Section.

## PUMP AND PAT DATA

### Available field data from the literature

The field data considered in this paper for predicting pump and PAT performance curves over the entire range of operation are derived from the literature. Barbarelli et al. [13] reported the performance data acquired experimentally from twelve different centrifugal pumps, running in both pump and PAT mode. The twelve pumps are characterized by  $\Omega$  values in the range of 0.16 - 1.15. It should be noted that the study [13] only provides head and efficiency data. Thus, power was calculated by the authors of this paper. The twelve pumps are characterized by the BEP values reported in Table 1; these values were calculated by the authors of this paper by interpolating the pump experimental data documented by Barbarelli et al. [13].

The physics-based model is challenged to reproduce the performance curves of a heterogeneous fleet of machines. In fact, the volume flow rate at the BEP ranges from 5.1 l/s to 73.7 l/s and the maximum efficiency increases from 43.6% to 83.2%. Otherwise, the head at the BEP decreases from 34.7 m to 8.1 m, passing from pump #1 to pump #12. Instead, power at BEP ranges from 1.1 kW (pump #4) to 11.7 kW (pump #11).

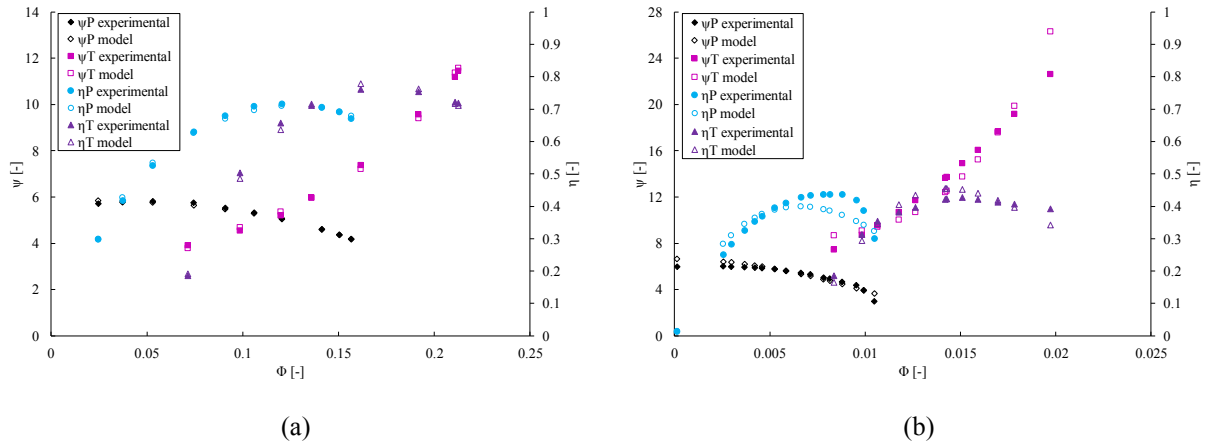
## RESULTS

### Performance curves

For the sake of brevity, Fig. 1 reports both pump and PAT experimental data reported in [13] and the values predicted by the physics-based model for the pump/PAT which allowed the lower (#9) and the higher (#1) OF values, calculated as in Eq. (14), i.e. the two pumps/PATs which are reproduced most/least accurately by the simulation model. It should be noted that nondimensional power is not sketched, since it is not reported in [13].

**TABLE 1 - Pump characteristics at the BEP**

Pump	$D$ [m]	$n$ , rps	$\Omega$ , -	$\underline{Q}$ , $10^{-3}$ m <sup>3</sup> /s	$H$ , m	$P$ , kW	$\eta$ , %
#1	0.335	24.17	0.16	6.58	34.72	5.12	43.57
#2	0.315	24.17	0.18	7.24	31.39	5.07	44.04
#3	0.250	24.17	0.24	6.87	19.62	2.42	54.94
#4	0.200	24.17	0.30	5.14	12.41	1.12	56.01
#5	0.250	24.17	0.38	16.18	19.39	4.85	65.80
#6	0.250	24.17	0.48	26.56	19.32	6.89	73.94
#7	0.160	24.17	0.54	9.71	8.53	1.20	68.43
#8	0.220	24.17	0.59	25.23	14.23	4.74	74.98
#9	0.200	24.17	0.65	23.52	12.04	3.85	72.37
#10	0.200	24.17	0.82	41.26	12.94	9.95	52.68
#11	0.200	24.17	1.06	73.72	13.44	11.72	83.15
#12	0.160	24.17	1.15	40.63	8.12	4.06	79.32



**FIGURE 1 – Experimental ([13]) vs. predicted nondimensional head and efficiency of pump/PAT #9 (a) and pump/PAT #1 (b) (full symbols: experimental data; empty symbols: simulation model)**

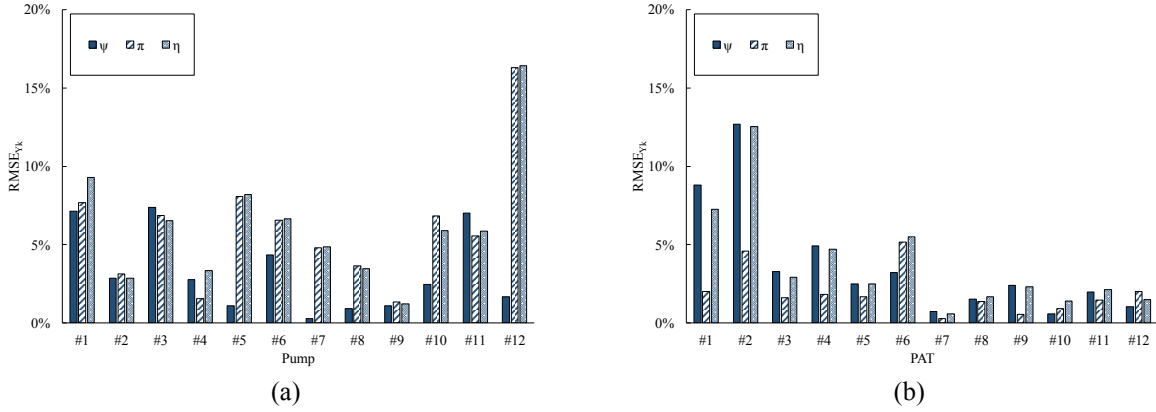
It can be observed that, even in the worst case (pump/PAT #1), both performance curves are physically sound over the entire range of operation. However, it has to be also observed that, in the worst case, some non-negligible deviations of head occur at the highest flow rates for both pump and PAT and, above all, the predicted pump efficiency curve significantly underestimates actual pump efficiency. Such deviations are quantitatively discussed below for all the considered pumps/PATs.

### Prediction accuracy

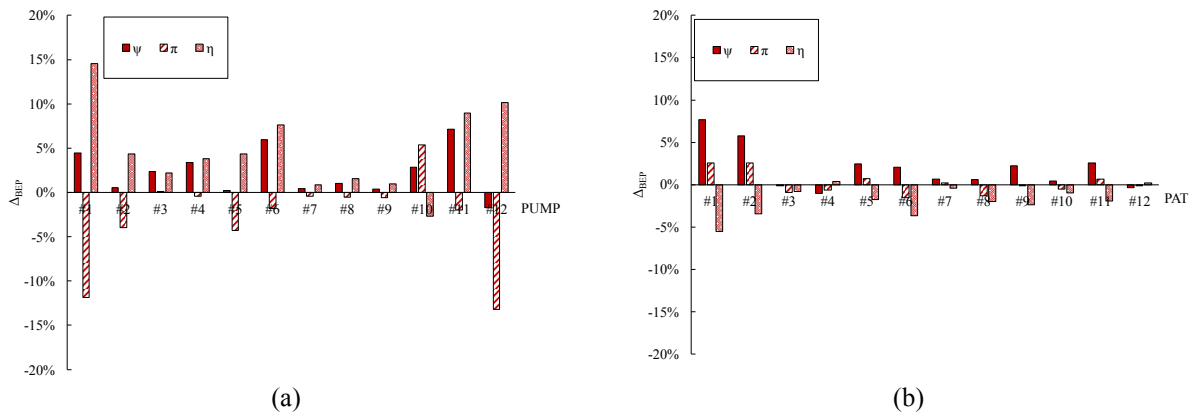
The prediction accuracy of the twelve pumps and PATs is assessed with respect to the original field data reported in [13]. The values of  $RMSE_{Y_k}$  are summarized in Fig. 2, both for pumps and PATs.

It can be highlighted that, with the exception of pump #12 and PAT #2, the RMSE values are almost independent of the considered machine and all nondimensional characteristics are calculated very accurately (RMSE values lower than 10% and, in several cases, even lower than 5%). In the two worst cases (pump #12 and PAT #2), the maximum RMSE is equal to 16.4% and 12.7%, respectively. Compared to typical prediction errors documented in the literature [10], the prediction accuracy of the simulation model is very good. Another relevant finding is that the physics-based model generally proves to simulate PAT performance curves with a higher accuracy compared to pump curves.

The accuracy of model prediction is also assessed at the BEP of both pump and PAT, as summarized in Figure 3. Negative deviations mean that the prediction model overestimates the experimental head/efficiency value at BEP. The highest absolute deviations for pump are 7.1% for head, 13.2% for power and 14.6% for efficiency, while the corresponding values for PAT are 7.7% for head, 2.6% for power and 5.5% for efficiency. Once again, by considering typical prediction errors at BEP [3, 13], such relative deviations always seem acceptable, if it is considered that the simulation model applied in this paper allows the estimation of the *complete* PAT performance curves.



**FIGURE 2** - RMSE for the twelve pumps (a) and PATs (b) with respect to experimental data [13] over the entire range of operation



**FIGURE 3** - Relative deviation of the predicted values of pump (a) and PAT (b) with respect to experimental data [13] at BEP

Finally, as a general comment, the physics-based model generally reproduces the BEP of PATs with a higher accuracy than the BEPs of the corresponding pump. Moreover, by comparing Fig. 2 and Fig. 3, it is clear that the model predicts the BEPs with a higher reliability compared to the entire head/efficiency curve, since their relative deviations at BEP are generally lower than or comparable to the RMSEs calculated on the entire range of operation.

## CONCLUSIONS

This paper presented the application of a physics-based simulation model, previously developed by the authors, to predict the performance curves of PATs on the basis of the performance curves of the respective pump. The simulation model was calibrated on experimental data taken from the literature, reporting both the pump and PAT behavior of twelve different turbomachines characterized by specific speed values in the range of 0.16 – 1.15. The model proved to be a powerful and reliable tool for estimating both pump and PAT performance curves over the entire range of operation. In fact, all the performance curves predicted by the simulation model were physically consistent over the entire range of operation. Moreover, the relative deviation of the model prediction with respect to the BEPs always seemed acceptable, compared to typical deviations of other methods available in the literature.

## NOMENCLATURE

$b$	width	$x$	parameter for pump/PAT model tuning
BEP	best efficiency point	$y$	model parameter
$c$	absolute velocity	$Y$	nondimensional performance parameter ( $\pi$ , $\eta$ , $\psi$ )

$d$	diameter	$Z$	hydraulic loss
$D$	pump nominal diameter	$\alpha$	angle between direction of circumferential and absolute velocity
$g$	gravitational acceleration	$\beta$	angle between relative velocity vector and negative direction of circumferential velocity
$H$	head	$\Delta$	relative deviation at the BEP defined as $(Y_{int}-Y_s)/Y_{int}$
$k$	index of pump/PAT ( $k=1, \dots, 12$ )	$\varepsilon$	wrap angle
$n$	rotational speed in [rps]	$\zeta$	loss coefficient
$N$	number	$\eta$	efficiency
OF	objective function	$\lambda$	angle between vanes and side disks
$P$	power	$\pi$	nondimensional power defined as $P/(\rho n^3 D^5)$
PAT	pump as turbine	$\rho$	density
$Q$	volume flow rate	$\phi$	nondimensional volume flow rate defined as $Q/(nD^3)$
RMSE	root mean square relative error	$\psi$	nondimensional head defined as $gH/(n^2 D^2)$
$s$	casing clearance	$\Omega$	nondimensional specific speed defined as $2\pi n Q^{0.5}/(gH)^{0.75}$
$u$	circumferential velocity		
<b>Subscripts and Superscripts</b>			
A	outlet casing	RR	disk friction
$ax$	axial	s	simulated
B	blade	s3	throttling
BEP	best efficiency point	sp	volute
e	experimental	st	stage
E	inlet casing	T	PAT
er	friction created by the components of axial thrust balance devices	th	theoretical
h	hydraulic, hydrostatic bearing	u	useful
$k$	index of pump/PAT ( $k=1, \dots, 12$ )	V	volumetric
La	impeller	$Y$	nondimensional parameter
Le	diffuser	$\eta$	efficiency
m	mechanical, meridional component	$\pi$	nondimensional power
P	pump	$\psi$	nondimensional head
Rec	recirculation		

## REFERENCES

1. Venturini, M., Alvisi, S., Simani, S., Manservigi, L., 2017, "Energy Production by Means of Pumps As Turbines in Water Distribution Networks", *Energies* 2017, 10, 1666; doi:10.3390/en10101666.
2. D. Novara, S. Derakhshan, A. McNabola, H. M. Ramos, 2017, "Estimation of unit cost and maximum efficiency for Pumps as Turbines" 9th Eastern European IWA Young Water Professionals conference - Budapest; 24-27 May.
3. S. Derakhshan and A. Nourbakhsh, 2008, "Experimental Study of Characteristic Curves of Centrifugal Pumps Working As Turbines in Different Specific Speeds", *Experimental Thermal and Fluid Science* 32:800–807.
4. S. Derakhshan and A. Nourbakhsh, 2008, "Theoretical, Numerical and Experimental Investigation of Centrifugal Pumps in Reverse Operation", *Experimental Thermal and Fluid Science* 32:1620–1627.
5. M. Renzi, M. Rossi, 2019, "A generalized theoretical methodology to forecast flow coefficient, head coefficient and efficiency of Pumps-as-Turbines (PaTs)", *Energy Procedia* 158:129-134.



6. M. Rossi, M. Renzi, 2018, “A general methodology for performance prediction of pumps-as-turbines using Artificial Neural Networks”, [Renewable Energy](#) 128:265-274.
7. M. Stefanizzi, M. Torresi, B. Fortunato, S. M. Camporeale, 2017, “Experimental investigation and performance prediction modeling of a single stage centrifugal pump operating as turbine”, [Energy Procedia](#) 126:589-596.
8. M. Rossi, A. Nigro, M. Renzi, 2019, “A predicting model of PaTs’ performance in off-design operating conditions”, [Energy Procedia](#) 158:123-128.
9. M. Rossi, A. Nigro, M. Renzi, 2019, “Experimental and numerical assessment of a methodology for performance of Pumps-as-Turbines (PaTs) operating in off-design conditions”, [Applied Energy](#) 248: 555-566.
10. M. Binama, W. T. Su, X. B. Li, F. C. Li, X. Z. Wei, Shi An, 2017, “Investigation on pump as turbine (PAT) technical aspects for micro hydropower schemes: A state-of-the-art review”, [Renewable and Sustainable Energy Reviews](#) 79:148-179.
11. M. Venturini, L. Manservigi, S. Alvisi, S. Simani, 2018, “Development of a physics-based model to predict the performance curves of pumps as turbines”, [Applied Energy](#), 231, pp. 343-354.
12. J. F. Gulich, Centrifugal pumps. Springer 3<sup>rd</sup> Edition 2010. Berlin Heidelberg.
13. S. Barbarelli, M. Amelio, G. Florio, 2017, “Experimental activity at test rig validating correlations to select pumps running as turbines in microhydro plants”. [Energy Conversion and Management](#) 149:781-797.
14. Optimization Toolbox™ User's Guide. The MathWorks, Inc. 3 Apple Hill Drive, Natick, MA 1760-2098. September 2016.

## APPENDIX

**TABLE A1** - Simulation model parameters

	Parameter	Meas. unit	Description
	<b>Pump</b>		
$x_1$	$d_1/d_2$	-	impeller inlet diameter / impeller outlet diameter
$x_2$	$b_2/d_2$	-	impeller outlet width / impeller outlet diameter
$x_3$	$d_3/d_2$	-	volute diameter / impeller outlet diameter
$x_4$	$b_3/b_2$	-	volute width / impeller outlet width
$x_5$	$s_{ax}/d_2$	-	axial casing clearance / impeller outlet diameter
$x_6$	$\varepsilon_{sp}$	degree	wrap angle of the inner volute
$x_7$	$\alpha_1$	degree	flow angle at impeller inlet
$x_8$	$\beta_{2B}$	degree	blade angle at impeller outlet
$x_9$	$\beta_{1B}$	degree	blade angle at impeller inlet
$x_{10}$	$\alpha_{3B}$	degree	volute cutwater camber angle
$x_{11}$	$\lambda$	degree	angle between vanes and side disks
$x_{12}$	$\zeta_{E1}$	-	inlet casing loss coefficient #1
$x_{13}$	$\zeta_{E2}$	-	inlet casing loss coefficient #2
$x_{14}$	$y_{er,P}$	-	parameter for estimating $P_{er}$ in a pump
	<b>PAT</b>		
$x_{15}$	$\phi_{BEP,T} / \phi_{BEP,P}$	-	Ratio of PAT/pump flow rate at the BEP
$x_{16}$	$\psi_{BEP,T} / \psi_{BEP,P}$	-	Ratio of PAT/pump head at the BEP
$x_{17}$	$\zeta_{La,BEP}$	-	PAT losses at the BEP
$x_{18}$	$\zeta_{Le,BEP}$	-	PAT diffuser or volute losses at the BEP
$x_{19}$	$YZ$	-	parameter for estimating hydraulic losses
$x_{20}$	$y_{S3,1}$	-	parameter #1 for estimating throttling losses $P_{s3}$
$x_{21}$	$y_{S3,2}$	-	parameter #2 for estimating throttling losses $P_{s3}$
$x_{22}$	$y_{er,T}$	-	parameter for estimating $P_{er}$
$x_{23}$	$y_{u,1}$	-	parameter #1 for estimating the useful power P
$x_{24}$	$y_{u,2}$	-	parameter #2 for estimating the useful power P