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The numerical simulation of steady and unsteady flows in channel networks using an Extended Riemann Problem at the junctions

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Abstract: In this work, an analysis of the numerical simulations of the channel flows in symmetrical and asymmetrical networks using the one-dimensional shallow water equations (SWE) was performed. Inner boundary conditions must be imposed at the junction nodes, and their numerical treatment is crucial to develop reliable mathematical models. Classical methods to provide the inner boundary conditions that are extensively based on the energy and the momentum balances at the junction nodes. However, such methods suffer their empirical formula based on experimental coefficients, which limit their applicability in practical applications (river flow, dam flow). As an alternative to such methods, we propose to solve an Extended Riemann Problem (ERP) at the junction nodes, consistently with the physical and mathematical properties of the SWE. No empirical coefficients are involved, and this approach can be easily used in practice. Furthermore, the ERP approach is supported by theoretical evidences (i.e., existence and uniqueness theorems), that ensure the consistency of the numerical scheme. This theoretical analysis can be found in literature for a star network of three identical channels, and it is extended to general network configurations by its authors. Considering subcritical conditions, different flow simulations are performed in symmetrical and asymmetrical confluences for both steady and unsteady flows. The numerical solutions are compared to experimental data from literature and analytical solutions obtained by the authors. The overall results show the suitability of the ERP method to successfully provide the inner boundary conditions for wide range of applications.

Keywords: Channel networks, junctions, Riemann problem, shallow water equations, subcritical flows.

1. Introduction

Channel networks are an important research topic in hydraulics (Chow 1959), in the context of either natural systems such as river basins or artificial networks such as irrigation systems. Extensive studies have taken into account the physical structure of the network flows, see for example (Gurram et al. 1997; Herrero et al. 2016; Hsu et al. 1998a; Taylor 1944) and the references therein. Providing a simulation tool to understand large channel networks is important from multiple different points of view. For example, in hydraulic engineering, one might be interested in designing a control strategy at the channel junctions to govern the whole network flow. Concerning the most important previous contributions, the classical methods to incorporate the junctions in a one-dimensional (1D) channel network in a numerical model were noticed to suffer from empirical formulations that limit their applicability. An example of such methods can be found in (Gurram et al. 1997; Hsu et al. 1998a; Shabayek et al. 2002).

In literature, both theoretical and numerical investigations of the junction networks are studied. Among those studies, Herrero et al. 2016 recently discusses the junction flow structure. Focusing on the junction flow features, the most significant parameters are the junction configuration, the angles between the branches, and the bed levels (Biron et al. 1996). The contributions in (Ghostine et al. 2010, 2012; Kesserwani et al. 2008) studied the junction flows numerically, providing 1D numerical simulations of such flows. However, the junction flow shows to have essentially a three-dimensional (3D) structure (Wang et al. 2007). Among the numerical studies, Kesserwani et al. (2008) provided sufficient information to understand the limitations of the classical junction models, studying the junction flows in a flume.

The 1D shallow water equations (SWE) are solved to model the branch flows. Since the 1D SWE are singular at the junction node, inner boundary conditions must be used to model the connection of the branches. Such boundary conditions are based on mass, energy, and momentum conservations, for consistency with the physical principles governing the SWE. The different approaches are denoted in literature Equality model (Akan 1981), Gurram model (Gurram et al. 1997), Hsu model (Hsu et al. 1998a), and Shabayek model (Shabayek et al. 2002). The Equality model shows the poorest results to reproduce the junction flows for Froude number (F) greater than 0.35, while the

other models (i.e., Gurrum, Hsu, and Shabayek models) show more or less acceptable results. However, those models are vulnerable, due to the presence of empirical coefficients that must be tuned and are not case-independent.

Recently, Elshobaki et al. (2017) provided the theoretical background to use the solution of an Extended Riemann Problem as inner boundary condition, as an alternative to the classical approaches. The Riemann approach is promising to solve symmetric and non-symmetric network with either continuous or discontinuous bottom for subcritical flows. However, the validation of such approach is missing in literature. Thus, the purpose of this work is to validate the Riemann approach for practical applications.

To achieve this aim, experimental data found in literature (Hsu et al. 1998a,b; Pinto Coelho 2015) are used. Unluckily, the experimental data are limited to subcritical steady flows in asymmetric confluences (Hsu et al. 1998a,b; Pinto Coelho 2015). Therefore, an unsteady flow test case is also used to validate the Riemann approach: the analytical solutions (Elshobaki et al. 2017) of the Extended Riemann Problem in a Y-shaped confluence.

In this work, the 1D SWE are solved for each branch using the Dumbser-Osher-Toro (DOT) Riemann solver (Dumbser and Toro 2011). The numerical scheme has to be completed with proper boundary conditions. The inflow-outflow boundary conditions are provided according to (Abbott 1966; García-Navarro and Savirón 1992) while the internal boundary conditions, needed at the junction node, are provided by the Riemann approach. It is worth noting that the application of the Riemann approach is not limited to a specific numerical scheme, see (Briani et al. 2016; Chang et al. 2016; Toro 2009).

The rest of the paper is organized as following: In section 2, the mathematical model and its numerical discretization are presented. In section 3, the Riemann approach is introduced to supply the internal boundary conditions for 1D simulations. The Riemann approach is solved iteratively using the hybrid Newton method (Powell 1970). The numerical results of steady and unsteady flow simulations are shown in section 4. Finally, conclusions about the results appear in section 5.

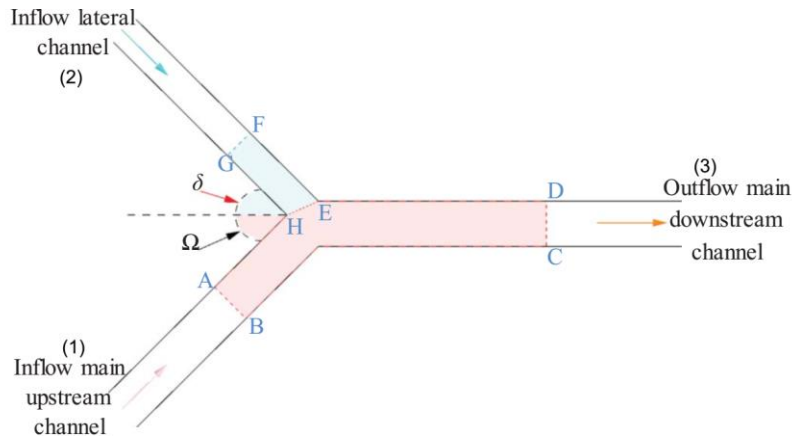


Figure 1. Channel network of three branches with equal width.

2. One Dimensional Shallow Water Equations in Channel Network

In this section the 1D SWE for a single open channel is briefly described.

2.1. Mathematical Model

The 1D SWE are written as follows:

$$\frac{\partial U}{\partial t} + A(U) \frac{\partial U}{\partial x} = 0 \quad (1)$$

Where U is the vector of conservative variables, $A(U)$ is the system matrix and x (m) and t (s) are the space and the time, respectively. The state variables and the system matrix are:

$$U = (h \ hu \ z), \ A(U) = (0 \ 1 \ 0 \ gh - u^2 \ 2u \ gh \ 0 \ 0 \ 0) \quad (2)$$

Where h m is the water depth u ms^{-1} is the flow velocity, z m is the bottom elevation; g m^2s^{-1} is the gravity. Equation (1) is written in quasi-linear form. Because the junction node spatial extension is small in comparison to the overall length in each branch, the friction can be neglected.

2.2. Numerical Method

In this paper, the finite volume method (Toro 2009) based on the Dumbser-Osher-Toro (DOT) solver (Dumbser and Toro 2011) is used to discretize the mathematical model. However, any other solvers such as the HLL, Rusanov, or Roe can be used. This model is summarized by the following equation:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x} (D_{i+\frac{1}{2}}^- + D_{i-\frac{1}{2}}^+) \quad (3)$$

Where the conservative variables are averaged over the i th spatial domain and n th time step. $\Delta x = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$ is the uniform spatial step and $\Delta t = t^{n+1} - t^n$ is the time step. Choosing a linear integration path $\psi(s)$ (Dumbser and Toro 2011) the parameter s belong to the interval $[0,1]$:

$$\psi(s) = \psi(U^-, U^+, s) = U^- + s(U^+ - U^-) \quad (4)$$

The fluctuations $D_{i\pm\frac{1}{2}}^\pm$ becomes:

$$D_{i\pm\frac{1}{2}}^\pm = \frac{1}{2} \left(\int_0^1 A(\psi(s)) ds \pm \int_0^1 |A(\psi(s))| ds \right) (U_{i+1} - U_i) \quad (5)$$

Using the Gauss-Legendre quadrature rule, Eq. (6) can be written as:

$$D_{i\pm\frac{1}{2}}^\pm = \frac{1}{2} \left(\sum_{j=1}^G \omega_j [A(\psi(s_j)) \pm |A(\psi(s_j))|] \right) (U_{i+1} - U_i) \quad (6)$$

where G is a given number of point in the quadrature in the interval $[0,1]$ with nodes s_j and weights ω_j , see (Stroud 1971). The scheme in Eq. (3) must be completed with boundary conditions. We treat each branch of the network as a single open channel. Therefore, there are two types of boundary conditions needed. The first type is the well-known inflow and outflow conditions. Since we consider only the subcritical flow, one condition is needed at the inflow boundary and one at the outflow boundary. The specific discharge is provided at the inflow boundaries. At the outflow boundary, the water depth is provided. Both boundary conditions are numerically treated according to the method of characteristics (Abbott 1966; Kesserwani et al. 2008). The second type is the inner boundary conditions, which connect the branches of the network at the junction. Here, it is proposed that those conditions are supplied by the solution of an extended Riemann problem (Elshobaki et al. 2017), defined at the junction. In the next section, the Riemann approach is analyzed, which provides the internal conditions at the junction (i.e., the junction model).

3. The Riemann Approach at the Junction

The Riemann Problem for the SWE in junction network is first investigated in (Goudiaby and Kreiss 2013) for symmetric network with continuous bottom and recently extended in (Elshobaki et al. 2017) for a non-symmetric network with a discontinuous bottom. The Extended Riemann Problem (ERP) is defined by Eq. (1) for each branch and coupled with piecewise initial constant states (h_o, u_o) in the three branches as a Riemann problem established at the junction of three channel and distinguished from the classical Riemann problem in a single channel. Considering the channel network shown in Fig. 1, the Riemann solution is obtained solving the following nonlinear system:

$$Q_1 + Q_2 = Q_3 \quad (7)$$

$$\frac{u_1^2}{2g} + h_1 + z_1 = \frac{u_2^2}{2g} + h_2 + z_2 \quad (8)$$

$$\frac{u_2^2}{2g} + h_2 + z_2 = \frac{u_3^2}{2g} + h_3 + z_3 \quad (9)$$

$$u_1 - u_{1o} = -f(h_{1o}, h_1) \quad (10)$$

$$u_2 - u_{2o} = -f(h_{2o}, h_2) \quad (11)$$

$$u_3 - u_{3o} = f(h_{3o}, h_3) \quad (12)$$

$$f(h_o, h) = \{2(\sqrt{gh_o} - \sqrt{gh}), h < h_o \quad (h_o - h) \sqrt{\frac{g}{2} \left(\frac{1}{h_o} + \frac{1}{h} \right)}, h \geq h_o \quad (13)$$

Where Q m^3s^{-1} is the volumetric water discharge, u is the water velocity. The sub-indices 1, 2, and 3 refer to the network branch as the upstream main channel, the lateral channel, and the downstream main channel, respectively.

The sub-index o refers to the initial value of the associated variable. Equation (7) is the continuity equation, while Eq. (8) and Eq. (9) are derived imposing the total head equality between branches as justified in (Valiani and Caleffi 2009) for a single open channel. The remaining equations are obtained using the Rankine-Hugoniot and Riemann invariant relationships. The system (7)-(13) is solved iteratively using hybrid Newton method (Powell 1970). Here, the Riemann approach that represent by Eqs. (7)-(13) is well established in terms of existence and uniqueness of the solution admitted as shown in (Elshobaki et al. 2017).

4. Results

4.1. Steady State Results

In this subsection, we give the simulation results of the water flow in a channel network with different junction angle δ and a fixed $\Omega = 0^\circ$. The test cases (Hsu et al. 1998a,b; Pinto Coelho 2015) were carefully chosen because most of the existing work in this field was based on them. Although other test cases are available, but they are omitted due to the space limit. Figure 2 shows the relation between the depth ratio and the discharge ratio, where the Riemann approach is used to provide the predicted depth and discharge at the junction. The numerical solutions were compared with experimental data (Hsu et al. 1998a,b). There is a good match between the Hsu et al. (1998a) data and the numerical solution produced by the Riemann approach, for junction angle of 30° , 45° , and 60° ; between the Hsu et al. (1998b) data and the Riemann approach for the junction angle of 90° . Table 1 shows the corresponding errors, defined as the non-dimensional difference between the numerical solution and the experimental data. The percentage error (E) is computed according to the following equation:

$$E = 100 \times \left| \frac{Y_e - Y_n}{Y_e} \right| \quad (11)$$

where Y_e is the depth ratio between the upstream main channel and the downstream main channel. Y_n is the computed depth ratio using the Riemann approach. According to Table 1, the errors are small in comparison with Hsu et al. (1998a) data ($E=3\%$) where the effect of the junction angle appears to be small. On the other hand, the error is doubled for the Hsu et al. (1998b) data with the 90° junction angle. However, the error remains acceptable according to the error threshold ($E=8\%$) suggested in (Kesserwani et al. 2008).

Figure 3 presents the comparison between the Pinto Coelho (2015) data and the Riemann approach, where the junction angle are 30° and 60° . A good match between the numerical solutions and the Pinto Coelho (2015) data is noticed. Table 1 shows that the error remains bounded below the critical error value (Kesserwani et al. 2008). It can be noticed also that the error for the junction angle of 60° is increased up to 5.6%, which is greater than the one in the previous test case with the same junction angle. This might be due to the different combination between the Froude number and the junction angle in each test case. However, the relevant effect of the junction angle does not overcome the Riemann approach to match experimental data in both cases.

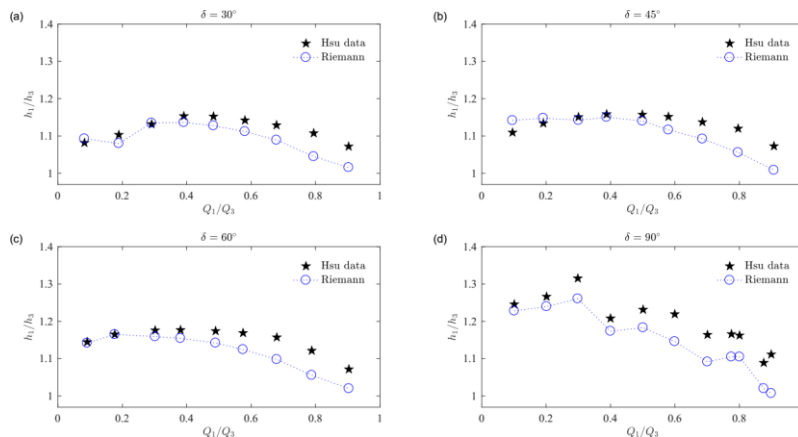


Figure 2. Numerical solutions obtained by Riemann approach (circles) compared with experimental data, for (Hsu et al. 1998a) experiments, for junction angle $\delta = 30^\circ, 45^\circ$ and 60° ; for (Hsu et al. 1998b) experiments, for $\delta = 90^\circ$ (stars).

Table 1. Percentage errors between the Riemann approach and experimental data (Hsu et al. 1998a,b).

Junction angle	Riemann approach
30°	2.68
45°	2.87
60°	2.88
90°	5.84

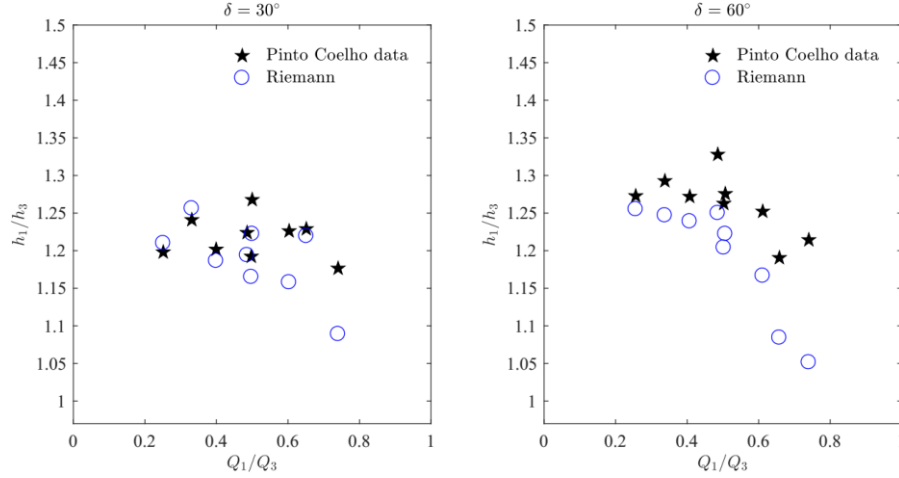


Figure 3. Numerical solutions obtained by Riemann approach (circles) compared with experimental data, for (Pinto Coelho 2015) experiments, for junction angle $\delta = 30^\circ$ and 60° (stars).

Table 2. Percentage errors between the Riemann approach and experimental data (Pinto Coelho 2015).

Junction angle	Riemann approach
30°	2.83
60°	5.62

4.2. Unsteady State Results

We present the numerical solution of the water flow in the channel network shown in Fig. 1 with $\delta = 45^\circ$ and $\Omega = 45^\circ$. The numerical solution is compared to the analytical one (Elshobaki et al. 2017). The simulation started with piecewise constant initial discharges and depths of $0.1 \text{ m}^3\text{s}^{-1}$ and 0.5 m in the main upstream channel and in the lateral channel, and $0 \text{ m}^3\text{s}^{-1}$ and 1.0 m in the main downstream channel. Figure 4 shows the simulation results, showing the evolution of the discharge in the spatial domain in each branch of the network. The analytical solution is plotted in solid lines and the numerical one in dot-dashed lines. The l_1 error is shown in Table 3, which is computed in according to the following equation:

$$e_k = \Delta x \sum_{i=1}^N |Q_k^*(x_i, t) - Q_k(x_i, t)| \quad (14)$$

where k is the branch index: $k = 1$ refers to the main upstream channel, $k = 2$ to the lateral channel, $k = 3$ to the main downstream channel. N refers to the number of mesh cells and Q^* is the analytical discharge according to (Elshobaki et al. 2017).

Figure 4 shows the perfect match between the numerical solution and the analytical one (Elshobaki et al. 2017). The l_1 error is very small in all the three branches, as reported in Table 3. This result proved the Riemann approach capability to predict the flow depth and the flow discharge at the junction network for unsteady flow. This result is not found in existing literature concerning classical methods.

Table 3. l_1 error in the computed discharge in each branch, comparing the Riemann approach and the analytical one (Elshobaki et al. 2017).

Branch	The Riemann approach
Branch ₁	4.2644E-3
Branch ₂	4.2644E-3
Branch ₃	8.8333E-3

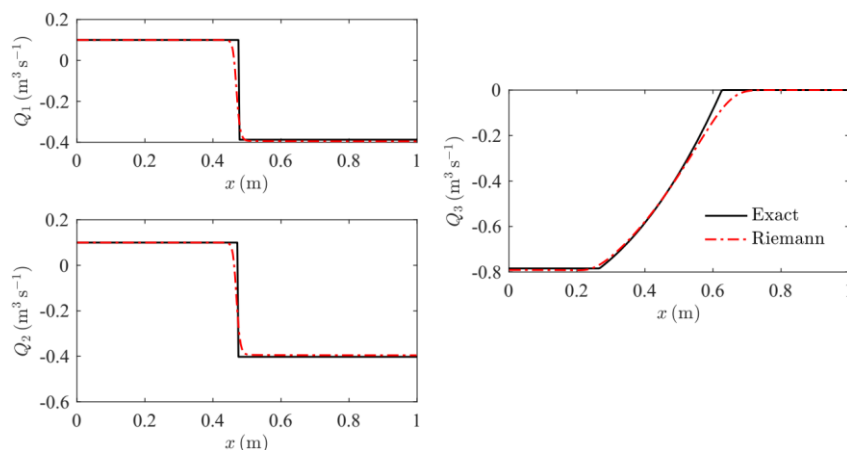


Figure 4. Analytical discharge profiles in each branch according to (Elshobaki et al. 2017), solid line, compared with the numerical solution using the Riemann approach, dash dot lines. Time is 0.2 s.

5. Conclusions

This paper uses an Extended Riemann problem (ERP) solution to model the inner boundary conditions for numerical simulation of open channel network flows. The classical methods presented in literature suffer their empirical formulations, which limit their practical applicability. On the contrary, the proposed ERP methodology does not depend on empirical formulations that require tuning parameters. Experimental data (Hsu et al. 1998a,b; Coelho 2015) are used to validate the ERP approach for steady state flow in asymmetrical confluences. Moreover, an analytical solution (Elshobaki et al. 2017) for unsteady flows is used to validate the ERP approach in Y-shaped confluences. Good results in matching both experimental data and analytical solution are shown, as the error for the steady flow simulation remains bounded below the critical 8% error value estimated in (Kesserwani et al. 2008). Very good matches between the numerical solution and the analytical one is obtained. Present results are valid for subcritical flows only, as supported by theoretical findings in (Elshobaki et al. 2017). However, obtaining similar results concerning supercritical flows in the future appears to be a reasonable target from the mathematical point of view.

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