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Network design through the phasing of construction approach

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Abstract

This paper presents a new approach for design of water distribution mains aimed at considering the phasing of construction. It makes it possible to identify, on prefixed time steps or intervals (for instance 25 years), the upgrade of the construction rendering the network able to satisfy, during the expected life of the system, growing nodal demands related to the increment in the population served. An optimization methodology was set-up and applied to a simple case-study, where two different scenarios were considered concerning the growth of the network. Results showed that this approach is able to yield better results when compared with the single flow design, because it enables short term construction upgrades to be performed while keeping a vision of the expected long term network growth.

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Keywords: water distribution networks; design; multi-objective optimization; phasing of construction.

1. Introduction

Most of the studies in the scientific literature related to the design of water distribution networks [1,2,3,4,5] do not give an answer to the practical problem of the phasing of construction, which is particularly relevant to distribution mains. In actual fact, they were developed laying on the two following restrictive assumptions:

- design is performed 'statically' by referring to one or more theoretical operating condition corresponding to a single design date (fixed water demands, often referred to the hour of maximum demand for a future scenario(s)

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positioned at the end of the assumed life cycle for the water distribution system, i.e. the heaviest loading condition for the water distribution system);

- all the construction is done in a single phase: there is no gradual growth/build-out in the system (this assumption clearly refers to a theoretical situation since real water distribution systems are usually subject to expansion or modifications related to the social, commercial and industrial evolution of the area).

However, the two assumptions reported above are inconsistent with the design and maintenance practice as is and should be performed in reality, i.e. 'dynamically' in a bid to follow the expansion of urban centers, as was proposed by Lansey et al. [6] and Kang and Lansey [7] in the context of water distribution systems and water reclamation systems respectively. In particular, as far as water distribution systems are concerned, Lansey et al. [6] set up a single objective methodology aimed at assessing optimal maintenance scheduling, without, however, presenting a thorough comparison with the maintenance based on single static design scenario. With this shortcoming in mind, this paper presents a multi-objective optimization method to account for the phasing of construction in the design of water distribution mains and shows the benefits of adopting this new design approach in contrast with basing design on a single static design scenario.

In the next sections, a methodology set up with the objective to address the problem of phasing of construction via an optimization process is initially described and then applied to a simple case-study (the two-loop network of Alperovits and Shamir [1]). The description of the results is followed by the conclusions. More details about the methodology proposed and the applications can be found in paper [8].

2. Methodology

2.1. Overview

The present methodology makes it possible to identify how a water distribution network can be designed and upgraded in phases in order to guarantee high level service to users when an increase in demand and an extension of the layout are predicted to occur in the (near) future. In order to set the phasing of the construction, it is first important to subdivide the whole construction period T into n phases of length Δt, with n being an integer number. While the overall logic can be applied to any combination of time step sizes, the adoption of a prefixed time step Δt entails that at the generic year $(k-1)\Delta t$ (at the beginning of the generic phase), with k being an integer number within the range $[1\text{: } n]$, the network is supposed to be upgraded considering the demand and layout predicted at year $k\Delta t$ (i.e. at the end of the current phase, alias at the beginning of the subsequent phase). In order to guarantee the efficiency of the network in the whole construction time, a number of upgrades equal to n (i.e. equal to the number of phases) has to be performed. The following optimization methodology, based on the use of a multi-objective genetic algorithm, can be applied in order to optimize network upgrades over time.

2.2. Network upgrade optimization

At the generic year $(k-1)\Delta t$ (i.e. at the beginning of the kth phase/interval) in order to supply water with acceptable service pressure in the network within the next Δt years, it is necessary to add $n_{p,k}$ pipes, among which $n_{p1,k}$ have to be inserted in new sites (where no pipes were present earlier) in order to reach new demanding nodes and $n_{p2,k}$ have to be laid in parallel to previously existing pipes. Since the whole construction period is divided into n upgrade phases, the decisional variables of the network upgrade problem are then the diameters to be adopted for the $n_p = \sum_{k=1}^{n} n_{p,k} = \sum_{k=1}^{n} n_{p1,k} + \sum_{k=1}^{n} n_{p2,k}$ pipes, to be chosen in a prefixed set of n_D diameters. It is worth remarking that at year 0 (i.e. at the beginning of the construction period), a certain number n_{p0} of pipes *may* already be present in the network and the numbers $n_{p1,1}$, $n_{p1,2}$,…, $n_{p1,n}$ of pipes to be inserted in new sites at the beginning of the first, second, … and n-th upgrade phase *are assumed to be known*. At year 0, i.e. at the beginning of the first upgrade phase, the maximum number of pipes $n_{p2,1}$ that may be laid in parallel to previously existing pipes is equal to n_{p0}; at year Δt, i.e. at the beginning of the second upgrade phase, the maximum number of pipes $n_{p2,2}$ that may be laid in parallel to previously existing pipes is equal to $n_{p0} + n_{p1,1}$, since the n_{p0} pipes already present before the beginning of the construction can be triplicated and the $n_{p1,1}$ pipes inserted at the beginning of the first upgrade phase can be duplicated; at year $2\,\Delta t$, i.e. at the beginning of the third upgrade phase, the maximum number of pipes $n_{p3,2}$ that may be laid in parallel to previously existing pipes is equal to $n_{p0} + n_{p1,1} + n_{p1,2}$; then, at the generic year $(k-1)\Delta t$, i.e. at the beginning of the kth upgrade phase, the maximum number of pipes $n_{p2,k}$ that may be laid in parallel to previously existing pipes is equal to $n_{p0} + \sum_{i=1}^{k-1} n_{p1,i}$. As far as the laying of parallel pipes is concerned, it is worth highlighting that parallel pipes do not necessarily have to be laid in the same street. The pipes may be laid in a parallel street or right-of-way that may not have existed at previous construction times.

A multi-objective optimization can then be performed in order to assess the n_p diameters in order to obtain optimal network upgrade configurations in the trade-off between cost and reliability. The whole present worth construction cost C, first objective function of the optimization process, can be evaluated as the sum of the present worth costs $C_{k\,\Delta t}$ of the n upgrades that is:

$$C = \sum_{k=1}^{n} C_{k\Delta t} \tag{1}$$

with:

$$C_{k\Delta t} = \frac{\sum_{m=1}^{n_{p1,k}} c_m L_m + \sum_{j=1}^{n_{p2,k}} c_{p,j} L_j}{\left(1+R\right)^{(k-1)\cdot\Delta t}} \tag{2}$$

where L is the length of the pipes which have to be introduced and c and c_p are the unit cost associated with the diameters of the pipes to be laid in new sites (as numerous as $n_{p1,k}$) and in parallel to previously existing pipes (as numerous as $n_{p2,k}$). In general, for each diameter, unit cost c_p can be increased with respect to c in order to take account of the fact that laying a pipe parallel to another pipe may be more expensive than installing the same pipe in a new site.

The second objective function of the optimization process, representative of network reliability, is the minimum pressure surplus IS *over the whole construction time*, which can be calculated as:

$$\mathrm{IS}_{\min} = \min_{k} \mathrm{IS}_{k\cdot\Delta t} \text{ with } k \text{ ranging between 1 and } n \tag{2}$$

where $\mathrm{IS}_{k\,\Delta t}$ is the pressure surplus that network features at the end of the kth upgrade phase, i.e. at time $k\Delta t$, given by $min(h_{k,r} - h_{des,k,r})$ where r ranges between 1 and nn_k (number of nodes in the network at time $k\Delta t$) and $h_{k,r}$ indicates the pressure head value obtained at the r-th network demanding node by means of a *demand-driven* network simulation model [9] considering nodal demands at time $k\,\Delta t$ and $h_{des,k,r}$ is the requested (desired) pressure head at the j-th node at the time $k\,\Delta t$. It is worth highlighting that formula (3) gives the same weight to the pressure surpluses at different times; in other words, no surplus present value monetization was considered in the objective function $\mathrm{IS}_{\min}$.

In this study, the multi-objective optimization, aimed at minimizing the cost C and maximizing pressure head surplus $\mathrm{IS}_{\min}$, is performed by means of the NSGAII genetic algorithm [10].

In the genetic algorithm, each individual of the population is encoded with n_p genes, representing the IDs of the pipe diameters to be installed in the network; in particular, $\sum_{k=1}^{n} n_{p1,k}$ genes take on values within the range 1-n_D; the other $\sum_{k=1}^{n} n_{p2,k}$ genes, instead, take on values within the range 0-n_D; taking into account value 0 in the latter case helps us considering the possibility that, in some sites, the parallel pipe does not have to be laid since the pipe(s) previously laid already meet the demands. Incidentally, the $\sum_{k=1}^{n} n_{p1,k}$ genes and the $\sum_{k=1}^{n} n_{p2,k}$ genes are consistently ordered (within each individual) with the phases considered in the construction period. It is worth highlighting that as a result of the encoding of individual genes, after the appearance of the generic pipe site at a certain year, the methodology provides for laying of at least one pipe in that site; in other words, the "no pipe option" in an existing pipe site is not contemplated by the methodology because we are assuming there are users along that line which would need service even though their demands are grouped at nodes.

During the optimization process, a penalty function $-pen(\mathrm{IS_{min}})$, with pen being a high number, can be introduced into objective function (2) in order to penalize network configurations with too low (negative) value of the pressure head surplus $\mathrm{IS_{min}}$ (values of $\mathrm{IS_{min}}$ far below value 0).

In the initial population of the genetic algorithm, it is convenient to insert two prefixed individuals; in particular, all the genes of the first prefixed individual are assigned the lowest values (= 1 or 0 whether the generic gene refers to one of the $n_{p1,k}$ or $n_{p2,k}$ pipes, respectively, to be laid at the kth time, whereas all the genes of the second prefixed individual are assigned the highest values, i.e. n_D). These individuals facilitate the convergence of the algorithm towards cheap and expensive solutions, respectively.

At the end of the optimization process, a Pareto front of optimal solutions in the space "total present worth cost C-$\mathrm{IS_{min}}$" is obtained. Each optimal solution encodes the diameters of the pipes that have to be laid in the various sites at the various time steps.

3. Applications

3.1. Case study

The case study considered here makes reference to the network layout of Alperovits and Shamir [1], made up of 6 nodes with outflow and 8 sites for pipe laying (Fig. 1a). The network is fed by n_0 = 1 reservoir (node 1), which presents a value of the head equal to 60 m with respect to the nodes with the lowest ground elevation (nodes 2 and 5). Nodal data in terms of ground elevation and demand are reported in Table 1 in paper [8]. All possible connections between nodes (i.e. the first pipe to be laid and the subsequent potential parallel pipes) and pipe Hazen Williams roughness coefficients were assumed equal to 1000 m and 130, respectively. The network is a traditional benchmark for the problem of network optimal design and is herein considered to be an example of a network of distribution mains.

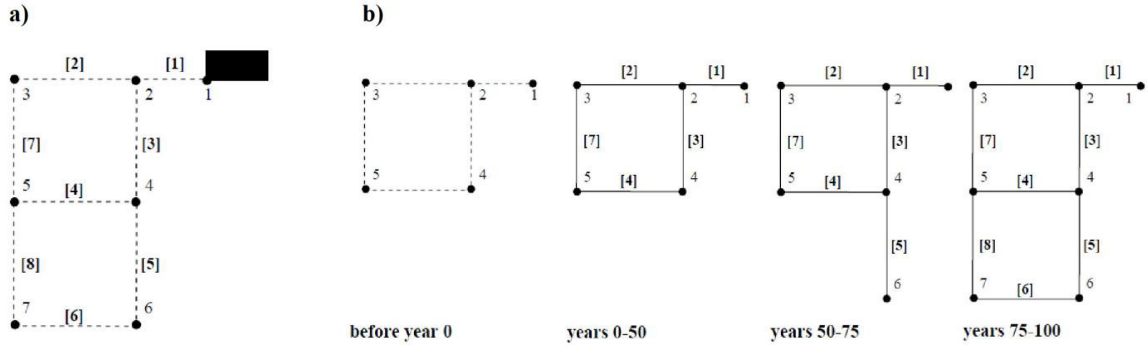

Fig. 1. (a) network of Alperovits and Shamir [1]; (b) phases of network construction.

The unit costs c relative to the pipe diameters which can be installed in the network are shown in Table 2 in paper [8]. These unit costs are multiplied by 1.2 raised to n_{par} in the case of network upgrades obtained by laying parallel pipes to pipes previously laid, where n_{par} is the number of parallel pipes already present in the site where the new pipe has to be positioned; this penalty makes it possible to take account of the fact that the insertion of a parallel pipe is more expensive and complicated than the insertion of the first pipe and such cost and complication increase as the number of pipes already laid grows.

The sample case study contains a number of assumptions which limit its general applicability:

- Costs, discount rate and demands are known with certainty. Engineers tend to use conservative designs to account for demand uncertainty and assume that cost uncertainty is consistent across alternatives.

- There are no pumps and therefore energy costs are not accounted for. This will mostly affect large transmission mains.
- Analysis of shutdowns and valving are not included. The problem addressed in this study relates to large transmission mains and often the smaller distribution mains can support system during temporary shutdown of transmission mains.
- There is only a single pressure zone so decisions about boundaries are not included.
- Only peak demand conditions are considered but leakage costs are not taken into account.
- Decrease in pipe resistance due to aging is not considered.

The effects of each of these assumptions will be analyzed in later studies.

In the case study, for the sake of simplicity, a uniform demand growth (without uncertainty) at network demanding nodes over a period of 100 years was considered; in particular, for each node the following linear demand growth model was considered:

$$D(t) = D_0\left[1+\frac{t-t_0}{100}\right] \tag{4}$$

where $D(t)$ is the demand at year t and D_0 is the demand at time t_0, initial time instant for the node.

In the applications described below, a scenario representative of a network whose topology changes across time is considered (Fig. 1b). In this scenario, expansion takes place during the construction period: whereas demanding nodes 2, 3, 4, 5 are always present in this scenario (t_0 = 0), demanding node 6 is added at year 50 (t_0 = 50) and demanding node 7 appears at year 75 (t_0 = 75); as to pipes, pipe sites 1, 2, 3, 4, 7 are always present in this scenario, pipe site 5 is added at year 50 and pipe sites 6 and 8 appear at year 75.

For the application of the genetic algorithm described in the "Methodology" section, three different kinds of optimization were performed (hereinafter indicated as optimizations I, II and III – see Table 3 in paper [8]) for each of the network scenarios presented above. In particular, optimization I was performed considering a T = 100 year long planning horizon. As regards network upgrades, a number of upgrade phases n = 4 and thus a time step Δt equal to 25 years were considered (this time step is a typical time duration considered in water systems planning). For cost conversion to value at year 0, a discount rate R = 4 % was adopted. Optimizations II, instead, concerned a T = 25 year long planning horizon. For network upgrades, a number of upgrade phases n = 1 and thus a time step Δt equal to 25 years were considered. Finally, optimization III was carried out using a T = 100 year long planning horizon, a number of upgrade phases n = 1 (and thus a time step Δt equal to 100 years). As a matter of fact, when used for optimizations II and III, the multi-step methodology described in the "Methodology" section collapsed in a traditional single step methodology (due to n = 1), where the single reference condition as to network layout and nodal demands is the situation at the end of the planning horizon considered (25 and 100 years). The rationale behind the choice to perform optimizations II and III was to create a base of comparison for optimization I, with the final objective to assess how phasing of construction affects network design.

It is worth stressing that each type of optimization was applied to each scenario. In particular, optimization I was performed assuming head pressure values h_{des} variable in space and time according to Table 5 in paper [8] (variations in h_{des} are representative of the fact that the average height of the buildings served by the node may vary in time), while optimization II and III were performed assuming the values for h_{des} of Table 5 at the end of the period of 25 years and 100 years, respectively, for the nodes served in these time instants.

In optimization I, a population of 640 individuals (= 20 × number of (potential) pipe sites × number of phases = 20 × 8 × 4) and a total of 1600 generations (= 2.5 × number of individuals) were considered; in optimizations II and III, which entail a lower number of individual genes (encoding only the diameters of the pipes that have to be laid in a single time step) than optimization I, both the population and the total number generations were reduced to one half (number of individuals = 320 and number of generations = 800). During the optimizations, penalty function $-pen(\mathrm{IS}_{\min})$, in which coefficient pen was set at 10^{15}, was applied to network configurations featuring $\mathrm{IS}_{\min} < -10$, thus affecting insignificantly optimization results relative to network configurations of the Pareto front featuring ISmin ≥ 0, which are the most interesting from the practical point of view.

3.2. Results

The Pareto fronts of solutions obtained by means of optimizations I, II and III of the Methodology described above are reported in the graphs a, b and c respectively in Fig. 2. Each graph reports the fronts relative to both network growth scenarios 1 and 2. Each of these fronts comprises optimal solutions in terms of trade off between total present worth cost and pressure surplus. In the graph in Fig. 2, only configurations featuring pressure surplus greater than or equal to 0 were considered. In particular, for graph 2a relative to optimization I, the total present worth cost is intended to be the sum C of the present worth costs C_k of the various upgrades whereas the pressure surplus is intended to be IS_{min}, i.e. the minimum value of pressure surplus over the whole construction period. For graph 2b relative to optimization II, the total present worth cost is intended to be C_{25}, i.e. the present worth cost of the single 25 year long upgrade whereas the pressure surplus is intended to be IS_{25}, i.e. the value of the (minimum) pressure surplus at the end of the single 25 year long upgrade. Finally, for graph 2 c relative to optimization III, the total present worth cost is intended to be C_{100}, i.e. the present worth cost of the single 100 year long upgrade whereas the pressure surplus is intended to be IS_{100}, i.e. the value of the (minimum) pressure surplus at the end of the single 100 year long upgrade.

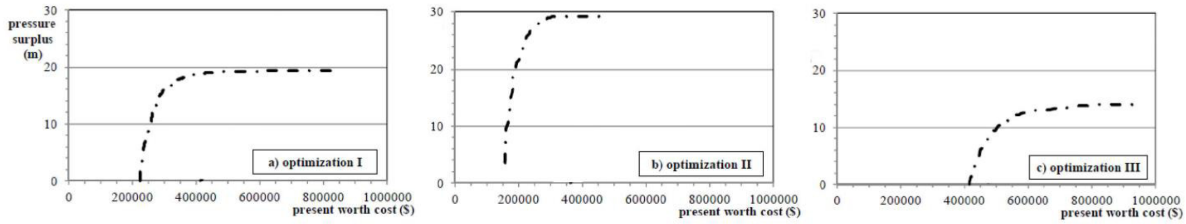

Fig. 2. Optimizations I (a), II (b) and III (c) – Pareto front of optimal solutions in the total present worth cost – pressure surplus space (see text).

As was expected, in all the cases an increase in the total present worth cost provides an increased network pressure surplus along the Pareto front. The curves in the graphs in Fig. 2 are (relatively) steep for low cost values and become almost asymptotic for high cost values; this means that at the beginning a small increase in costs enables higher reliability to be achieved easily whereas the highest surplus values are obtainable only thanks to a large expense increase.

Table 1 of this paper reports pipes installed across time for the network configuration featuring total present worth costs = 232 500 $ and IS_{min} = 3.6 m derived from optimization I (one of the solutions of the Pareto front in Fig. 2a, selected in order to enable comparison with optimizations II and III for prefixed value of pressure surplus), along with costs and present worth costs of the various upgrades and the pressure surplus values across time; for the initial upgrade, a cost equal to 160 000 $ and a value of IS_{25} = 9.2 m are reported. The comparison between the first construction upgrade of optimization I and the optimization II configuration obtained for a value of IS_{25} close to IS_{min} (taken from the Pareto front of graph b in Fig. 2 - see next to last column of Table 1) points out that network initial configuration of optimization I presents a larger pipe 3 and a smaller pipe 7. In all, it features higher cost C_{25} and pressure surplus IS_{25} since it is slightly oversized in a bid to cope with network expansion and demand increase across the 100 year construction period and not only to meet the demands for the first 25 years (which is the unique objective of optimization II). The comparison between the final configuration of optimization I and the optimization III configuration obtained for a value of IS_{100} close to IS_{min} (taken from the Pareto front of graph c in Fig. 2 - see last column of Table 7) points out that optimization III configuration has almost identical values of IS_{100} with respect to optimization I configuration, with a total present worth cost of 434 600 $. The latter value is much greater than the total present worth cost of the optimization I network configuration (sum of the present worth costs = 232 500 $).

Summing up, the results of the applications carried out in this paper showed that optimization I, which considers a long time horizon and phasing of construction, gives advantage with respect to optimizations II (short time horizon and single step construction) and III (long time horizon and single step of construction). In particular, with respect to optimization II, it provides engineers with the possibility to adopt at the first construction step piping that turn out to be useful in the long time horizon. With respect to optimization III, it gives engineers the possibility to defer construction, thus reducing the present worth costs at year 0.

As regards the practical use of the methodology presented in this paper, it is worth noting that, even if each optimal solution comprises diameters to be laid in the various sites at the various time steps (i.e. at the beginning of the various phases), in practice the engineers can use the methodology presented with the objective to make decisions for the near future, i.e. for the first phase of system growth (25 years for the simulations of this paper); however by having a vision of the long term growth of the system, the engineer can make better decisions. By having a long term idea of development, capacity can be provided such that decisions make in the current year will fit logically into the long term plan.

Table 1. Solution extracted from the Pareto front produced by the optimization I with ISmin = 3.6 m. Corresponding solutions (with IS25 and IS100 close to ISmin) produced by the optimizations II and III – Diameters (mm) of pipes added in network upgrades at the beginning of each time step. Costs and present worth costs of the various upgrades. Pressure surplus IS at the end of each time step.

Time step [years]		Opt. I 0-25	25-50	50-75	75-100	Opt. II 0-25	Opt. III 0-100
Diameters [mm] of pipes added	pipe 1	356	406	0	508	356	559
	pipe 2	203	0	152	0	203	254
	pipe 3	305	0	457	0	254	559
	pipe 4	254	152	0	0	254	305
	pipe 5	0	0	356	305	0	457
	pipe 6	0	0	0	356	0	356
	pipe 7	102	0	0	0	152	102
	pipe 8	0	0	0	102	0	102
Costs ($)		160 000	96 720	165 560	244 824	156700	434600
Pres. worth cost C_k [$]		C_{25}	C_{50}	C_{75}	C_{100}	C_{25}	C_{100}
		160 000	36 281	23 296	12 923	156 700	434 600
		C = 232 500					
Surplus IS [m]		IS_{25}	IS_{50}	IS_{75}	IS_{100}	IS_{25}	IS_{100}
		9.2	3.6	5.3	3.9	3.5	3.7
		IS_{min} = 3.6 m					

4. Conclusions

In this paper a new network design approach based on the phasing of construction was presented. In situations where nodal demands increase in time without uncertainty, this approach entails identifying, on prefixed time steps, the optimal network upgrades that allow the network to keep on supplying water with acceptable service pressure to users. Designing in a single phase without considering long term implications, results in a system that may need inefficient upgrades at the end of the design period or excessive initial investments in capacity that may not be required for a long (uncertain) time in the future. In this study, network upgrades are identified via an optimization performed by a multi-objective genetic algorithm, which aims at minimizing the costs of the upgrades converted into value at the initial time and at maximizing the minimum pressure surplus over time.

Applications showed that making use of such approach leads to better results than the traditional design approach, based on a single construction phase, for the following reasons:

- it allows engineers to design the short term upgrades, which are supposed to guarantee a prefixed level of reliability, while keeping an idea of the long term network growth and expansion;

- for long time horizon, it turns out to be cost effective; in fact, by partially deferring construction, the community is able to put aside resources that can be more effectively allocated to alternative uses. Furthermore, it is more flexible since it always gives engineers the possibility to recalibrate subsequent upgrades whether real development is far from expected.

Concluding, it is also worth highlighting that the methodology presented in this paper only concerns the dynamic design of water distribution networks and is supposed to be of assistance to engineers in order to perform actual pipe laying in a sustainable way, i.e. also trying to take future development into account. However, methodologies for optimal maintenance and rehabilitation can be superimposed on network design configurations obtained following the present approach in order to schedule the repairs and replacements of the pipes which are being laid during network development.

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