



# FORUM ACUSTICUM EURONOISE 2025

## MULTI-DENSITY INVERSION CHARACTERISATION METHOD FOR FIBROUS MATERIAL

Andrea Santoni<sup>1\*</sup>

Francesco Pompoli<sup>1</sup>

Cristina Marescotti<sup>1</sup>

Patrizio Fausti<sup>1</sup>

<sup>1</sup> Department of Engineering – University of Ferrara, Ferrara, Italy

### ABSTRACT

This paper describes a novel inversion methodology for determining the transport parameters of the Johnson-Champoux-Allard (JCA) equivalent fluid model, developed to provide a quick and easy-to-implement characterisation tool. The proposed approach, based on an optimisation algorithm, requires only the experimental evaluation of the normal incidence sound absorption coefficient on fibrous material samples, prepared from loose fibres, at various compression ratios. Unlike existing inversion methods, the proposed approach links the variation of transport parameters to changes in the material's apparent density. The validation presented in the paper considers experimental data measured on two types of fibres: hemp fibres and glass fibres. Validation of the proposed method shows good agreement with well-established techniques and demonstrates its effectiveness across a broad range of fibrous materials.

**Keywords:** *acoustic optimization, inversion approach, material characterisation, fibrous material, sound absorption*

### 1. INTRODUCTION

This paper presents a methodological approach for characterising the transport parameters necessary to describe the acoustic behaviour of fibrous materials, using measurements of the normal incidence sound absorption coefficient,

\*Corresponding author: andrea.santoni@unife.it.

**Copyright:** ©2025 Andrea Santoni et al. This is an open-access article distributed under the terms of the Creative Commons Attribution 3.0 Unported License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

in accordance with the transfer function method described in ISO 10534-2. The main objective of this study is to develop a simple and reliable procedure for characterising fibrous materials and optimizing their density. This allows for a significant reduction in the prototyping phases typically required during the design and optimization of sound-absorbing panels. Although several inversion techniques have already been developed for the same purpose, the proposed method is the only one capable of correlating variations in transport parameters with material density. Conventional inversion techniques perform separate minimisations for each material or sample, followed by a possible data interpolation [1]. The developed inversion methodology is based on the semi-phenomenological model of Johnson-Champoux-Allard (JCA) [2, 3], which requires five transport parameters as input data to compute the complex density and the dynamic bulk modulus of an equivalent dissipative fluid: airflow resistivity  $\sigma$ , porosity  $\phi$ , tortuosity  $\alpha_\infty$ , and the viscous and thermal characteristic lengths  $\Lambda$  and  $\Lambda'$ . This paper presents the main validation results of the proposed methodology by considering two types of fibres: hemp fibres and glass fibres. However, the proposed methodology is also applicable for the characterization of various fibrous materials, including both natural fibres and those traditionally used in acoustic applications, as detailed in reference [4].

### 2. MATERIALS AND METHODS

The proposed inversion characterization approach requires the experimental measurement of the normal incidence sound absorption coefficient  $\alpha_{n,exp}$ , in accordance, for example, with ISO 10534-2, on samples at different compression ratios, which implies different bulk densities of the fibrous material. This methodology is partic-





ularly suitable for investigating loose fibres: the fibres are weighed and placed in the sample holder of an impedance tube, where they can be easily compressed using a movable rigid backing. An acoustically transparent mesh is used to restrain the sample and prevent sliding along the tube, as shown in Figure 1. The sound absorption coefficient



**Figure 1.** Loose hemp fibres and their installation in the plane wave impedance tube.

cient  $\alpha_{n,\text{exp}}$  is then measured for different combinations of sample thickness  $h$  and density  $\rho$  of the samples. Increasing the compression ratio of the sample results in an increase in density and a reduction in open-cell porosity  $\phi$ , which can be evaluated knowing the fibre's density  $\rho_s$  as:

$$\phi = 1 - \frac{\rho}{\rho_s} \quad (1)$$

The material's airflow resistivity  $\sigma$  can be approximated as a function of porosity  $\phi$  and the effective radius  $r_e$  of an equivalent 2D monodisperse fibrous medium [5]:

$$\sigma = \frac{\eta}{(2r_e)^2} \times \frac{\sqrt{1 - (1 - \phi)}}{0.21 \left( \frac{0.71}{1 - \phi} - 3\sqrt{\frac{0.71}{1 - \phi}} + 3 - \sqrt{\frac{1 - \phi}{0.71}} \right)} \quad (2)$$

The remaining transport parameters used in the JCA model can be expressed analytically as functions of porosity  $\phi$  or the effective radius  $r_e$ . The latter is one of the design parameters of the multi-variable function to be optimized. The high-frequency limit of tortuosity  $\alpha_\infty$  is computed using Archie's empirical law based on the material's porosity  $\phi$ :

$$\alpha_\infty = \left( \frac{1}{\phi} \right)^\beta = \left( \frac{1}{\phi} \right)^{0.9574} \quad (3)$$

While the constant  $\beta$  could be generally treated as an additional design parameter of the multi-variable optimisa-

tion, dependent on the microstructural characteristics of the fibrous material, in this case a fixed value  $\beta = 0.9574$  was adopted, independent of the fibres' effective radius  $r_e$  [5]. The scale-of-pores geometrical quantities representing viscous and thermal characteristic lengths are determined using empirical equations proposed by Pompoli [6]:

$$\Lambda = A_1 r_e (1 - \phi)^{-A_2} \quad (4)$$

$$\Lambda' = A_3 r_e (1 - \phi)^{-A_4} \quad (5)$$

The unknown coefficients  $A_i$  (with  $i = 1, 2, 3, 4$ ) represent, along with the effective radius  $r_e$ , the optimisation design variables to be determined through a minimisation algorithm. This algorithm computes the normal incidence sound absorption coefficient using the JCA model  $\alpha_{n,\text{JCA}}$ , expressing the transport parameters through Eqs. (1) to (5). The optimisation goal is to determine a set of design variables

$$x = \{r_e, A_1, A_2, A_3, A_4\},$$

that minimise an objective function  $\mathcal{F}(x)$  defined in terms of the normalized difference between the experimental and JCA-predicted sound absorption coefficients:

$$\mathcal{F}(x) = \sum_{i=1}^n \frac{1}{n} \sum_{l=1}^m \frac{1}{m} |\alpha_{n,\text{exp},i,l} - \alpha_{n,\text{JCA},i,l}| \quad (6)$$

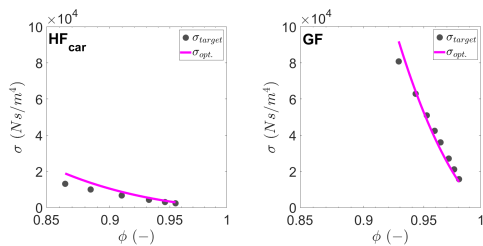
The minimisation of  $\mathcal{F}(x)$  reduces the discrepancy between the experimental and JCA-predicted normal incidence sound absorption coefficients, summed over all the frequencies  $m$  and all the investigated densities  $n$ , by adjusting the design variables the transport parameters depend upon. The optimal solution can be computed using different solvers, while specifying search space boundaries, constraints of the optimization problem, and, in some cases, an initial guess for the design variables. The results presented in the following section were obtained using a MATLAB implementation of the proposed approach, employing a nonlinear programming solver. Further details on the implementation and the influence of different solvers are provided in reference [4].

### 3. EXPERIMENTAL RESULTS

This paragraph presents the main result validating the proposed methodology and assessing its reliability by analysing the accuracy of both the transport parameters obtained from the optimization process and the computed



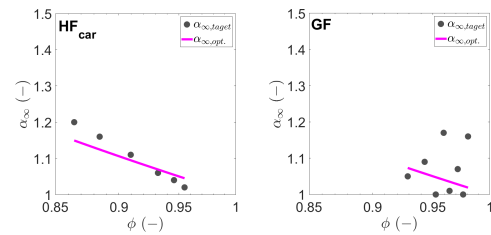
normal incidence sound absorption coefficient. Two different types of fibres are considered: natural hemp fibres subjected to a carding process ( $HF_{car}$ ) and traditional glass fibres ( $GF$ ). The transport parameters obtained through the proposed approach, expressed as functions of material density, are compared with values derived from well-established inversion techniques based on measurements of a single sample [1]. Figure 2 compares the airflow resistivity curve  $\sigma_{opt.}$ , obtained using the proposed optimization methodology, with the target values  $\sigma_{target}$ , which are derived from the dynamic density of each sample, experimentally evaluated using the three-microphone transfer matrix method. This comparison follows the relationship proposed by Olny and Panneton [7]. The comparison highlights a good approximation of the target values within the investigated density range. The airflow resistivity curve evaluated for glass fibres ( $GF$ , shown on the right-hand side of Figure 2) interpolates the values determined at discrete densities considered in the experimental testing of the samples. For hemp fibres ( $HF_{car}$ , on the left-hand side of Figure 2), a slight deviation is observed between the airflow resistivity obtained as a function of density through the optimization algorithm and the discrete values derived from the measured complex density. The optimized airflow resistivity curve slightly overestimates the target values due to the simplifying assumptions in Eq. (2). The assumed condition of a two-dimensional unidirectional fibre orientation, with fibres aligned perpendicularly to the airflow, is generally met by traditional fibrous materials, where fibre dimensions and orientation can be designed, but it does not accurately represent the random distribution of natural fibres. Although tortuosity is not dependent on design variables,



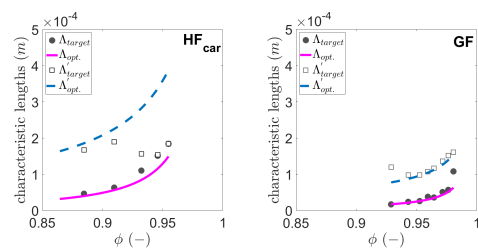
**Figure 2.** Comparison between the airflow resistivity curve obtained from the proposed method and the values derived from an experimental approach: left hemp fibres ( $HF_{car}$ ), right glass fibres ( $GF$ ).

since it is directly defined solely as a function of poros-

ity, which varies with different compression ratios, Figure 3 compares the density-dependent curve  $\alpha_{\infty, opt.}$ , used in the adopted model, with the discrete values  $\alpha_{\infty, target}$  obtained from a well-established inverse characterization approach. This comparison aims to demonstrate the reliability of Eq. (3) in estimating tortuosity as a function of porosity. For the  $HF_{car}$  material, the optimized tortuosity curve closely approximates the discrete tortuosity values. Although the two trends exhibit a slightly different slope, the values obtained from both approaches remain very close for any given porosity, especially considering the limited variation range of this parameter. For the  $GF$  material, the inverted values exhibit significant scatter, making it difficult to identify a clear trend. Nevertheless, the optimized curve remains within the range of scatter across the entire porosity range. Figure 4 presents sim-



**Figure 3.** Comparison between the tortuosity curve obtained from the proposed method and the values derived from an experimental approach: left hemp fibres ( $HF_{car}$ ), right glass fibres ( $GF$ ).

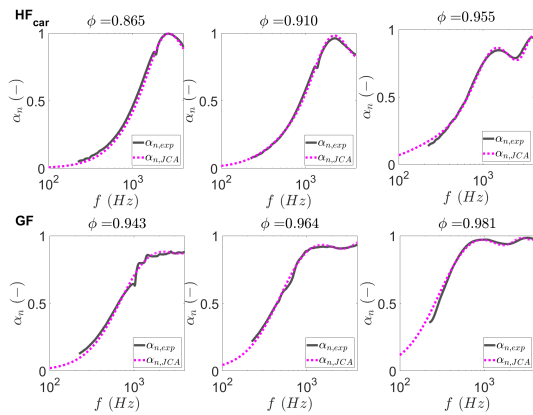


**Figure 4.** Comparison between the characteristic length curves obtained from the proposed method and the values derived from single inversions: left hemp fibres ( $HF_{car}$ ), right glass fibres ( $GF$ ).

ilar comparisons between the optimized  $\Lambda_{opt.}$  and  $\Lambda'_{opt.}$  curves obtained using the proposed methodology and the values of the characteristic lengths  $\Lambda_{target}$  and  $\Lambda'_{target}$  derived from traditional inversion methods. For both  $HF_{car}$



# FORUM ACUSTICUM EURONOISE 2025



**Figure 5.** Comparison between the experimental and the JCA-predicted normal incidence sound absorption coefficients: top hemp fibres ( $HF_{car}$ ), bottom glass fibres ( $GF$ ).

and  $GF$  materials, the method provides results consistent with those obtained through single-sample inversion characterization. In general, with single inversions, the trend of the viscous characteristic length  $\Lambda$  as a function of porosity is more easily identifiable than that of the thermal characteristic length  $\Lambda'$ . Consequently, in the optimized curves, a higher consistency with the target values is observed for the viscous characteristic length  $\Lambda$ . Finally, the accuracy of the method is assessed by comparing the experimental normal incidence sound absorption coefficient with the frequency-dependent curves predicted through the JCA model by using the transport parameters characterized by the proposed methodology. Figure 5 illustrates this comparison for different porosity values, for both hemp and glass fibres. In both cases, the results highlight the high accuracy of the JCA model, confirming the reliability of the transport parameters determined using the proposed method.

## 4. CONCLUSIONS

This article presented an innovative, fast, and reliable methodology for characterising the macrostructural parameters of fibrous materials, providing a valuable tool for the design and optimisation of sound-absorbing panels while significantly reducing the prototyping phases. The proposed inverse characterization methodology integrates an optimization algorithm with the Johnson-Champoux-Allard (JCA) model and can be easily implemented in any

acoustics laboratory, as it requires only the experimental measurement of the normal incidence sound absorption coefficient for samples with different densities. Unlike other inversion methods, the proposed approach allows for the correlation of transport parameter variations with the apparent density of the material. The results presented, referring to two different types of fibres, demonstrate that the methodology provides reliable estimates of the transport parameters essential for the acoustic optimisation of the material. The methodology can be applied to a wide range of fibrous materials used for acoustic applications and is particularly advantageous when working with loose fibres.

## 5. REFERENCES

- [1] P. Bonfiglio and F. Pompoli, "Inversion problems for determining physical parameters of porous materials: Overview and comparison between different methods," *Acta Acustica united with Acustica*, vol. 99, no. 3, pp. 341–351, 2013.
- [2] D. L. Johnson, J. Koplik, and R. Dashen, "Theory of dynamic permeability and tortuosity in fluid-saturated porous media," *Journal of fluid mechanics*, vol. 176, pp. 379–402, 1987.
- [3] Y. Champoux and J.-F. Allard, "Dynamic tortuosity and bulk modulus in air-saturated porous media," *Journal of applied physics*, vol. 70, no. 4, pp. 1975–1991, 1991.
- [4] A. Santoni, F. Pompoli, C. Marescotti, and P. Fausti, "Characterization of fibrous media transport parameters from multi-compression-ratio measurements of normal incidence sound absorption," *The Journal of the Acoustical Society of America*, vol. 157, no. 2, pp. 1185–1201, 2025.
- [5] F. Pompoli and P. Bonfiglio, "Definition of analytical models of non-acoustical parameters for randomly-assembled symmetric and asymmetric radii distribution in parallel fiber structures," *Applied Acoustics*, vol. 159, p. 107091, 2020.
- [6] F. Pompoli, "Acoustical characterization and modeling of sustainable posidonia fibers," *Applied Sciences*, vol. 13, no. 7, p. 4562, 2023.
- [7] X. Olyny and R. Panneton, "Acoustical determination of the parameters governing thermal dissipation in porous media," *The Journal of the Acoustical Society of America*, vol. 123, no. 2, pp. 814–824, 2008.