Blue Gravity Waves from BICEP2?

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We present new constraints on the spectral index n_T of tensor fluctuations from the recent data obtained by the BICEP2 experiment. We found that the BICEP2 data alone slightly prefers a positive, "blue", spectral index with $n_T = 1.36 \pm 0.83$ at 68% c.l.. However, when a TT prior on the tensor amplitude coming from temperature anisotropy measurements is assumed we get $n_T = 1.67 \pm 0.53$ at 68% c.l., ruling out a scale invariant $n_T = 0$ spectrum at more than three standard deviations. These results are at odds with current bounds on the tensor spectral index coming from pulsar timing, Big Bang Nucleosynthesis, and direct measurements from the LIGO experiment. Considering only the possibility of a "red", $n_T < 0$ spectral index we obtain the lower limit $n_T > -0.76$ at 68% c.l. ($n_T > -0.09$ when a TT prior is included).

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I. INTRODUCTION

The recent detection of B-mode polarization made by the BICEP2 experiment [1] clearly represents one of the major discovery in cosmology in the past twenty years. While the BICEP2 result clearly needs to be confirmed by future experiments, it is timely and important to fully analyze the BICEP2 data and to identify all possible inconsistencies at the theoretical level.

In this brief note we focus our attention on the spectral index of tensor fluctuations n_T . Indeed, a crucial prediction of inflation is the production of a stochastic background of gravity waves ([2]) with a slightly tilted spectrum,

$$n_T = -2\epsilon , \qquad (1)$$

where $\epsilon = -\dot{H}/H^2$ denotes a slow roll parameter from inflation (*H* is the Hubble rate during the inflationary stage).

In standard inflation ϵ is strictly positive [3] and in the usual parameter estimation routines, the tensor spectral index is assumed to be "red", or negligible.

However, in recent years, a set of inflationary models has been elaborated where the spectral index of tensor modes could be positive, $n_T > 0$, i.e. "blue". A first attempt to compare these models with observational data has been made in [4].

The main theoretical problem for the production of a blue spectrum of gravitational waves (BGW) is that the stress-energy tensor must violate the so-called Null Energy Condition (NEC). In a spatially flat FRW metric, a violation of NEC indeed corresponds to the inequality $\dot{H} < 0$ and is ultimately the reason for the red tensor spectrum in standard inflation.

Models that violates NEC have been already presented. For example, in the so-called super-inflation models [9] where inflation is driven by a component violating the NEC a BGW spectrum is expected. Models based on string gas cosmology as in [5], where scalar metric perturbations are thought to originate from initial string thermodynamic fluctuations [6], also can explain a BGW background. A BGW spectrum is also a generic prediction of a class of four-dimensional models with a bouncing phase of the universe [8]. To induce the bounce, the stress-energy tensor must violate the null energy condition (NEC). G-inflation [12], has a Galileon-like nonlinear derivative interaction in the Lagrangian with the resultant equations of motion being of second order. In this model, violation of the null energy condition can occur and the spectral index of tensor modes can be blue. BGW may also be present in scalar-tensor theories and f(R) gravity theories.

It is therefore timely to investigate the constraints on the tensor spectral index n_T from the BICEP2 data. Strangely enough, no constraint on this parameter has been presented by the BICEP2 collaboration while, as we discuss in the next section, we found that the BICEP2 data could provide interesting results on this parameter.

II. ANALYSIS METHOD

Our analysis method is based on the Boltzmann CAMB code [14] and a Monte Carlo Markov Chain (MCMC) analysis based on the MCMC package cosmomc [13] (version December 2013). We have implemented in the MCMC package the likelihood code provided by the BICEP2 team (we just use BB data). and considered as free parameters the ratio of the tensor to scalar amplitude r at $0.01hMpc^{-1}$, defined as $r_{0.01}$, and the tensor spectral index n_T . We prefer to use the pivot scale at $k = 0.01hMpc^{-1}$ since the BICEP2 data is most sensitive to multipole $l \sim 150$ and using the approximate formula $l \sim 1.35 \times 10^4 k$.

All the remaining parameters have been kept fixed at the Planck+WP best fit values for the LCDM+r scenario (see [15]).

Moreover, since the tensor amplitude should also be

consistent with the upper limits on r coming from measurements of the temperature power spectrum, we have assumed a prior of $r_{0.002} < 0.11$ at 95% c.l. (see [16]). We refer to this prior as the "TT" prior.

Note that the TT prior is taken at much larger scales, $k = 0.002hMpc^{-1}$ than those sampled by the BICEP2 experiments. As we show in the next section this prior is extremely important for the constraints on n_T .

III. RESULTS

Case	$r_{0.01}$	n_T
n_T free	0.19 ± 0.06	1.36 ± 0.83
TT prior $+n_T$ free	0.18 ± 0.05	1.67 ± 0.53
$n_T < 0$	0.22 ± 0.06	$n_T > -0.76$
TT prior $+n_T < 0$	0.15 ± 0.03	$n_T > -0.09$

TABLE I. Constraints at 68% c.l. on $r_{0.01}$ and n_T parameters for the cases described in the text. A blue spectral index $(n_T > 0)$ is strongly suggested when a TT prior of $r_{0.002} < 0.11$ at 95% c.l. is included in the analysis.

The results of our analysis are reported in Table I and Figure 1. We consider four cases: n_T free, n_T free but with the TT prior, n_T assumed to be negative $(n_T < 0)$ and n_T assumed to be negative plus the TT prior.

We can derive the following conclusions:

- The BICEP2 data alone slightly prefers a positive spectral index. The case $n_T = 0$ is consistent with the data in between two standard deviations.
- When a TT prior of $r_{0.002} < 0.11$ at 95% c.l.. is assumed, the BICEP2 data strongly prefers a blue spectral index with $n_T \leq 0$ excluded at more than three standard deviations.
- If we restrict the analysis to negative n_T we obtain a lower limit of $n_T > -0.76$ at 68% c.l. $(n_T > -0.09$ in case of the TT prior).

IV. CONCLUSIONS

In this brief note we have presented new constraints on the spectral index n_T of tensor fluctuations from the recent data obtained by the BICEP2 experiment. We found that the BICEP2 data alone slightly prefers a positive, "blue", spectral index with $n_T = 1.36 \pm 0.83$ at 68% c.l.. However, when a TT prior on the tensor amplitude coming from temperature anisotropy measurements is assumed we get $n_T = 1.67 \pm 0.53$ at 68% c.l., ruling out a scale invariant $n_T = 0$ spectrum at more than three standard deviations. Considering only the possibility of a "red", $n_T < 0$ spectral index we obtain the lower limit $n_T > -0.76$ at 68% c.l. $(n_T > -0.09$ when a TT prior is included).

These results are at odds with current upper limits on the tensor spectral index coming from observations of pulsar timing, Big Bang Nucleosynthesis, and from direct upper limits from the LIGO experiment (see e.g. [17]). Considering $r_{0.002} = 0.2$ and using the method adopted in [17] we found the current upper limits on n_T : $n_T \leq 0.81$, $n_T \leq 0.29$ and $n_T \leq 0.15$ at 68% c.l. from pulsar timing, LIGO and BBN respectively. The LIGO and BBN limits are in strong tension with the BICEP2+CMB value. Therefore a positive spectral index does not provide an acceptable solution to the tension between the BICEP2 data and current upper limits on r from temperature anisotropies. This indicates either the need of including extra parameters (as the running of the scalar spectral index [1] or extra neutrino species [18]) to relax current bounds on $r_{0.002}$ from temperature anisotropies or the presence of unresolved systematics in current CMB data.

During the submission of this paper other works appeared discussing the possibility of a BGW from BI-CEP2 (see [19]) but without presenting numerical constraints on n_T and an independent analysis of the BI-CEP2 data. We also like to point out the discussion on the cosmocoffee.info website where results similar to ours have been presented by Antony Lewis.

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FIG. 1. Constraints on the n_T vs $r_{0.01}$ plane for the 4 cases discussed in the analysis. No prior on n_T (Top Left), No prior on n_T but TT prior on $r_{0.002}$ (Top Right), $n_T < 0$ (Bottom Left), $n_T < 0$ and TT prior on $r_{0.002}$ (Bottom Right)

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