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Incorporating Fairness Into the Gateway-Based Risk Mitigation Policy for Hazmat Transport

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ABSTRACT

In hazardous material transport on road networks, two conflicting objectives must be addressed simultaneously: minimizing risk and minimizing cost. Risk mitigation policies may yield as a secondary outcome uneven flow distribution on the network. This study empowers an existing risk mitigation policy based on gateways (GBP) to improve fairness. According to GBP, each vehicle is obliged to traverse a compulsory node (a gateway) on its minimum cost itinerary from origin to destination. Gateways must be located on a few network nodes and assigned to vehicles to minimize total risk, yielding a bi-level optimization problem. GBP already proved able to reduce total risk by opening just a few gateways and to a limited detriment of total cost. However, gateways may end up acting as flow concentrators, thus hampering equity. This study aims to bridge the gap between risk mitigation and fairness. To this aim, we generalize the multi-commodity flow formulation of the problem by imposing a capacity constraint on the nodes, discuss its impact on the model structure, and experimentally investigate whether it is possible to achieve a more equitable risk distribution and how total risk and total cost are affected.

1 | Introduction

The transportation of hazardous materials (hazmat) in Western countries is subject to strict regulations in order to prevent environmental damage and protect the public from exposure to potential hazards. While accidents involving hazmats seldom occur during transportation, consequences can be rather severe (so-called *low-probability-high-consequences* events) as much as those due to accidents at fixed installations [1]. Road transport is probably the most critical of all transport modes, as hazmat vehicles mix with private vehicles on the same network infrastructure and may even travel through urban areas. Hazmat regulation mainly pertains to government agencies and international organizations, which have promulgated a large body of rules regarding material classification, packaging requirements, and vehicles safety standards. On the other hand, routing guidelines are much less specific, leaving this task to local authorities, despite of the fact that defining the itinerary of each shipment from origin to destination on the road network is a major task in terms of both number of vehicles involved and the entity of potential harm. Indeed, a recent survey in Italy revealed that more than 78,000 vehicles are involved in hazmat transport, 10,000 of which daily move goods such as diesel, petrol or kerosene all over the country. Likewise, dangerous goods transported (in tonne-kilometers) on the European road network represent around 4% of the total road freight transport [2].

To regulate routing decisions, authorities may act at different levels and by way of different policies. They may either dictate mandatory routes for specific shipments (*direct control*) or, in case they lack the power to impose specific routes on individual carriers, may issue ad hoc rules that drivers must

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comply with when selecting their itinerary (*indirect control*). The latter include: (i) forbidding access to sensitive areas or through specific links, (ii) toll charging on some other links, and (iii) requiring vehicles to go through mandatory checkpoints. The so called gateway based policy (GBP in the following) that we address in this article falls into the third case, and it aims at diverting vehicles away from risky itineraries by enforcing detours through given locations, that is, the gateways, that are selected in such a way that the new routes will be safer. To enforce rule compliance, authorities may leverage technological innovations such as tracking systems and sensors, which allow real time monitoring [3]. However, when setting up any regulation, the most challenging aspect is to take into account carrier reaction. Indeed, carriers typically pursue cost minimization. Therefore, in an unregulated scenario shipments follow minimum-cost itineraries, while, when rules are present, carriers exploit any degree of freedom to be compliant at minimum cost. However, the new itinerary is not necessarily safer than the former one. This introduces the issue of dealing with different stakeholders in a hierarchical position who pursue potentially conflicting objectives, so that evaluating the impact of a given policy requires solving hard bi-level optimization problems. Moreover, transport policies should preserve economical viability and avoid excessive cost deterioration which could jeopardize carrier compliance. In fact, the regulation of hazardous materials transportation should strike a balance between carriers perspective and public safety, facilitating the movement of essential goods while protecting the people and the environment.

Beside dealing with the impact on total risk and total cost of risk mitigation policies, scientific research investigated the fairness of the policy induced itineraries, intended as an equitable spread of risk among the population. Ensuring fairness involves addressing potential disparities in the distribution of risks associated with hazmat transportation. Hence, hazmat vehicles itineraries should be planned and designed in such a way as to avoid placing a disproportionate burden on certain communities. This means avoiding the concentration of risk in certain areas, which, without loss of generality, may be implemented by setting an upper bound to the vehicle flow at a certain network granularity, that is, either on the arcs, or at the nodes, or at different sub-networks.

In this article, we try to merge risk mitigation and fairness in the framework of the above mentioned GBP. In detail, according to GBP, each o/d pair is assigned a compulsory node (a gateway) that drivers are obliged to traverse at the minimum cost on the path from origin to destination. The gateway location problem (GLP) consists of two hierarchical decisions: the first one is to select a limited number k of gateway locations from a larger candidate set; the second one is to assign one gateway to each o/d pair so that the sum of the risk of the new itineraries is minimized. First introduced in [4], GBP already proved effective at reducing total risk to little cost deterioration [5] and its efficacy was experimentally tested even for small values of k [6]. Nevertheless, GBP intrinsically induces flow concentration at each of the k locations that have been selected to host a gateway. Indeed, by definition, (at least) all shipments which have been assigned to the gateway hosted on a certain location will travel through it. Therefore, potential hazmat traffic concentration at the gateways poses a fairness issue in case of uneven distribution. Think for example of the extreme case of all shipments being assigned to the same gateway: all hazmat shipments that move on the network from their origin to their destination would cross the location hosting that gateway. Even though this is an unlikely situation, at present GBP includes no safeguard to prevent such solutions, nor the entity of this drawback has been assessed, which motivate this study.

This study provides the following contributions:

i. it assesses the magnitude of the unfairness related to the GBP by quantifying flow concentration at the gateways in the current GLP solutions and in so doing it brings to the forefront a weak aspect of the gateway based methodology which could pose a serious obstacle to its practical deployment;

ii. it proposes to hedge against this weakness by enforcing an upper bound to the quantity of flow passing through the nodes. In particular, it exploits the bi-level multi-commodity flow model developed for GLP to introduce two capacitated versions, that is, capacity at the gateways and capacity at each node, yielding the Capacitated GLP (C-GLP) which has never been studied before;

iii. it shows that both capacitated versions of GLP retain the same structural properties that allow single level reformulation; this fact suggests that realistic instances of both problems can be solved by state of the art MILP solvers with no need for developing ad hoc solution approaches, and are therefore easily available to the public authorities willing to adopt the GBP;

iv. it tests the increase of the computational burden due to the capacity constraints, stressing the model for high values of the parameters, to provide solid ground to the previous statement regarding the ability to solve GLP on real networks;

v. regarding solution quality, since any risk reduction policy in hazmat transport should be evaluated from a multi-criteria perspective, this study experimentally assesses the deterioration in terms of risk reduction capability and cost increase due to the introduction of the capacity constraints in order to quantify what must be traded for an increase in fairness.

The computational experiments will be carried out on the network of Ravenna, Italy, largely adopted as a test bed in the hazmat literature [7].

The rest of the article is organized as follows: in Section 2, the main studies addressing fairness and risk mitigation in hazmat routing are recalled; in Section 3, the multi-commodity flow model for GLP is presented and two capacitated variants are introduced; computational experiments are reported and discussed in Section 4, while Section 5 is devoted to conclusions.

2 | Literature

In the following, the main studies addressing fairness in hazmat routing are recalled, as well as the most effective risk mitigation policies. However, we make no claim to be exaustive with respect to such a rich body of literature, as we focus on problems with the same structure as our, that is, either single or multi-commodity shipments delivered by truck, each one described by a single origin—destination pair and a quantity of material, that share the same road network in the same period with no uncertainty. Specifically, we do not discuss risk mitigation policies based on dispatching trips over different times that is, exploiting time-based road closure policies as in [8], nor we consider transportation modes other than road, as rail, air or water, as well as multi-modal trips such as in [9, 10], nor studies involving the location of production or consumption sites which determines the origin or the destination of the shipments, as much as the location for emergency response centers on the network as in [11, 12]. Likewise, we do not consider vehicle routing problems where multiple clients are visited along one route from and to a depot. Discussions on risk assessment are also not contemplated. The reader willing to gain a broader perspective on hazmat routing may refer to very recent and less recent reviews such as [13–15].

Particular care will be given to the criteria which lead the search for the best routes, both explicitly (in the objective function at some stage of the solution approach) and implicitly (when used to model side effects of a regulation based solution approach), how a trade-off is reached when more than one criterion is considered, and whether the solution approach is exact or not. Furthermore, we will distinguish between *local routing*, intended as considering multiple shipments between a single o/d pair, each allowed to follow a different itinerary, and *global routing*, intended as several carriers that share the same network on which different hazmat shipments are routed at the same time between several o/d pair, and all goods shipped between the same o/d pair follow the same path. We will also refer to these two cases as the *single commodity* or the *multi commodity* case, respectively. The GBP falls in the multi commodity case.

2.1 | Risk Mitigation Policies in Local Routing

When routing multiple shipments between the same o/d pair, total risk minimization would be obviously achieved by channelling all shipments onto the same minimum risk route. Since this would clearly be unfair, hazmat traffic should be split on different itineraries to avoid overloading certain links. In this case, we speak of local routing or single commodity flow problems.

A pioneering study [16] developed a MILP formulation and a heuristic approach for computing a minimum total risk set of routes which spread the risk equitably in the geographical region embedding the transportation network, by ensuring that the risk levels of any two zones in the region differ for less than a given threshold. While developed for local routing, the proposed methodology could be adapted to global routing as well.

Other studies extend prior investigations into computing a set of dissimilar paths for a given o/d pair [17]. These studies aim to strike a balance between the degradation of the objective function, usually the cost, and the requirement for dissimilarity, employed as an indicator of equity. Recently, Moghanni, Pascoal, and Godinho [18] developed new linear integer programming formulations for the kth shortest dissimilar paths that account for different measures of similarity, complemented by a polynomial algorithm yielding loopless paths. In [19] the authors improve on [17] in the way the selection step is performed once the kth shortest paths have been computed, that is, by solving a p-dispersion problem.

In [20] the initial set of paths is computed by taking into account both cost and risk, that is, a multi-criteria algorithm returns a set of Pareto-Optimal paths. Then, the selection step is implemented by a p-dispersion algorithm, where dissimilarity takes into account all the area potentially affected by an accident on the path. In [21] a set of k paths of minimum total risk is computed such that the maximum risk sustained by the population on each network link is below a given threshold, considering a distance-dependent risk propagation function which determines how much the (population living along the) link is affected by accidents occurring on nearby links.

In [22] a trade-off between cost, risk, and fairness is considered: k paths are selected with high spatial dissimilarity among the efficient ones with respect to cost and risk minimization, in order to guarantee an equitable spatial distribution of risk. In [23] different hazmat types are routinely shipped through a urban network with uneven population density along various simultaneous routes and in presence of exogenous risk sources. These are computed by extending the single-product, single-shipment, hazmat routing model developed by Sherali et al. in 1997 (see [24]) so that risk is equitably spatially distributed by imposing a threshold on the risk any population segment is exposed in the long term.

Note that searching for spatial dissimilarity among cost efficient paths in local routing not only optimizes total cost but also reduces the cost gap among different shipments, with an eye on fairness among carriers.

A final remark concerns fairness among trucks, that is achieved in [25] by spreading the shipments between the same o/d pair both geographically and along time to overcome the disadvantages of drivers following the longest routes. Due to a lack of historical data to estimate the risk of each link, the problem is modeled and heuristically solved as a Stackelberg game between the transportation company and a demon willing to target the shipment which would cause the worst damage.

All these studies assume that the authority has the ability to enforce routing decisions on carriers and oblige them to follow specific itineraries.

2.2 | Risk Mitigation for Global Routing

The governing authority possesses several options to influence the routing decisions of minimum-cost-driven carriers without prescribing specific itineraries, that is, by closing specific road segments, by imposing caps on hazmat traffic flow on certain links or sections of the network, by implementing toll pricing, or by mandating a detour through a check point (a gateway) along the o/d path.

Many papers address the **link closure** policy (also known as hazmat routing network design, HRND), in which the authority interdicts some network links to hazmat traffic, aiming at minimizing total risk, while carriers select minimum cost routes on the residual network. Bi-level leader/follower models capture the authority/carrier relationship.

Kara and Verter were the first to formulate this bi-level model in [26] which they solved to optimality by linearizing the Karush–Kuhn–Tucker (KKT) conditions of the linear relaxation.

The challenge posed by the HRND bi-level model is addressed in different ways in other studies.

Exact approaches include: [27] where the authors handle stability by perturbing the cost coefficients of the inner problem objective function making the carriers risk aware in adversarial scenario, and propose a more compact one level reformulation of the problem; [28] where Gzara proposes a family of valid cuts and an exact cutting plane algorithm for the combinatorial formulation of the bilevel model which proved very fast. More recently, Fontaine and Minner [29] solve the problem by Benders Decomposition once the KKT conditions have been applied. This proves particularly efficient in case of several commodities.

The HRND problem was generalized in [30], by addressing several transport modes (including different vehicles) and a fair risk distribution. Regarding the first issue, while each commodity (an o/d pair and a demand) can be split among a few transport modes, each shipment will pick a single mode, that is, inter-modal transportation is forbidden. Moreover, the authority may forbid the transit on a link for selected modes. The second issue concerns risk balancing on the population. The influence of an accident occurred on a given link, for a given commodity, using a given mode, over (the population of) a zone, is supposed to be known. Rather than minimizing total risk, the authors experiment with a few classical equity measures among population centers as the objective function, such as minimizing: maximum risk, average and maximum deviation to mean, average and maximum deviation among all pairs. Since pursuing risk equilibration typically leads to risk increase on some agents (here, population centers), a piece-wise linear penalty is added for each population center in case its risk is greater than some thresholds. The approach was tested on the Sioux Falls network (24 nodes—76 arcs) partitioned into 6 population centers. As expected, pure risk equilibration significantly increases total risk. The authors argue that adding a constraint on the level of equilibration would undermine the KKT transformation, while imposing such a constraint on each carrier would clash with the assumption of cost driven carriers. Therefore a bi-criteria objective function is proposed which is a weighted sum of the total risk and the average deviation among all pairs. On the contrary, as discussed in Section 3.2, the fairness driven strategy we are proposing in this article can be fully integrated into the bi-level model without compromising its structure.

Other studies on HRND avoid the challenge posed by the bi-level model and propose heuristic approaches. Erkut and Alp [7] formulate it as a minimum risk Steiner tree with (greedily selected) additional edges. Erkut and Gzara [31] propose a heuristic method able to find stable solutions. Verter and Kara [32] state the problem in terms of path selection: given a pre-computed set of routes on the risk-cost trade-off boundary, ordered by carriers preferences (i.e., cost), a set of links must be interdicted so that the sum over all carriers of the risk of their preferred feasible path is minimum. Since individual carriers may be protected from excessive cost increase by a proper path selection (by discarding itineraries much more expensive than the minimum cost path), this approach may be considered as addressing fairness for individual carriers. The approach is tested on the Ontario highway networks, the largest instance made of 176 nodes and 205 links.

A second strategy the authority can carry out to indirectly reroute hazmat flow is by **toll pricing**. Marcotte et al. in [33] were the first ones to propose the use of tolls, traditionally adopted to regulate traffic congestion, as a tool to reroute carriers far from populated areas. Tolls add to the cost of the links, thus influencing carriers route choice. First a bi-level problem is proposed, and then it is reduced to a mixed-integer single-level problem. On a problem instance from Western Ontario, Canada, the toll based policy proved more effective in risk mitigation than HRND.

Marcotte et al. disregard the effects on risk due to the interaction between hazmat and regular vehicles. This gap is filled by later studies such as [34–36].

In particular, Wang et al. in [35] introduced the notion of dual-toll pricing (regular and hazmat tolls) to account for such an interaction and used a new risk measure to consider duration-population-frequency of hazmat exposure, while assuming a linear travel delay for analytical tractability. The problem is formulated as a Mathematical Program with Equilibrium Constraints (MPEC) which is solved separately at each stage. The approach is tested on a 46 nodes—70 arcs road network from Albany, New York state and 3 hazmat o/d pairs.

In [36] the toll paid by a carrier on a link also depends on the total risk induced on that link by all the carriers' route choices. Thus, the inner problem of the bi-level model is a Nash equilibrium problem among carriers, while at the outer level the government authority set tolls such that they minimize both the network total risk and the maximum link total risk, to address also risk equity. As carriers are allowed to split their shipment across different routes, their feasible region is a relaxation of [33]. The problem is solved heuristically and tested on the Ravenna network w.r.t. the aggregate risk function [31].

In [34], Esfandeh et al. improve on [35] by considering a nonlinear travel delay function in the dual-toll problem. The bi-level model tackles risk minimization at the upper level while the lower level models the toll driven optimal flow, where regular vehicles will take the user equilibrium flow and each hazmat carriers will optimize time and cost. Two cases are considered, when all links are tollable and when only a subset can be tolled, and a heuristic approach for each case is provided. The approach is tested on the Sioux Falls network (24 nodes and 76 arcs) with random demand and at most 20 o/d hazmat pairs.

Ke, Zhang, and Bookbinder [37] enrich the dual toll policy by integrating into the definition of risk a multi-degree fuzzy incident rate, population exposure, and travel time. Beside minimizing total risk also the maximum risk on a link is minimized in a bi-objective function. The problem is solved heuristically, by GA, and tested on the Sioux Falls network as well as on a 32 nodes - 102 arcs network extracted from the city of Nanchang, China.

The toll strategy has been further generalized in [38], where carriers are assumed to be bounded rational decision-makers whose route choice behavior is affected by a link-based perception error. Risk equity between different links is pursued by constraining the maximum link risk. The proposed bi-level programming model, aiming to minimize both total tolls and maximum total risk, is solved by combining particle swarm optimization and k-shortest paths.

Masoud, Kim, and Son [39] argue that both HRND and toll pricing, when implemented independently as standalone policies, may not demonstrate the same level of effectiveness in practice as they seem to suggest on paper. HRND is questioned in terms of carriers extra cost, while toll pricing is criticized for its limitations to existing tollways or its need for an extensive deployment of toll stations or sensors. They propose to integrate dual toll pricing and HRND into a three-level model which embeds Wardrop's principles of traffic equilibrium. The approach is tested on the San Antonio freeway and toll way system, modeled as a network with 10 nodes.

Finally, Sun et al. in [40] consider the 35-commodity instances on the Ravenna network used in [26] beside many others, taking as a risk measure the cost of accidents times the accident probability on the arc. They analyze the impact of HRND on the cost increase for each individual o/d pair and found that (i) 10 out of 35 were already on the minimum risk path, and have not been affected by link closure; (ii) in 5 cases risk increased, for three of them below 1% and above 18% for the other two; (iii) cost increase for those that have been rerouted ranges from 10% to 57%, showing a great inequality. The authors propose to add a maximum cost increase for each o/d pair, thus addressing at the same time total risk, total cost and cost fairness, and compute its impact on total risk and total cost in various scenarios. Since cost increase and risk reduction do not follow a linear trend, the threshold for individual cost increase should be carefully calibrated.

The impact of uncertain risk coefficient on HRND has been addressed in [41], where uncertainty on each link for each shipment as well as uncertainty on each link across all shipments are heuristically tackled.

A much lighter and technologically undemanding control system is based on **gateways**, intended as compulsory checkpoints for hazmat vehicles on which policy GBP is based. GBP aims at diverting vehicles away from their risky shortest path from origin to destination by assigning to each vehicle a compulsory gateway to be crossed along the itinerary. Apart from this requirement, there is no other compliance, so each vehicle will follow the shortest route through the assigned gateway, a so-called shortest *gateway path* [42]. GLP consists of locating k gateways and assigning one gateway to each vehicle so that total risk is minimized. This relation is captured by a bilevel multi-commodity flow model whose formulation was introduced in [4] where the GBP was first presented. GLP was further investigated in [6] and its resilience to variable demand was experimentally assessed in [43]. When tested on the 35 commodities on the Ravenna network, the itineraries yielded by the GBP provide a high quality tradeoff between risk and cost [5] for all the proposed risk functions, approaching the cost-risk Pareto frontier. However, fairness was not considered.

It should be mentioned that many other studies have addressed the multi-objective nature of hazmat routing in the multi-commodity case but took for granted that the authority can enforce routes on carriers. For example, in [44] goal programming was used to handle the conflict among cost and different risk types, while equity was managed by adding capacity to the arcs of the network. A different perspective is taken in [45] where total risk and risk equity are both addressed. Total risk is minimized while guaranteeing equitable risk spreading by solving a bi-level model by linearizing the KKT optimality conditions of the inner problem. Regional and local authorities are the two stakeholders with conflicting objectives: regional authority (the follower of the bi-level model) aims to minimize total risk, while local authorities (the bi-level model leader) want to lower the risk over their local jurisdictions, thus pursuing equity in terms of minimization of the maximum local risk.

A (selection of) the aforementioned studies is summarized in Table 1, to show at a glance their main features, emphasizing which solution quality criteria are handled.

In conclusion, four stakeholders involved in decision-making can be identified, namely: (i) global production system, which aims at total cost minimization of the transport system, (ii) governing authority which pursues total risk minimization, (iii) local authority which promotes equity in spacial risk distribution, (iv) individual drivers that would like to bound the shipment cost increase, w.r.t. the unregulated scenario, that each of them has to face for sake of risk reduction. The four criteria reported in Table 1 in columns 2–5 in this order, namely *Cost*, *Risk*, *Fairness (population)*, and *Fairness (carrier)*, reflect this view, while showing the attempts to partially synthesize the perspective of these stakeholders. In particular, regarding the rule based approaches for global routing, we can see a lack of contributions concerning fairness integration, that is, aiming at embodying total risk, total cost, and risk equity all at once, in the respective risk mitigation policy. We aim to partially fill this gap by empowering the GBP, which already handles total risk and total cost, with additional constraints to edge against inequity in spacial risk distribution.

3 | Multicommodity Flow Models for the Capacitated GLP

In this section, we recall the uncapacitated multi-commodity flow model for GLP and introduce two capacitated variants, obtained by imposing an upper bound on the flow allowed through each gateway, as well as through each other node. Other papers have exploited capacity to enforce equity, as [16, 45]. Note that capacity at a node imposes an upper bound on the flow of each arc incident on that node that is no higher than the node capacity itself, while link capacity induces a much higher upper bound on the flow traversing its two end nodes when their degree is greater than 2.

Consider the networks depicted in Figure 1, where arc cost coefficients are drawn in red and arc risk coefficients are drawn in

TABLE 1 | The main features of the most significant cited studies are listed, including: Which solution quality criteria are handled (cost, risk, and fairness w.r.t. uneven spatial distribution of risk or unequal cost increase for individual drivers); if the problem involves a single o/d pair or many; if vehicle itineraries are either the result of drivers' reaction to some regulations (*policy* option, such as link closure (HRND), toll pricing, or gateway, for global routing) or are directly computed and enforced by a single decision-maker; if the problem is solved exactly, heuristically, or both; and the size (number of nodes) of the networks on which computational experiments have been carried out.

References	Cost	Risk	Fairness (population)	Fairness (carrier)	Single/multi-commodity	Rules	Exact/heuristic	Node cardinality
[44] (Zografos, 1989)	✓	✓	✓	—	M	Goal programming	H	8
[16] (Gopalan, 1990)	—	✓	✓	—	S	—	E+H	50
[26] (Kara, 2004)	✓	✓	—	—	M	HRND	E+H	48
[20] (Dell'Olmo, 2005)	✓	✓	✓	✓	S	—	H	699
[7] (Erkut, 2007)	✓	✓	—	—	M	HRND	H	105
[32] (Verter, 2008)	✓	✓	—	✓	M	HRND	H	48
[33] (Marcotte, 2009)	✓	✓	—	—	M	tolls	E	48
[45] (Bianco, 2009)	—	✓	✓	—	M	Link capacity	E+H	311
[4] (Bruglieri, 2011)	✓	✓	—	—	M	Gateways	E	105
[36] (Bianco, 2016)	✓	✓	✓	—	M	Tolls	H	105
[40] (Sun, 2016)	✓	✓	—	✓	M	HRND	E	105
[37] (Ke, 2020)	✓	✓	✓	—	M	Tolls	H	32
[25] (Mohri, 2020)	✓	✓	—	✓	S	—	H	22
[30] (Fontaine, 2020)	✓	✓	✓	—	M	HRND	H	24

Note: The first author and the publication year have been reported for the sake of clarity.

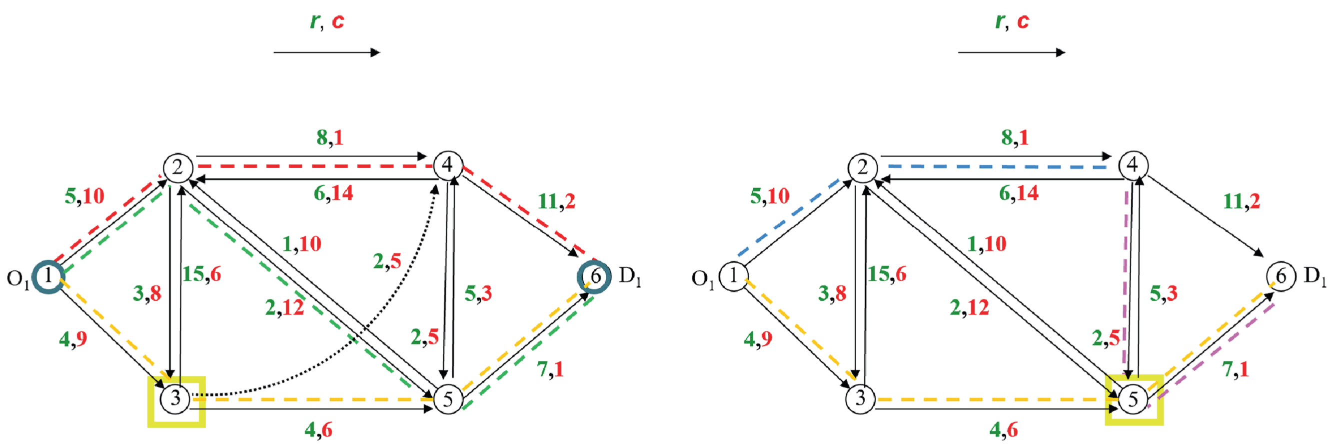

FIGURE 1 | In both networks, cost coefficients are in red, risk coefficients are in green. Consider carrier 1, shipping from node 1 to 6. On the left hand side, the itinerary in the unregulated scenario is shown in dotted red, the minimum risk path is shown in dotted green, and the gateway path w.r.t. node 3 is the dotted yellow path. In such a case, (i.e., $z_3^1 = 1$), if the extra arc (3, 4) were added (depicted as a black dotted arc), the carrier would have a second gateway path with the same cost but a much higher risk, and the bilevel problem would not be stable. When a second carrier (network depicted on the right hand side), shipping from 4 to 6, has to share the only open gateway this must be moved to node 5: Its gateway path is shown as the purple dotted line. On the right, the exemption of a third carrier, shipping from 1 to 4, is also shown, and its path is depicted in dotted blue line.

green. Carrier 1 ships from node 1 to node 6 ($o_1 = 1, d_1 = 6$). As shown on the network on the left hand side, in an unregulated scenario, that is, when the authority prescribes no rules, the carrier would follow its minimum cost itinerary, namely path (1, 2, 4, 6) depicted in dotted red, with cost equal to 13 and risk value 24, while the safest itinerary is path (1, 2, 5, 6), depicted in dotted green, with cost 23 and risk 14. If the carrier were assigned a gateway located at node 3, its gateway path would be (1, 3, 5, 6), depicted as the dotted yellow line, with cost 16 and risk 15. Had the extra arc (3, 4) been added, that is, the black dotted arc with cost 5 and risk 2, carrier 1 would have a second gateway path w.r.t. node 3, with a much higher risk. In such a case, the bilevel problem would not be stable.

Now, consider the network on the right hand side of Figure 1, and suppose that a second carrier has to ship from node 4 to node 6; in the unregulated scenario it would follow its minimum cost path (4, 6) with cost 2 and risk 11. If only one gateway could be opened ($k = 1$), this should be relocated from node 3 to node 5 to achieve optimality. Indeed, carrier 1 would stay on the same path, while carrier 2 would decrease the transportation risk from 11 to 9 while increasing the cost by 4 by moving along the dotted purple path. Moreover, consider a third carrier, shipping from 1 to 4. This carrier would be exempted since no gateway could decrease its risk below the one of its minimum cost path.

3.1 | The Uncapacitated Multi-Commodity Flow Model for GLP

For sake of clarity, we recall the bilevel multi-commodity flow model that implements the GBP and its reduction to a single level MILP.

Consider a set of commodities $V = \{1, \dots, n_V\}$, each associated with an origin/destination pair o_v and d_v, and let φ_v be the quantity of hazardous materials that have to be shipped from o_v to d_v along a single itinerary (in short, the demand of v). In the following, the terms "commodity," "vehicle" and "carrier" will be used interchangeably.

The road network is modeled as an oriented graph $G = (N, A)$, with $o_v, d_v \in N \ \forall v \in V$. Cost and risk coefficients, denoted as $c_{ij} > 0, r_{ij} \geq 0$ respectively, are given $\forall (i,j) \in A$. Note that there is no capacity, neither on the arcs nor at the nodes. Let $N^G \subseteq N$ denote the set of candidate sites to host gateways, with potentially $N^G = N$, let k denote the number of nodes to be chosen in N^G for the purpose of hosting a gateway each, and let $gtw(v)$ denote the gateway assigned to vehicle v, that is, vehicle v must go through node $gtw(v)$ on its way from o_v to d_v by following an itinerary (a gateway path) made of two minimum cost paths. In particular, given $h = gtw(v)$, let $\overline{p}_v^h$, denote the upstream gateway path, that is, the shortest path from o_v to h, while $\underline{p}_v^h$ denotes the downstream gateway path, that is, the shortest path from h to d_v, for each carrier v. Moreover, let ρ_v^c (ρ_v^r) be the c-optimal (r-optimal) path from o_v to d_v for each $v \in V$, where c stands for cost and r stands for risk. When, for a given vehicle v, none of the gateways can lower the risk with respect to ρ_v^c, then vehicle v is exempted (see Figure 1 on the right hand side).

Consider the following binary variables:

i. variables y_h represent the selection of location h where a gateway will be activated $\forall h \in N^G$, that is, $y_h = 1$ denotes an open gateway at node h, while $y_h = 0$ means that no gateway has been located at node h;

ii. variables z_h^v represent gateway assignment, that is, $z_h^v = 1$ assigns the gateway located at node h to vehicle v, that is, $z_h^v = 1$ if and only if $gtw(v) = h$, for each $h \in N^G$, $v \in V$, while $z_h^v = 0$ means that the gateway at node h is not assigned to vehicle v;

iii. variables γ_v represent the possibility for vehicle v not to be rerouted whenever the detour by a gateway would increase the risk of the shipment with respect to the risk related to the minimum cost path; indeed, when $\gamma_v = 1$ no gateway is assigned to vehicle v so that it will travel along its minimum cost path; in such a case we say that vehicle v has been *exempted*; exemption is modeled by allowing the upstream gateway path to reach destination;

iv. variables $\overline{x}_{ij}^v$ and $\underline{x}_{ij}^v$ are the multi-commodity flow variables for the upstream and the downstream sub-path, respectively. For all vehicles $v \in V$, a binary variable $\overline{x}_{ij}^v$ is defined for each $(i,j) \in A$ so that, when $\gamma_v = 1$, the upstream sub-path can reach the destination node d_v; on the opposite, variable $\underline{x}_{ij}^v$ is defined for all $(i,j) \in A$: $i \neq o_v$, since no downstream sub-path may start from the origin o_v.

TABLE 2 | Main symbols and parameters of the GLP.

Symbol	Definition
$G = (N, A)$	The directed graph modeling an abstraction of the road network:
N	A set of nodes, at the intersection of a set of links
$A \subseteq N \times N$	A set of arcs
c_{ij}, r_{ij}	Non negative cost and risk of arc $(i,j) \in A$
$N^G \subseteq N$	The set of nodes where a gateway could be located
h	A generic gateway location
k	The number of gateways to be activated
$V = \{1, \dots, n_V\}$	Set of vehicles
(o_v, d_v)	Origin—destination pair for each vehicle $v \in V$, with $o_v, d_v \in N$
φ_v	Demand of vehicle $\forall v \in V$
$gtw(v)$	A notation to denote the node where the gateway assigned to vehicle v is located

TABLE 3 | Main variables of the bilevel GLP model.

Variable	Definition
$y_h \in \{0,1\}$	Binary variable associated with the selection of candidate location h, $\forall h \in N^G$
$z_h^v \in \{0,1\}$	Binary variable associated with the selection of gateway h to vehicle v, $\forall h \in N^G, \forall v \in V$
γ_v	Binary variable associated with the exemption of vehicle v, $\forall v \in V$
$\overline{x}_{ij}^v$	Binary flow variables on arc (i,j) for the upstream gateway path of vehicle v, $\forall v \in V$, $\forall (i,j) \in A$
$\underline{x}_{ij}^v$	Binary flow variables on arc (i,j) for the downstream gateway path of vehicle v, $\forall v \in V$, $\forall (i,j) \in A$

Tables 2 and 3 summarize constants, symbols, and the main variables for the GLP.

A bilevel uncapacitated multi-commodity flow formulation for GLP is provided below:

$$P^{BL.GLP} : \min \sum_{v \in V} \varphi_v \sum_{(i,j) \in A} r_{ij} (\overline{\xi}_{ij}^v + \underline{\xi}_{ij}^v) \quad \text{subject to} \quad (1)$$

$$\sum_{h \in N^G} z_h^v = 1 - \gamma_v \quad \forall v \in V \quad (2)$$

$$y_h \geq z_h^v \qquad \forall h \in N^G, \forall v \in V \quad (3)$$

$$\sum_{h \in N^G} y_h = k \quad (4)$$

$$z_h^v \in \{0,1\} \qquad \forall h \in N^G, \forall v \in V \quad (5)$$

$$y_h \in \{0,1\} \qquad \forall h \in N^G \quad (6)$$

$$\gamma_v \in \{0,1\} \qquad \forall v \in V \quad (7)$$

where

$$\overline{\xi}_{ij}^v,\ \underline{\xi}_{ij}^v \in argmin\ P^{SP}: \quad (8)$$

$$\min \sum_{v \in V} \sum_{(i,j) \in A} c_{ij}(\overline{x}_{ij}^v + \underline{x}_{ij}^v) \qquad \text{subject to}$$

$$\delta^+_{o_v}(\overline{x}^v) = 1 \qquad \forall v \in V \quad (9)$$

$$\delta^-_h(\overline{x}^v) - \delta^+_h(\overline{x}^v) = z_h^v \qquad \forall h \in N^G, \forall v \in V \quad (10)$$

$$\delta^-_i(\overline{x}^v) - \delta^+_i(\overline{x}^v) = 0 \qquad \forall i \neq o_v, d_v,\ i \notin N^G, \forall v \in V \quad (11)$$

$$\overline{x}_{ij}^v \geq 0 \qquad \forall (i,j) \in A, \forall v \in V \quad (12)$$

$$\delta^-_{d_v}(\overline{x}^v) = \gamma_v \qquad \forall v \in V \quad (13)$$

$$\delta^-_{d_v}(\underline{x}^v) = 1 - \gamma_v \qquad \forall v \in V \quad (14)$$

$$\delta^+_h(\underline{x}^v) - \delta^-_h(\underline{x}^v) = z_h^v \qquad \forall h \in N^G, \forall v \in V \quad (15)$$

$$\delta^+_i(\underline{x}^v) - \delta^-_i(\underline{x}^v) = 0 \qquad \forall i \neq o_v, d_v,\ i \notin N^G, \forall v \in V \quad (16)$$

$$\underline{x}_{ij}^v \geq 0 \qquad \forall (i,j) \in A, \forall v \in V \quad (17)$$

As usual, $\delta^+_i(\overline{x})$ stands for $\sum_{(i,j) \in FS(i)} \overline{x}_{ij}$ and $\delta^-_i(\overline{x})$ for $\sum_{(j,i) \in BS(i)} \overline{x}_{ji}$ where $FS(i)$ and $BS(i)$ denote respectively the forward and backward star of node i; the same notation holds for variables $\underline{x}_{ij}$. Moreover, $\overline{\xi}_{ij}^v$ and $\underline{\xi}_{ij}^v$ denote the optimal value of the binary flow variables of the upstream and downstream gateway path of vehicle v, $\forall v \in V$.

Note that variables z_v^h are decision variables at the outer level, while they act as right-hand side coefficients of the flow balance constraints at the inner level. Indeed, when node $h \in N^G$ is selected to host the gateway assigned to vehicle v, that is, $z_h^v = 1$, then node h acts as the destination of the upstream gateway path as well as the source node of the downstream one.

Cardinality constraints (4) impose that exactly k gateways are installed at that many locations, while semi-assignment constraints (2) assign to each commodity one open gateway but for the case $\gamma_v = 1$. This option captures the case in which no gateway assignment is able to reduce the risk of a commodity v with respect to the unregulated scenario, such as when $r_{\rho_v^c} = r_{\rho_v^r}$, that is the risk of the minimum cost path is the same as the risk of the minimum risk path. In such a case a fake gateway located at the destination node is assigned to the vehicle. This guarantees that the GLP solution will never increase the risk of the unregulated scenario. Constraints (3) ensure that a gateway must be open in order to be assigned. Observe that the integrality constraints (6) on the y_h variables could be relaxed since they are implied by constraints (5). Furthermore, the inner problem is separable and it corresponds to the solution of $2n$ shortest path problem. Denote them respectively as $SP_h^v(\overline{x})$ (constraints (9–13)) and $SP_h^v(\underline{x})$ (constraints (14–17)).

The unimodularity of the flow balance constraint matrix allows us to reformulate the problem as a one level optimization problem by exploiting linear programming duality, and restate the objective function of the inner problem in terms of its optimality conditions. In particular, let us introduce $\pi^+_{o_v v}$, π^+_{hv} and π^+_{iv} as the dual variables associated with flow balance constraints (9–11), and $\pi^-_{d_v v}$, π^-_{hv} and π^-_{iv} as those associated with (14–16), respectively. Dual feasibility constraints of the two shortest path problems $SP_h^v(\overline{x})$ and $SP_h^v(\underline{x})$ are formulated as usual in (18) and (19) and (20) and (21), respectively.

Note that, since the rank of the flow balance constraint matrix is the number of nodes minus one, there is one degree of freedom in setting the value of the dual variables ((19) and (21)):

$$\pi^+_{jv} - \pi^+_{iv} \leq c_{ij} \qquad \forall (i,j) \in A,\ \forall v \in V \quad (18)$$

$$\pi^+_{o_v v} = 0 \qquad \forall v \in V \quad (19)$$

$$\pi^-_{iv} - \pi^-_{jv} \leq c_{ij} \qquad \forall (i,j) \in A,\ \forall v \in V \quad (20)$$

$$\pi^-_{d_v v} = 0 \qquad \forall v \in V \quad (21)$$

Optimality is reached when the feasible solutions of the primal and the dual problem have the same objective function value, as stated below.

$$\sum_{(i,j) \in A} c_{ij}\overline{x}_{ij}^v = \sum_{h \in N^G} \pi^+_{hv} z_h^v \qquad \forall v \in V \quad (22)$$

$$\sum_{(i,j) \in A} c_{ij}\underline{x}_{ij}^v = \sum_{h \in N^G} \pi^-_{hv} z_h^v \qquad \forall v \in V \quad (23)$$

Let us introduce a new set of variables $\{\omega_h^{+v}, \omega_h^{-v} \geq 0\ \forall h \in N^G,\ \forall v \in V\}$, due to represent the cost of the optimal upstream and downstream gateway path, respectively, by way of the following constraints: Constraints (24) and (25) linearize optimality conditions (22) and (23), while $\omega_h^{+v} \leq \pi^+_{hv} z_h^v$ and $\omega_h^{-v} \leq \pi^-_{hv} z_h^v$ is enforced by (26–29), so that (26–29) in synergy with (24) and (25) yield the equalities $\omega_h^{+v} = \pi^+_{hv} z_h^v$ and $\omega_h^{-v} = \pi^-_{hv} z_h^v$.

$$\sum_{(i,j) \in A} c_{ij}\overline{x}_{ij}^v = \sum_{h \in N^G} \omega_h^{+v} \qquad \forall v \in V \quad (24)$$

$$\sum_{(i,j) \in A} c_{ij}\underline{x}_{ij}^v = \sum_{h \in N^G} \omega_h^{-v} \qquad \forall v \in V \quad (25)$$

$$\omega_h^{+v} \leq L_h^{+v} z_h^v \qquad \forall h \in N^G\ \forall v \in V \quad (26)$$

$$\omega_h^{+v} \leq \pi^+_{hv} \qquad \forall h \in N^G\ \forall v \in V \quad (27)$$

$$\omega_h^{-v} \leq L_h^{-v} z_h^v \qquad \forall h \in N^G\ \forall v \in V \quad (28)$$

$$\omega_h^{-v} \leq \pi^-_{hv} \qquad \forall h \in N^G\ \forall v \in V \quad (29)$$

L_h^{+v} and L_h^{-v} can be set equal to the cost of any path from o_v to h and from h to d_v. In this way, ω_h^{+v} is set to 0 if the corresponding gateway is not assigned to commodity v, and it is bounded from above by the dual variable associated with node h, otherwise. The one level MILP reformulation is:

$$P^{1L.GLP}: \min \sum_{v \in V} \varphi_v \sum_{(i,j) \in A} r_{ij}(\overline{x}_{ij}^v + \underline{x}_{ij}^v) \quad \text{s.t. } (2\text{–}21), (24\text{–}29) \quad (30)$$

Note that the existence of alternative minimum cost paths with different levels of risk may make the model unstable, since the optimal solution of the inner problem is not guaranteed to be unique. However, the model becomes stable by perturbing the cost coefficients of the arcs. In particular, by adding the arc risk coefficients (once normalized by a given constant) to the cost coefficients, we represent a collaborative behavior of the driver, who will choose the minimum risk path among the minimum cost ones. Likewise, by subtracting them from the cost coefficients, an adversarial behavior is represented, as the driver would choose the riskiest path among the minimum cost ones, as discussed in [4]. Similarly, a collaborative (cost-aware) or adversarial behavior of the leader can be obtained by perturbing the risk coefficients (by summing or subtracting the normalized cost coefficients, respectively). This last feature can be exploited to stress the risk mitigation potentials of the proposed solution approach in the most challenging scenario, that is, an adversarial behavior of both the leader and the followers.

3.2 | Adding Capacity to the GLP

Starting from the observation that the GBP obliges the flow to go through the selected k gateways, the first attempt is to impose capacity constraints only on the nodes selected to host a gateway, with respect to the commodities for which the node acts as a gateway. This is achieved by the following constraint where Δ^h is the gateway capacity:

$$\sum_{v \in V} \varphi_v z_h^v \leq \Delta^h \; y_h \qquad \forall h \in N^G \tag{31}$$

However, a gateway h may also be traversed by the flow of a commodity whose gateway is different from h. A stronger variant of (31) imposes an upper bound on the sum of all the flow that traverses a gateway, as in (32):

$$\sum_{v \in V} \varphi_v \sum_{j:(h,j) \in A} (\overline{x}_{hj}^v + \underline{x}_{hj}^v) \leq \Delta^h \; y_h + D(1 - y_h) \quad \forall h \in N^G \tag{32}$$

where $D = \sum_{v \in V} \varphi_v$.

The first model variant we consider refers to the MILP model (30) and (32).

In order to ensure a more comprehensive spatial distribution of risk, capacity is imposed on each node, yielding the more constrained variant of the problem, modeled by (30) and (33), where Δ^i is the node capacity:

$$\sum_{v \in V} \varphi_v \sum_{j:(i,j) \in A} (\overline{x}_{ij}^v + \underline{x}_{ij}^v) \leq \Delta^i \qquad \forall i \in N \tag{33}$$

Since both (32) and (33) are global constraints that tie together all commodities, at a first glance one might expect them to affect the separability of the inner problem, thus forbidding the one level reformulation of both capacitated models. Indeed, in the minimum cost multi-commodity flow problem, the addition of capacity forces some commodity to be routed along a different route than the minimum-cost one. On the contrary, in the GLP model, constraints (32) and (33) affect just the gateway assignment while the itinerary from o_v to $gtw(v)$ and the one from $gtw(v)$ to d_v for any commodity and for any gateway are optimal w.r.t. cost. This allows to add either (32) or (33) to the single level reformulation (30). More formally:

Theorem 1. *The bilevel integer linear programming model for the Capacitated GLP defined by (1–17), (33) can be reformulated as a one level ILP model.*

Proof. For each selection of k gateways and for each assignment of the selected gateways to the n_V vehicles, the inner problem (8–17) computes the n_V gateway paths, each made of two minimum cost paths. Either these gateway paths comply with capacity constraints (33), or the associated value for the y_h and z_h^v variables is not feasible. Therefore, constraints (33) are not part of the inner problem, but rather restrict the feasible values that y_h and z_h^v may assume. It follows that the inner problem retains the integrality property. □

Note that in the uncapacitated variant of the GLP any k-tuple of gateways yields a feasible solution, also due to the exemption, whereas this no longer applies in the capacitated variant.

4 | Computational Results

4.1 | Data Set

The network of the city of Ravenna, Italy, has been chosen as a benchmark by several studies in the hazmat literature such as [6, 7, 29]. Not only the graph represents a real network, but risk coefficients on the arcs are available with respect to three different risk functions that provide a different perspectives, namely the *on-arc*, *around-arc*, and *aggregate* risk functions. Moreover, the city layout is a critical one for many reasons: (i) the city lies on the cost of the Adriatic Sea, and an industrial harbor is located not far from the city center, where hazmats may arrive by boat and are stocked at large depots; (ii) natural gas is produced offshore and it is processed once it reaches the coast; (iii) there are few industrial districts and about 130 fixed plants, mainly operating in the field of petrochemical, agricultural and inorganic products, and food industry [46] located in the outskirt of Ravenna which could require hazmats as production inputs and act as destination points; (iv) the city is at the crossroad of three main road infrastructure heading north, south, and west with respect to the urban center. At the same time, the area around the port is densely populated, there is a protected (naturalistic) area on the north shore while the south shore is a popular holiday destination. The abstract representation of the Ravenna road network is made of 104 nodes and 134 links (268 arcs).

Given the network, regarding the commodities, we consider the instances with $\{20, 30, 40, 50, 60\}$ vehicles used in [27, 31]. Namely, for each value of n_V, the test bed consists of ten instances with demand randomly generated in $[10, \ldots 100]$ with uniform distribution. Note that previous studies on the GBP also included computational experiments on the Ravenna network and 35 o/d pairs, whose demand varies between 25 and 50,186 (# of shipments) with an average of 4217. In this study, those instances have been disregarded because of their high variance in demand distribution and since the flow of the same commodity is unsplittable. Setting a meaningful value for the capacity, which must obviously be greater than the largest demand, would be tricky. At the same time, a large capacity threshold (such as being able to accommodate the two largest commodities on the same node) would not impact on the routing of commodities with demand from average to little. In our opinion flow splitting should be allowed in such cases, on behalf or risk equity.

4.2 | KPIs, Model Variants, and Experimental Setting

In this section, we introduce the key performance indicators (KPIs) according to which the obtained solutions are evaluated and we describe the computational experiments.

The selected KPIs allow us to evaluate both the quality of the obtained solutions and the efficiency of the model used to generate them. The solution quality is evaluated in terms of the three perspectives discussed above, that is, safety, cost-effectiveness, and fairness intended as a fair risk spacial distribution. Specifically, the safety (risk) and cost-effectiveness (cost) of the GLP solutions are positioned in the risk (cost) range given by the two extreme scenarios, that is, the over-regulated scenario in which each carrier is obliged to follow the minimum risk path and the unregulated scenario in which each carrier travels along the minimum cost path, as depicted in Figure 1. Thus, the risk and cost of these two paths give the risk and cost range. To be more precise, as already observed, we can have alternative minimum risk paths with different costs and the same occurs for the minimum cost paths which can exhibit different levels of risk.

In our experiments, among the alternative minimum risk paths, we selected the one with the minimum cost that corresponds to the scenario in which the leader is cost-aware. In regards to cost, among the alternative minimum cost paths, we selected the maximum risk path that corresponds to the scenario in which the follower is adversative. The fairness of the solutions is measured in terms of *overflowed* nodes, that is, those nodes that are traversed by a quantity of flow exceeding the node capacity. Specifically, we report both the number of nodes for which the capacity constraint is violated (nOFN) and the entity of the violation (%OF). The latter is computed as the ratio of the excess flow traversing a node to the node capacity for the most affected node. Finally, the efficiency of the selected model is measured in terms of computational time expressed in CPU seconds. Summarizing, name and description of the KPIs are provided in Table 4.

We tested the following three model variants: (i) Uncap which is the uncapacitated version, (ii) CapGtw in which the capacity constraint is imposed only on the nodes selected as open gateways, and (iii) CapNode in which the capacity constraint is imposed on all the network nodes.

TABLE 4 | KPIs: Name and description.

Name	Description
nOFN	Number of overflowed nodes (capacity is exceeded)
%OF	Entity of overflow for the most affected node
RiskPos	Risk positioning in the risk range
CostPos	Cost positioning in the cost range
Time	Computational time (expressed in CPU seconds)

In regards to the parameter, the number k of gateways to open varies in {3,4,5,6,7} while node capacity $C(k)$ depends on k and is defined as follows:

$$C(k) = \begin{cases} (1+\delta)\sum_{v\in V}\varphi_v/k & \text{if } k \in \{3,4\} \\ (1+\delta)\sum_{v\in V}\varphi_v/4 & \text{if } k \in \{5,6,7\} \end{cases}$$

where δ represents a tolerance and is fixed to 0.5 in the experiments. Note that tolerance is necessary, in particular for $k \leq 4$, to keep the number of infeasible instances under control. Indeed, $\delta = 0$ refers to an ideal solution in which it would be possible to distribute the flow equally over each gateway or node despite of the fact that commodity flows are not splittable. Moreover, some nodes may be a necessary bottleneck for some o/d pairs due to the network sparsity. In addition, in GBP, the carrier routes cannot be modified other that varying within the set of gateway paths.

Summarizing, we run the three model variants (Uncap, CapGtw, and CapNode) on the possible combinations of risk function (aggregate, around-arc, and on-arc), number $|V|$ of commodities in $\{20, 30, 40, 50, 60\}$, 10 instances for each value of $|V|$, 5 values for k ($k \in \{3, 4, 5, 6, 7\}$ summing up to 2250 experiments presented and discussed in the next section.

The numerical analysis has been performed on a PC equipped with an AMD Ryzen 9 3950x 16-core processor ×32 and 32 Gb of RAM. The optimization models have been coded in C++ and solved using the IBM ILOG Cplex 20.1 solver imposing a time limit equal to 7200 s.

4.3 | Analysis and Discussion

In this section, computational results are presented and discussed separately for each of the three risk functions. First,

TABLE 5 | For each risk function and each value of the number of commodities ($|V|$), the average, minimum, and maximum values of the number of overflowed nodes in the minimum risk path are reported.

Risk	|V|	avg	min	max
Aggregate	20	15.18	1	28
	30	12.96	0	26
	40	16.54	0	23
	50	16.24	0	26
	60	14.50	0	24
Around-arc	20	16.54	0	23
	30	5.42	0	16
	40	5.84	0	13
	50	6.52	0	13
	60	4.92	0	13
On-arc	20	7.06	0	19
	30	5.36	0	17
	40	6.96	0	14
	50	6.16	0	18
	60	6.36	0	17

TABLE 6 | Aggregate risk function.

|V|	Model		Overflowed nodes		RiskPos	CostPos	Time
			nOFN	%OF			
20	Uncap	avg	4.12	41.85	18.02	54.53	9.52
		min	0	2.23	8.80	24.39	5.99
		max	12	107.47	27.70	142.17	25.75
	CapGtw	avg	3.68	37.86	18.70	54.40	104.88
		min	0	2.23	8.80	25.72	5.75
		max	11	80.01	27.70	131.21	1816.88
	CapNode	avg	0	0.00	20.31	42.32	34.21
		min	0	0.00	11.35	12.60	10.80
		max	0	0.00	30.51	115.57	69.49
30	Uncap	avg	3.32	45.87	23.76	55.33	18.03
		min	0	2.56	11.50	27.35	10.20
		max	8	98.73	40.75	114.90	31.11
	CapGtw	avg	2.72	49.44	23.77	56.95	25.54
		min	0	0.73	11.50	32.02	10.19
		max	8	109.26	40.75	119.13	168.97
	CapNode	avg	0	0.00	25.14	40.94	243.72
		min	0	0.00	14.14	21.36	33.08
		max	0	0.00	40.75	94.99	2484.43
40	Uncap	avg	3.36	43.71	27.72	51.75	39.93
		min	0	0.75	11.32	22.27	21.57
		max	14	97.75	47.82	107.66	206.97
	CapGtw	avg	2.86	33.94	28.79	48.44	591.16
		min	0	6.86	15.70	24.03	20.47
		max	11	78.66	47.82	96.04	7200.72
	CapNode	avg	0	0.00	31.76	37.90	1449.75
		min	0	0.00	17.38	23.51	92.81
		max	0	0.00	53.50	72.93	7202.90
50	Uncap	avg	2.04	26.89	26.35	47.59	53.96
		min	0	0.00	14.32	20.78	28.81
		max	6	83.55	39.44	89.81	146.53
	CapGtw	avg	1.86	24.52	26.39	47.01	79.35
		min	0	0.00	14.61	19.35	32.90
		max	5	72.74	39.44	89.55	376.25
	CapNode	avg	0	0.00	27.02	36.18	750.62
		min	0	0.00	16.44	17.39	68.41
		max	0	0.00	39.49	69.39	7201.50
60	Uncap	avg	2.52	25.85	23.89	55.70	62.53
		min	0	0.00	14.21	14.29	34.48
		max	13	114.68	38.33	107.17	178.61
	CapGtw	avg	1.94	24.22	23.90	53.88	88.96
		min	0	0.00	14.21	19.10	39.72
		max	5	85.95	38.33	89.36	331.01
	CapNode	avg	0	0.00	24.31	39.19	1357.59
		min	0	0.00	15.40	12.30	86.39
		max	0	0.00	38.35	70.35	7203.30

Note: For each value of the number of commodities ($|V|$) and model, the average, minimum, and maximum values of the following metrics are provided: The number of overflowed nodes and the entity of the violation—in percentage, for the most affected node, risk, and cost position of the gateway path in the range given by risk and cost of minimum risk and cost paths, computational time expressed in seconds.

TABLE 7 | Around-arc risk function.

|V|	Model		Overflowed nodes nOFN	%OF	RiskPos	CostPos	Time
20	Uncap	avg	3.26	31.11	10.35	56.47	6.91
		min	0	2.87	1.07	32.43	4.24
		max	10	78.84	30.67	85.74	12.22
	CapGtw	avg	2.76	27.21	10.57	56.60	12.51
		min	0	2.87	1.29	33.90	4.42
		max	10	78.84	30.67	85.74	39.50
	CapNode	avg	0	0.00	16.65	54.37	29.53
		min	0	0.00	7.25	26.63	10.94
		max	0	0.00	30.67	76.49	102.85
30	Uncap	avg	1.62	19.05	12.44	61.80	12.46
		min	0	0.53	4.58	43.51	7.19
		max	5	44.93	27.17	89.79	34.40
	CapGtw	avg	1.44	17.25	12.51	61.86	16.67
		min	0	0.53	4.58	43.51	6.61
		max	5	44.93	27.17	86.07	73.66
	CapNode	avg	0	0.00	13.79	58.59	56.91
		min	0	0.00	5.58	43.51	16.42
		max	0	0.00	27.17	82.21	179.22
40	Uncap	avg	2.2	26.59	11.64	65.96	25.09
		min	0	3.62	5.50	38.78	12.21
		max	7	64.22	22.92	102.85	69.40
	CapGtw	avg	2.06	33.08	11.82	67.93	39.24
		min	0	6.73	5.98	38.78	16.31
		max	6	64.22	22.92	102.85	204.40
	CapNode	avg	0	0.00	15.31	61.03	583.53
		min	0	0.00	7.98	38.78	26.92
		max	0	0.00	38.92	88.23	7201.41
50	Uncap	avg	1.42	12.56	14.74	59.94	32.16
		min	0	0.00	5.18	38.29	19.89
		max	4	46.71	29.22	82.09	66.09
	CapGtw	avg	1.34	12.85	14.79	61.08	47.45
		min	0	0.00	5.18	38.29	27.32
		max	4	46.71	29.22	84.25	150.23
	CapNode	avg	0	0.00	16.18	57.44	485.72
		min	0	0.00	6.19	38.29	48.88
		max	0	0.00	31.22	84.25	4389.88
60	Uncap	avg	1.24	9.16	15.39	65.07	37.68
		min	0	0.00	5.67	42.27	22.71
		max	5	36.85	30.53	98.93	73.54
	CapGtw	avg	1.12	10.67	15.42	65.95	58.06
		min	0	0.00	5.67	42.27	31.79
		max	5	36.85	30.53	98.93	159.20
	CapNode	avg	0	0.00	16.34	61.12	464.15
		min	0	0.00	6.07	42.11	64.10
		max	0	0.00	30.53	91.76	7207.16

Note: For each value of the number of commodities ($|V|$) and model, the average, minimum, and maximum values of the following metrics are provided: The number of overflowed nodes and the entity of the violation—in percentage, for the most affected node, risk, and cost position of the gateway path in the range given by risk and cost of minimum risk and cost paths, computational time expressed in seconds.

TABLE 8 | On-arc risk function.

|V|	Model		Overflowed nodes		RiskPos	CostPos	Time
			nOFN	%OF			
20	Uncap	avg	4.2	45.57	12.23	65.07	5.49
		min	0	2.41	5.26	28.68	3.08
		max	16	97.36	25.85	91.65	13.36
	CapGtw	avg	3.44	40.20	13.21	63.31	29.93
		min	0	2.41	5.26	28.68	3.45
		max	11	76.72	25.85	90.38	340.91
	CapNode	avg	0	0.00	31.57	51.75	20.81
		min	0	0.00	5.47	28.68	4.72
		max	0	0.00	75.73	79.92	120.76
30	Uncap	avg	2.9	29.68	12.13	68.91	9.69
		min	0	1.36	6.46	43.06	4.68
		max	16	68.53	24.73	92.20	26.37
	CapGtw	avg	2.26	26.82	12.38	67.96	138.46
		min	0	1.36	6.46	43.06	4.80
		max	9	55.71	24.73	92.20	2053.25
	CapNode	avg	0	0.00	23.99	60.13	201.88
		min	0	0.00	6.51	26.05	7.27
		max	0	0.00	58.84	92.20	2476.54
40	Uncap	avg	3.98	24.32	9.32	69.87	14.54
		min	0	0.42	3.63	40.51	7.61
		max	11	76.48	19.26	89.21	29.92
	CapGtw	avg	2.92	23.97	9.86	68.08	220.69
		min	0	0.42	3.63	31.94	6.75
		max	9	76.48	19.72	89.21	3058.61
	CapNode	avg	0	0.00	18.55	63.08	138.68
		min	0	0.00	5.93	32.00	9.68
		max	0	0.00	41.74	89.21	918.78
50	Uncap	avg	2.9	15.42	13.45	68.80	22.39
		min	0	0.00	5.80	48.55	12.63
		max	10	47.13	20.94	94.11	39.34
	CapGtw	avg	2.74	14.14	13.63	67.30	150.39
		min	0	0.00	6.13	42.59	13.80
		max	10	47.13	20.94	92.16	1803.21
	CapNode	avg	0	0.00	20.64	62.31	610.76
		min	0	0.00	6.21	39.74	36.07
		max	0	0.00	36.43	92.13	7016.56
60	Uncap	avg	2.92	21.36	11.56	63.26	31.99
		min	0	0.00	7.86	38.68	17.12
		max	9	58.06	22.00	92.04	55.71
	CapGtw	avg	2.78	18.92	12.31	62.72	868.75
		min	0	0.00	7.86	38.68	23.27
		max	9	51.39	22.00	92.04	7202.30
	CapNode	avg	0	0.00	17.08	59.31	817.05
		min	0	0.00	7.86	43.13	38.81
		max	0	0.00	30.98	92.04	6660.16

Note: For each value of the number of commodities ($|V|$) and model, the average, minimum, and maximum values of the following metrics are provided: The number of overflowed nodes and the entity of the violation—in percentage, for the most affected node, risk, and cost position of the gateway path in the range given by risk and cost of minimum risk and cost paths, computational time expressed in seconds.

we present results supporting the fact that the minimum risk path might not be a good option in defining risk mitigation policies when ensuring fairness is crucial. Second, we give an overview of the computational results showing average values of the selected KPIs. Third, we give more detailed results showing the impact of the k value on the selected KPIs. This latter analysis is reported for $|V| = 30$ which is one of the intermediate values of $|V|$ in these experiments, and close to the original number of commodities—35.

Table 5 presents the average, minimum, and maximum values of the number of overflowed nodes in the over-regulated scenario. Each row reports information relevant to the 10 instances considered for a given value of $|V|$ and for a given risk function. The aggregate risk function is on average the risk function exhibiting the greatest number of overflowed nodes - which can be as high as 28. This is probably because when aggregate risks are considered, several zero-risk arcs are likely to appear in several minimum-risk paths thus concentrating flows in some specific network node. Indeed, the aggregate risk function is the one characterized by the greatest number of zero-risk arcs.

Note that, for each risk function, the maximum number of overflowed nodes tends to decrease as $|V|$ increases. We believe it is due to the fact that, as the capacity threshold increases proportionally with $\sum_{v \in V} \varphi_v$, the absolute value of the tolerance δ increases too.

Tables 6–8 report average, minimum, and maximum values of all the KPIs defined in Table 4, respectively for aggregate, around-arc, and on-arc risk functions. In rows labeled min and max, observe that the instance for which the minimum or maximum value of a KPI is obtained is not necessarily the same across all the KPIs.

As a general comment, we report that CapNode can fail to provide a feasible solution with the chosen tolerance ($\delta = 0.5$). This occurs 9, 13, and 17 times for aggregate, around-arc, and on-arc risk functions, respectively. The total number of failures is thus 39 over 750 runs. All the failures occur for $|V| = 20$ and they significantly reduce or disappear as the tolerance increases.

In regards to computational time, the time limit is exceeded occasionally when CapGtw or CapNode are used. For CapGtw, it happens 2 times on the combination aggregate and $|V| = 40$, and 4 times on the combination on-arc and $|V| = 60$. For CapNode, the figures are 7 times on the combination aggregate and $|V| = 40$, one time on the combination aggregate and $|V| = 50$, 4 times on the combination aggregate and $|V| = 60$, one time on the combination around-arc and $|V| = 40$, and one time on the combination around-arc and $|V| = 60$.

In regards to KPI CostPos, we observe that the cost of the GLP solution can exceed the cost range because we do not control cost deterioration. With the aggregate risk function, this occurs 13 times for UnCap, 7 for CapGtw, and 2 for CapNode. With the

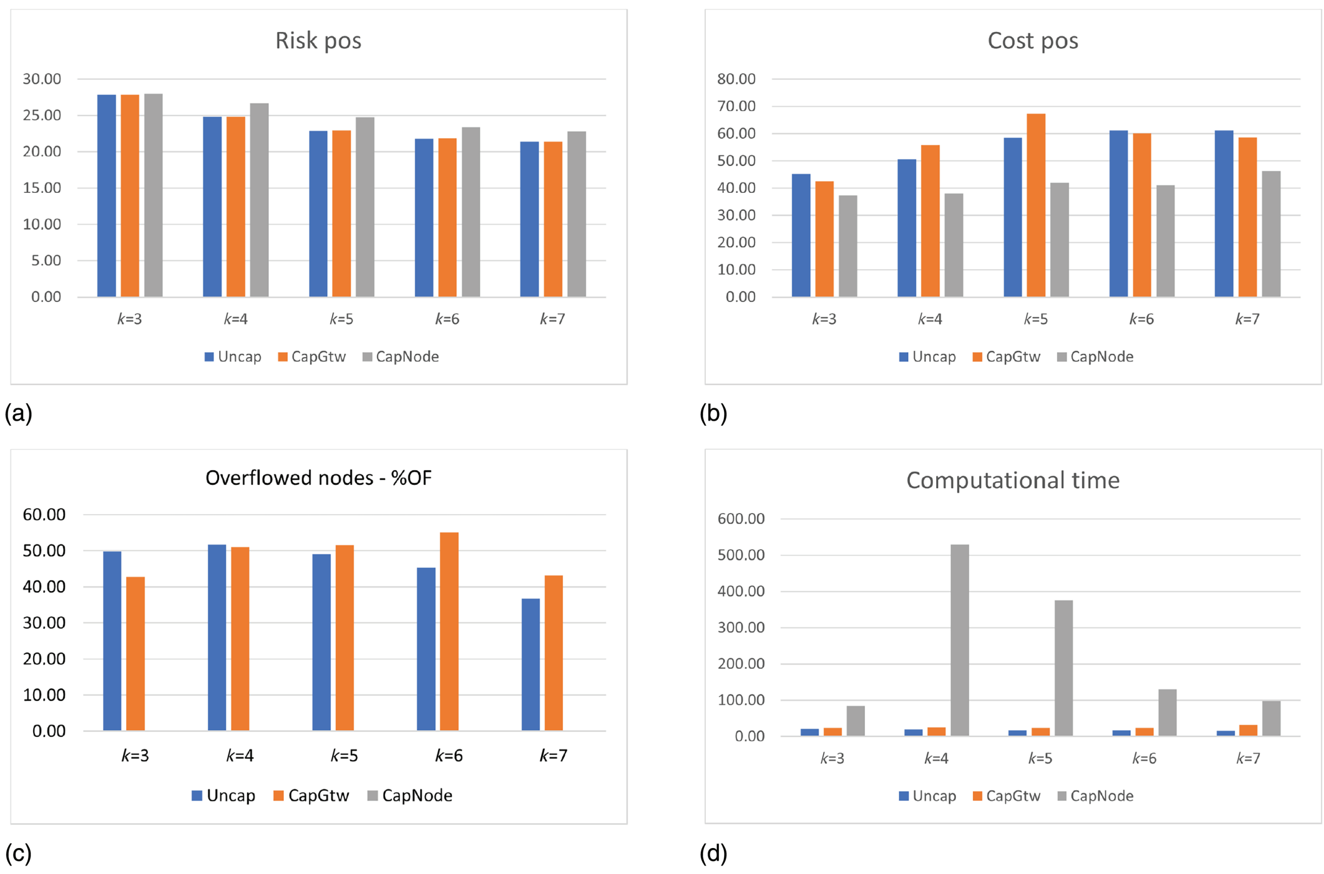

FIGURE 2 | Aggregate risk function. (a) Risk positioning—average. (b) Cost positioning—average. (c) Percentage capacity violation of the most affected node—average. (d) Computational time—average.

around-arc risk function, this occurs 1 time for UnCap and 1 time for CapGtw. With the on-arc risk function, it never occurs.

Across the three risk functions and the five values of $|V|$, the following general observations can be made: (i) Uncap is not satisfactory in pursuing fairness—%OF can indeed be quite high; (ii) CapGtw solves only partially this criticality—it is only able to slightly reduce the number of overflowed nodes as well as %OF, which in two cases is even worse for CapGtw than for Uncap; (iii) Uncap and CapGtw perform quite the same concerning RiskPos and CostPos; (iv) CapNode is well able to manage fairness with a limited increase of RiskPos—only with on-arc risk function and $|V| = 20$ the average increase of RiskPos is not negligible and is about 20%; (v) as a side effect, CapNode allows for controlling better cost-effectiveness to the other models; (vi) as expected, the computational time may remarkably increase when capacity constraints are imposed, and their impact seems to be greater for aggregate and around-arc risk functions than for the on-arc risk function. Interestingly, imposing the capacity constraint on all the nodes (CapNode) does not necessarily worsen the performance as opposed to the case in which the capacity is imposed only on a subset of nodes (CapGtw).

We conclude the section by giving a pictorial representation of the average (over the 10 instances) impact of k value on RiskPos, CostPos, %OF, and computational time when the number of vehicles is fixed ($|V| = 30$) separately for the three risk functions, respectively in Figures 2–4.

With the aggregate risk function (Figure 2), we can observe the following: (i) the risk level obtained with CapNode is very close to the one obtained with Uncap and the increase in RiskPos never exceeds 2% (Figure 2a); (ii) the cost of the solution is quite stable across k when CapNode is used, while it can vary remarkably with the other two models (Figure 2b); (iii) CapGtw is not always a viable option to reach fairness (see Figure 2c for $k \in \{5, 6, 7\}$); (iv) the computational times are negligible for Uncap and CapGtw, while they can be high for CapNode especially when intermediate values of k are selected (Figure 2d). However, for $k = 3$, CapNode provides very good solutions for all the KPIs.

With the around-arc risk function (Figure 3), we again observe a limited increase of RiskPos for all the k values when CapGtw or CapNode is used. For CapNode, the average computational time is much smaller than for the aggregate risk function and it reaches the maximum value for $k = 5$. There are however values of k for which the computational time is still acceptable even with CapNode. Finally, we observe that $k = 3$ allows us to obtain a good trade-off between solution quality and efficiency.

With the on-arc risk function (Figure 4), incorporating fairness can result in a significant increase in risk level, while from the

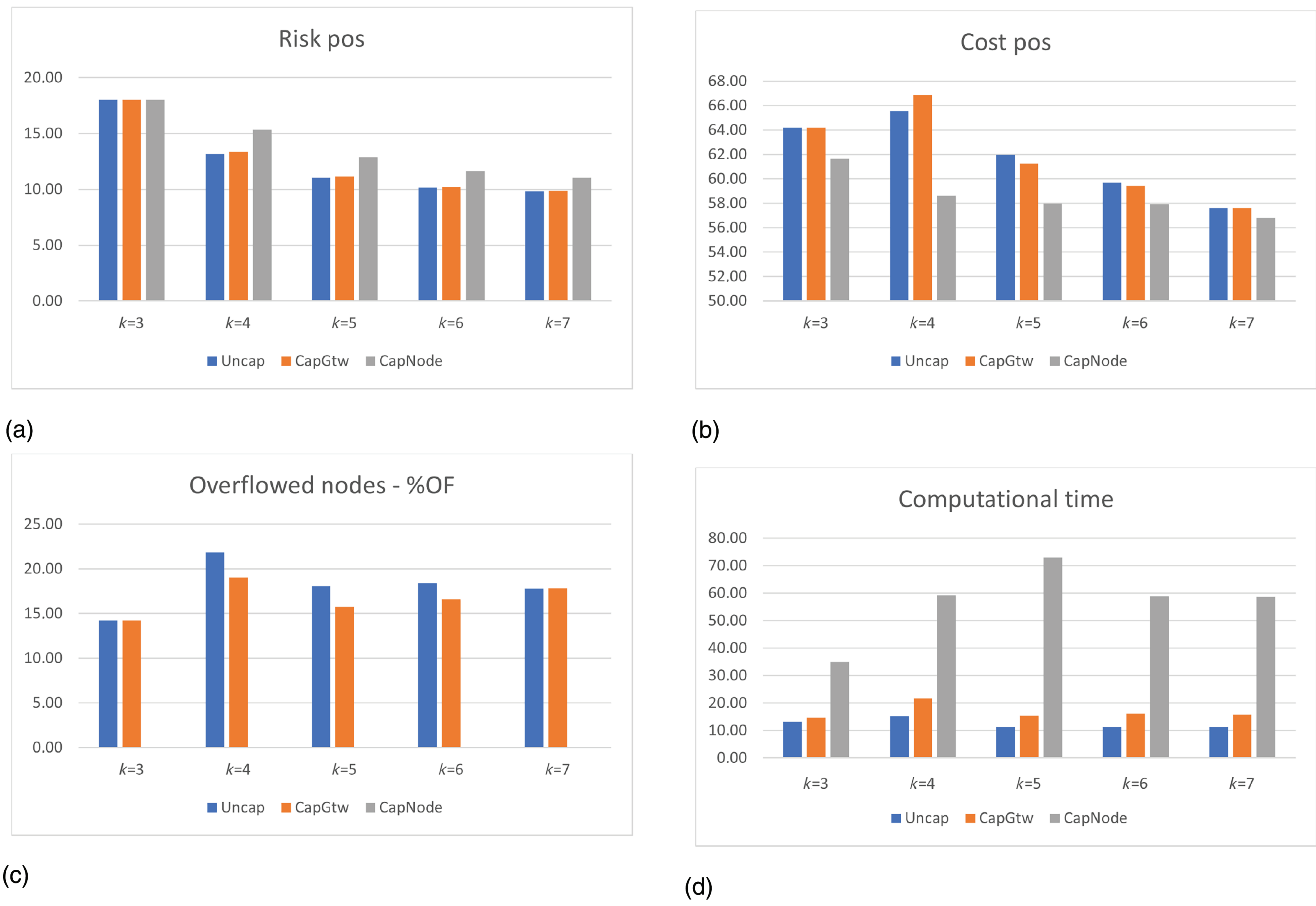

FIGURE 3 | Around-arc risk function. (a) Risk positioning—average. (b) Cost positioning—average. (c) Percentage capacity violation of the most affected node—average. (d) Computational time—average.

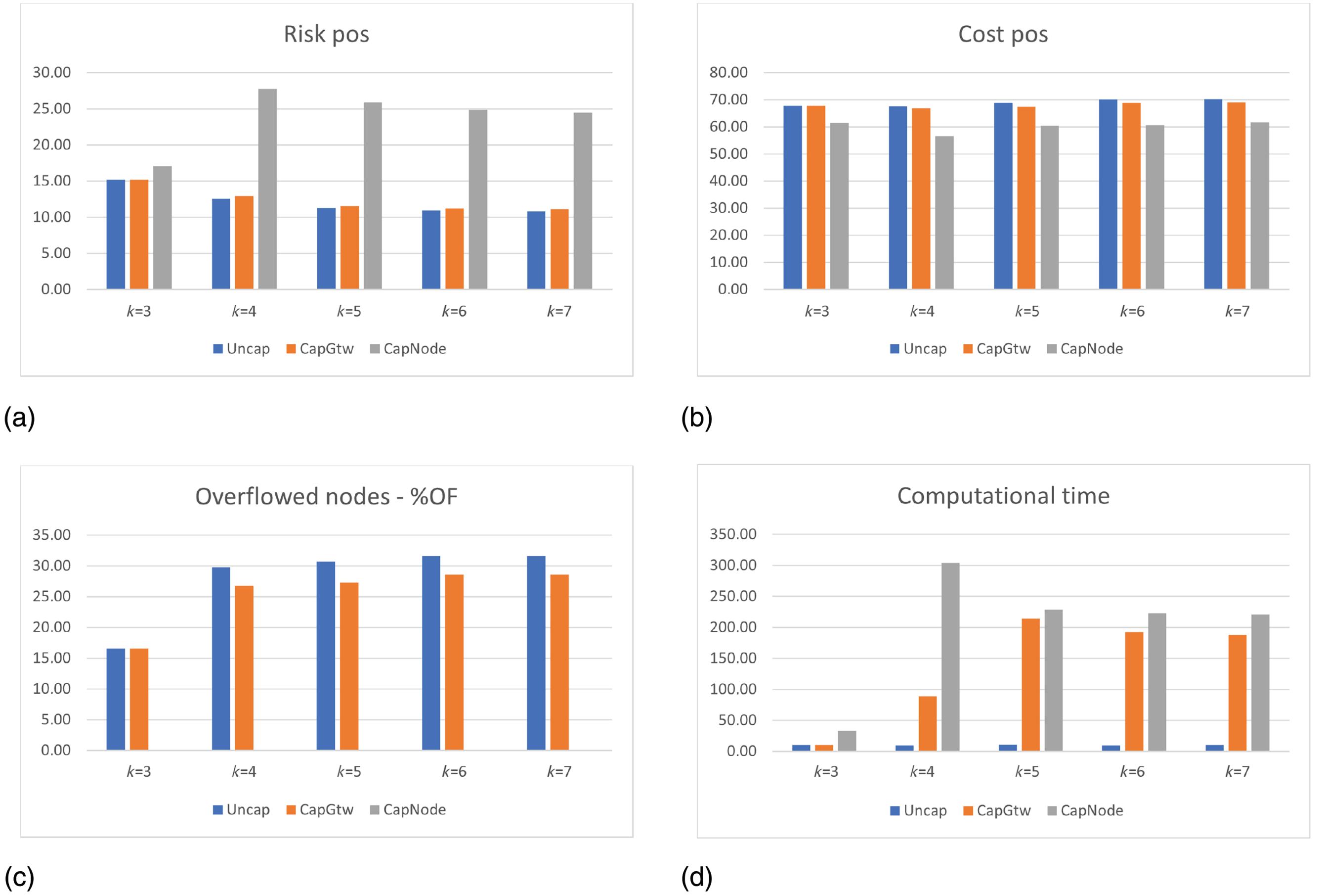

FIGURE 4 | On-arc risk function. (a) Risk positioning—average. (b) Cost positioning—average. (c) Percentage capacity violation of the most affected node—average. (d) Computational time—average.

cost-effectiveness perspective, CapNode is still better than the other two models. However, there is a value of k ($k = 3$) for which all the KPIs considered are satisfactory. The computational times required by CapNode are in an intermediate position to the other two risk functions.

5 | Conclusions

This study assesses the need for, and the viability of, introducing a fairness oriented component into the GBP for risk mitigation in hazmat transport on road networks. This is a necessary step since GLP solutions, despite of being next to the cost/risk Pareto frontier [5], show some criticalities with respect to spacial distribution of risk whose concentration may reach particularly high levels at some nodes, and not just at gateway nodes. Moreover, only few papers on global routing deal at the same time with total cost, total risk, and risk equity as well. Therefore it is important to widen the set of tools that public authorities may adopt to forge their risk mitigation policies aimed at rerouting hazmat flows by regulations. We propose to choose to impose capacity on the nodes as a proxy for an equity measure of risk spreading, that is, we put an upper bound on the maximum flow quantity allowed to traverse each node.

Note that we suppose that flow commodities differ on their o/d pair but behave in the same way with respect to potential risk. Therefore, total flow on the arcs (as well as on the nodes) can be taken as a proxy for risk, as already suggested in [44]. Nevertheless, w.l.o.g. our approach is able to handle different kinds of hazmat flows as well as different risk coefficients for each hazmat kind, on each link and node. In that case, the capacity constraint on the nodes could be rewritten as the maximum cumulative risk allowed, but the mathematical structure of the MILP model would remain basically the same.

Previous works had chosen to set capacity on the arcs or a bound on the total risk accumulated on each zone. Our choice, on the one hand clearly dominates imposing on the arcs the same capacity we set on the nodes, on the other hand, capacity on the nodes increases the granularity of the control with respect to a capacitated zone strategy, and we think the former is more consistent with hazmat transport in urban and sub-urban areas than the latter.

The results of experimentation conducted on widely used instances in the literature show that adding capacity on nodes proved to be a viable solution to pursue fairness preventing unequal spacial risk distribution. Ensuring fairness requires greater computational times with respect to the uncapacitated variant and comes at the cost of increasing total risk. The deterioration of risk, however, is generally small and depends very much on the risk function used. However, fairness can be achieved even with a limited number of gateways, which positively influences both the efficiency of the approach and the risk deterioration. Interestingly, it has been observed that imposing fairness also brings benefits in controlling costs.

From the methodological point of view, we remark that the GBP is able to encompass the capacity constraints on gateways as well as on each network node while retaining all the properties that allow single level reformulation. At the same time, we are aware of the many research directions that are still open that a generalization of the GBP may yield, some of which are currently under study.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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