Applied Mathematical Sciences, Vol. 17, 2023, no. 9, 413 - 420
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2023.917446

Non-Renewable Input and Waste Stock: Optimal Transition Towards Circularity

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Abstract

Natural resource stocks are drastically shrinking and the contemporary debate on depletion of reserves highlights their crucial role over time. This paper deals with the management of exhaustible resources when recycling and extraction are employed, as well as the short-term effects of the economic transition towards circularity on consumption and welfare.

Mathematical modeling may support effective plans for maximizing social welfare and protecting the environment. Concerning the production of a certain good, an optimal control model is studied to allocate labor between mining and recycling over a finite time horizon. A suitable scrap value function allows for reducing waste and maintaining natural stock in the forthcoming future. Well-posedness of the problem is proved and some qualitative features of the optimal solution are stated.

Mathematics Subject Classification: 49J15, 49K15, 34H05

Keywords: Optimal control, Optimality conditions, Exhaustible resource, Recycling, Input allocation

1 Introduction

The international authorities' agenda focuses on sustainability and the circular economy since it aims to manage waste, to safeguard natural resources, and to encourage production by employing recycled and second-market materials. The resources that are exhaustible or depletable are most at risk. They are produced by natural processes, but their creation rate is extremely slow on human timescales; as a result, this kind of resource is classified as non-renewable (cfr. [3], [7], [8], [9])). The overwhelming majority of non-renewable resources are crucial components of industrial production; but their natural stock is always fixed and finite, making it impossible to fully meet the demand for these resources on the world market. In this respect, a key challenge for a transition to sustainability and circular economy is the achievement of effective waste management and recycling. In this framework, we focus on both economic and environmental issues. Indeed, we aim to maximize the utility that results from consumption; at the same time, we concentrate on an environmental issue meant to strike a balance between the preservation of the natural resource, whose depletion could cause economic and environmental degradation, and the reduction of waste accumulation through recycling. These concerns are addressed by employing classical optimal control theory to optimize social welfare in terms of both current utility and future environmental damages. The subject of optimal control models dealing with recycling of natural resources is a major issue in the literature. Several authors in this regard focus primarily on municipal waste management and recycling processes (for instance, see [1], [2], [4]).

In this framework, we deal with a specific optimal strategy to control the use of both non-recycled and recycled depletable resources under a finite time horizon perspective. We develop an original model starting from the approach in [5], where the economy is studied by analysing its steady state over the infinite time horizon. Our original assumption of a fixed and finite temporal threshold is driven by United Nations and European Union rules which demand that certain environmental targets be met by a certain date in order to implement the ecological transition. However, taking into account a finite time horizon also makes it possible to investigate the short-term dynamics of the management of non-renewable resources and the corresponding recyclability. Rapid solutions might help the ecological transition and lessen environmental harm in the near future.

2 Production dynamics and optimal program

We assume that a final consumption good is produced by employing a non-renewable resource which is purchased from two different sectors. On one hand, the recycling sector exploits the existing waste stock and obtains the resource itself by a recycling process; on the other hand, the non-recycling sector is related to a conventional production of the depletable resource. Both recycled and non-recycled inputs are employed for the production of the final good C over a time horizon $[0, T]$, with given length $T > 0$. In this respect, the production depends on the non-recycled material denoted by $V(t)$ and on the recycled material $R(t)$, which is recovered from waste, according the following CES function

$$C(t) = \left[(\theta_V \, V(t))^\rho + (\theta_R \, R(t))^\rho\right]^{1/\rho}, \tag{1}$$

where ρ, θ_V and θ_R are time-invariant, with $0 < \theta_V, \theta_R < 1$. As usual, parameter ρ is related to the elasticity of substitution between non-recycled and recycled materials: due to the fact that the quality of recycled and non-recycled resources are different, we assume $0 < \rho < 1$ meaning that the quality of the non-recycled input exceeds the quality of the recycled one.

As a further assumption, the considered economy is endowed with a fixed amount of labor $L > 0$, which is devoted to get both inputs. We denote by $l(t)$ the labor demand employed in the conventional sector at every time t, thus $L - l(t)$ corresponds to the labor devoted to recycling. Our aim consists of efficiently allocating $l(t)$ in the admissible set

$$\mathcal{A} = \{l : [0, T] \to \mathbb{R} \; : \; 0 \leq l(t) \leq L, \; \forall t \in [0, T]\},$$

under the assumption of constant returns to scale in both non-recycling and recycling sectors such that

$$V(t) = m_V \, l(t), \qquad R(t) = m_R \, (L - l(t)), \tag{2}$$

with fixed labor productivities $m_V > 0$ and $m_R > 0$. Concerning the non-recycled input, it is related to the natural stock $S(t)$ of resource whose dynamics evolves as

$$\dot{S}(t) = -m_V \, l(t), \qquad S(0) = S_0, \tag{3}$$

where $S_0 > 0$ represents the initial amount of the available non-recycled input. Furthermore, the consumption process generates waste which can be partially saved and recycled for the production. We denote by $W(t)$ the cumulative amount of waste at any time t and suppose that the recycling process and the waste production occur at the same time. Let γ_V and $1 - \gamma_R$ be the waste generation rate of the non-recycled and the recycled materials, respectively, under the assumption that $0 < \gamma_V, \gamma_R < 1$. Then waste accumulation evolves according to the following equation

$$\dot{W}(t) = \gamma_V\, m_V\, l(t) - (1-\gamma_R)\, m_R\, (L - l(t)), \qquad W(0) = W_0, \tag{4}$$

where $W_0 > 0$ is the initial stock of recyclable waste inherited at $t = 0$ from the past. By integration of both (3) and (4) over the whole time horizon, we get

$$S_0 - m_V\, LT \leq S(t), \qquad W_0 - (1-\gamma_R) m_R\, LT \leq W(t).$$

Thus, we suppose that time horizon length T satisfies the following condition

$$T \leq \min\left\{\frac{S_0}{m_V\, L}, \frac{W_0}{(1-\gamma_R) m_R\, L}\right\}, \tag{5}$$

which assures $S(t) \leq 0$ and $W(t) \geq 0$ for any $t \in [0, T]$.

In this framework, a crucial issue is achieving the maximum welfare benefit from economic activities provided that the waste recycling capability increases and the resource stock is maintained for future time. Thus, the social planner program aims to efficiently allocate a restricted amount of labor for maximizing the social welfare thorough the objective function

$$J(l) = \int_0^T e^{-\delta t} U(C(l(t)))\, dt + \nu\, e^{-\delta T} S(T) - \mu\, e^{-\delta T} W(T), \tag{6}$$

where $\delta > 0$ represents the constant discount rate over time and

$$U(C(l(t))) = \frac{[C(l(t))]^{1-\sigma}}{1-\sigma} = \frac{[(\theta_V m_V l(t))^\rho + (\theta_R m_R (L - l(t)))^\rho]^{(1-\sigma)/\rho}}{1-\sigma},$$

is the consumers' utility deriving from the consumption of the final produced good. The scrap function depends on the weights $\nu, \mu > 0$ and models the future environmental damage related to both recycled and non-recycled stocks. Actually, the first term $\nu\, e^{-\delta T} S(T)$ can be interpreted as the value of an integral of the utility flow related to the future damage associated to the exhaustible resource employment, starting from time T with a stock $S(T)$. As a counterpart, the second term $-\mu\, e^{-\delta T} W(T)$ can be thought as the value of an integral of the future utility flow related to the future damage from waste disposal site starting from time T with a waste stock $W(T)$.
Under assumption (5), the optimal control model consists of searching for a labor $l^* \in \mathcal{A}$ such that

$$J(l^*) = \max_{l \in \mathcal{A}} J(l), \tag{7}$$

subject to state equations (3) and (4).

3 Optimal labor allocation

An optimal strategy for allocating the labor between the sectors can be characterized by defining the current value Hamiltonian as

$$H(l, S, W, \varphi_V, \varphi_R) = \frac{[C(l)]^{1-\sigma}}{1-\sigma} + \varphi_V\,[-m_V\,l] + \varphi_R\,[\gamma_V m_V l - (1-\gamma_R)m_R(L-l)],$$

where the costate variables φ_V and φ_R are the shadow prices of the non-recycled resource and recyclable stock, respectively. Due to the Pontryagin Maximum Principle, an optimal solution $l^*(t)$ together with state variables $S(t)$, $W(t)$ and costate ones $\varphi_V(t)$, $\varphi_R(t)$ must satisfy the following optimality necessary conditions:

(i) $l^*(t)$ maximizes $H(l(t), S(t), W(t), \varphi_V(t), \varphi_R(t))$, provided that $l^*(t) \in \mathcal{A}$;

(ii) the state dynamics is modelled by (3)-(4), where $l(t)$ is replaced by $l^*(t)$, and costate variables are continuous functions with piecewise continuous derivatives satisfying the following equations

$$\dot{\varphi}_V(t) = \delta\varphi_V(t), \qquad \varphi_V(T) = \nu\, e^{-\delta T},$$

and

$$\dot{\varphi}_R(t) = \delta\varphi_R(t), \qquad \varphi_R(T) = -\mu\, e^{-\delta T}.$$

By integration, we obtain

$$\varphi_V(t) = \nu e^{-\delta(2T-t)}, \qquad\qquad \varphi_R(t) = -\mu e^{-\delta(2T-t)}, \tag{8}$$

for all $t \in [0, T]$. We notice that $\varphi_V(t) > 0$ since the non-recycled resource stock is a "good" for the society, so that the larger the stock of non-recycled resource, the better is for the society with the aim to preserve Earth resources. On the other hand, $\varphi_R(t) < 0$ since the waste stock may represent a "bad" for the society as it harms the environment.

In order to maximize the Hamiltonian function, we focus on the derivative

$$\frac{\partial H}{\partial l} = C(l)^{1-\sigma-\rho} \cdot \Upsilon(l) - \Phi(\varphi_V, \varphi_R),$$

where we set $\Phi(\varphi_V, \varphi_R) = \varphi_V\, m_V - \varphi_R\,[\gamma_V m_V + (1-\gamma_R)m_R]$ with φ_V and φ_R evaluated by (8), and

$$\Upsilon(l) = (\theta_V\, m_V)^\rho l^{\rho-1} - (\theta_R\, m_R)^\rho (L-l)^{\rho-1}.$$

It is not so difficult to verify that $\partial H/\partial l$ is a continuous function with respect to l such that $\partial H/\partial l \to +\infty$ when $l \to 0^+$ and $\partial H/\partial l \to -\infty$ when $l \to$

L^-. Then, there exists a given labor level $l^* \in \mathcal{A}$ where $\partial H/\partial l$ nullifies. Furthermore, the second derivative is given by

$$\frac{\partial^2 H}{\partial l^2} = [C(l)]^{1-\sigma-2\rho} \left(-\sigma\,[\Upsilon(l)]^2 + (\rho-1)L^2[\theta_V\, m_V\, \theta_R\, m_R]^\rho[l(L-l)]^{\rho-2}\right).$$

Due to the fact that $\partial^2 H/\partial l^2 < 0$, then Hamiltonian function is strictly concave in l; therefore, l^* represents a maximum value for H. Thus, by evaluating $S(t)$ and $W(t)$ the solutions of (3)-(4) where $l(t)$ is replaced by $l^*(t)$, we obtain that $(l^*, S, W, \varphi_V = \nu e^{-\delta(2T-t)}, \varphi_R = -\mu e^{-\delta(2T-t)})$ satisfies the Maximum Principle statements (i)-(ii). It follows that l^* constitutes an optimal solution to problem (7), according to well-known and classical results of optimal control theory (see for instance Theorem 4.6.4 in [6]). In addition, since the Hamiltonian function is strictly concave in l, then it can admit no more than one maximum value: as a conclusion, there exists a unique optimal labor $l^* \in \mathcal{A}$ solving problem (7). In this way, the optimal control model well-posedness is stated.

4 Waste stock accumulation

In the transition towards circularity, a crucial issue is related to understanding whether the waste stock accumulates and increases or not. In this respect, we note that $\dot{W}(t) \geq 0$ in the case when the optimal labor allocation exceeds the level defined by the following threshold

$$\overline{L} = \frac{(1-\gamma_R)m_R}{\gamma_V m_V + (1-\gamma_R)m_R} L. \tag{9}$$

In other words, $W(t)$ increases over a given time interval if $l^*(t) \geq \overline{L}$; on the other hand, $W(t)$ is decreasing in time in the opposite case. Therefore, the comparison between the optimal labor l^* and $\overline{L}$ allows for establishing waste accumulation dynamics over time. More precisely, it is possible to consider the following two different situations.

(a) Under the assumption that

$$\Upsilon(\overline{L}) = (\theta_V\, m_V)^\rho \overline{L}^{\rho-1} - (\theta_R\, m_R)^\rho (L-\overline{L})^{\rho-1} > 0,$$

we set

$$\overline{\tau} = \frac{[C(\overline{L})]^{1-\sigma-\rho}\,\Upsilon(\overline{L})}{\nu m_V + \mu(\gamma_V m_V + (1-\gamma_R)m_R)} > 0,$$

and define $\bar{t} = 2T + ln(\overline{\tau})/\delta$. For any $t \leq \bar{t}$ we get

$$\frac{\partial H}{\partial l}(\overline{L}, S(t), W(t), \varphi_1(t), \varphi_2(t)) > 0;$$

therefore, due to the fact that $\partial H/\partial l$ is decreasing in l, the previous inequality implies $l^*(t) \geq \overline{L}$. Starting from this argument, the following different statements hold:

- If $\overline{\tau} \leq e^{-\delta 2T}$, then $\bar{t} \leq 0$; it follows that $\dot{W}(t) \leq 0$ over the whole time horizon and the waste stock reduces at every time.
- If $e^{-\delta 2T} < \overline{\tau} < e^{-\delta T}$, then $0 < \bar{t} < T$; as a consequence, we obtain $\dot{W}(t) \geq 0$ for $t \in [0, \bar{t}]$. It implies that waste accumulates until $\bar{t}$, after that waste reduces for the remaining time in $]\bar{t}, T]$.
- If $e^{-\delta T} \leq \overline{\tau}$, then $T \leq \bar{t}$. Thus, $\dot{W}(t) \geq 0$ and waste accumulates at any time.

(b) In the opposite situation, the assumption

$$\Upsilon(\overline{L}) = (\theta_V\, m_V)^\rho \overline{L}^{\rho-1} - (\theta_R\, m_R)^\rho (L - \overline{L})^{\rho-1} \leq 0,$$

yields $l^*(t) \leq \overline{L}$ for all $t \in [0, T]$, since $\partial H/\partial l$ is decreasing in l and

$$\frac{\partial H}{\partial l}(\overline{L}, S(t), W(t), \varphi_1(t), \varphi_2(t)) \leq 0.$$

It follows that $\dot{W}(t) \leq 0$ over the whole time horizon; thus the waste stock reduces over the whole horizon $[0, T]$.

5 Future purpose

This work represents the starting point for a deeper study where the social planner aims to allocate labor in an efficient way so that the social welfare is maximized by accounting for the environmental damages both in the current time horizon and in the forthcoming future. The resulting optimal control model is more complicated, and its well-posedness is more difficult to analyze. As a future purpose, we also aim to apply this approach as a predictive tool in order to investigate the management of depletable resources, such as timber materials and biofuels, in the framework of the sustainable transition.

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Received: May 11, 2023; Published: June 1, 2023