DOI: 10.61782/fa.2023.0144

forum acusticum 2023

ON THE SOUND TENSION AND ACTION-REACTION LAW IN ACOUSTICS

Domenico Stanzial[1*] **Carlos E. Graffigna**[2] **Isabella Masina**[3]

[1] CNR-IMM Via Gobetti 101, 40129 Bologna c/o Energy Acoustics Lab. DFST-University of Ferrara, Italy
[2] Dept. of Basic and Tech. Sciences, Nat. University of Chilecito, 9 de Julio 22, La Rioja, Argentina
[3] Physics and Earth Sciences Dept. - INFN, University of Ferrara, Via Saragat 1, 44122 Ferrara, Italy

ABSTRACT

After a brief introduction to the theory and physical interpretation of Lagrangian force density in general sound fields, this article focuses on its stationary average property providing the general expression of the action-reaction law for acoustic fields. This fundamental property allows to define the tension field of sound, which turns out to be easily measured as the gradient of the average potential energy density. The acoustic tension field has been then analytically calculated and visualized for quasi-stationary wave fields and divergent spherical waves. Moreover 2-D graphics comparing the behaviors of sound energy trajectories and tension fields are here reported.

Keywords: *Wave stress tensor, Acoustic tension, Inertial and elastic force densities, Visualization of model fields*

1. INTRODUCTION

As known from literature [1, p. 241-252], [2], the momentum density vector of any sound field

$$\mathbf{q}(\boldsymbol{x},t) := \frac{p\boldsymbol{v}}{c^2} \tag{1}$$

where p and $\boldsymbol{v}$ are the pressure and particle velocity solutions of the d'Alembert equation, is related to the wave-stress tensor [1] (i.e. the momentum density flux)

$$W := T^{ij}(\boldsymbol{x},t) = \\ = \begin{bmatrix} \rho_0 v_1 v_1 - L & \rho_0 v_1 v_2 & \rho_0 v_1 v_3 \\ \rho_0 v_2 v_1 & \rho_0 v_2 v_2 - L & \rho_0 v_2 v_3 \\ \rho_0 v_3 v_1 & \rho_0 v_3 v_2 & \rho_0 v_3 v_3 - L \end{bmatrix}$$

by the continuity equation

$$\frac{\partial \mathbf{q}}{\partial t} + \boldsymbol{\nabla} \cdot W = 0\,.$$

Provided that the vector $-\boldsymbol{\nabla} \cdot W$ is interpreted as a force density $\mathbf{f}(\boldsymbol{x},t)$, the above equation gives the quantitative expression of the acoustic momentum conservation law. This can be proved by calculating $\mathbf{f}(\boldsymbol{x},t)$ as the partial time derivative $\mathbf{q}_t$ of the momentum density, i.e.

$$-\boldsymbol{\nabla} \cdot W = \frac{\partial \mathbf{q}}{\partial t} = \frac{1}{c^2}\frac{\partial (p\boldsymbol{v})}{\partial t} = \frac{1}{c^2}\, p\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{c^2}\frac{\partial p}{\partial t}\boldsymbol{v} =: \mathbf{f}\,. \tag{2}$$

The above formal expression highlights that the force density is made up of two vector components: one $\mathbf{f}_{\boldsymbol{v}}(\boldsymbol{x},t)$ in the direction of the particle velocity $\boldsymbol{v}$ and the other one $\mathbf{f}_p(\boldsymbol{x},t)$ in that of its time derivative $\boldsymbol{v}_t$. Note also that both components of the wave-force field $\mathbf{f}(\boldsymbol{x},t) = \mathbf{f}_p + \mathbf{f}_{\boldsymbol{v}}$ are expressed at the second order of the concatenated acoustic fields p and $\boldsymbol{v}$. In equation (2), $\boldsymbol{v}_t$ is nothing but the usual inertial Euler's acoustic force per unit mass

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{\boldsymbol{\nabla} p}{\rho_0} \tag{3}$$

acting on the air particle, while p_t expresses the elastic force of the medium through the following form of

Corresponding author: *domenico.stanzial@unife.it.*

[1] $L = w_k - w_p$ is the Lagrangian density of sound (see (14) and [2, Eq. 35])

Hooke's law

$$p = -\rho_0 c^2 \boldsymbol{\nabla} \cdot \boldsymbol{r} \tag{4}$$

where $\boldsymbol{r}(\boldsymbol{x}, t) = \xi \mathbf{i} + \eta \mathbf{j} + \zeta \mathbf{k}$ is the displacement vector such that $\boldsymbol{r}_t = \boldsymbol{v}$ and $\rho_0 c^2 = \gamma P_0$ is the adiabatic bulk modulus. Since the divergence of the displacement

$$\boldsymbol{\nabla} \cdot \boldsymbol{r} = \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} = \frac{\Delta \mathscr{V}}{\mathscr{V}} \tag{5}$$

accounts for the dilatation of the air particle volume per unit volume, the explicit expression of p_t appearing in (2) is obtained from (4) as

$$\frac{\partial p}{\partial t} = -\rho_0 c^2 \boldsymbol{\nabla} \cdot \boldsymbol{v} \,. \tag{6}$$

Finally, by replacing $\boldsymbol{v}_t$ and p_t from (3) and (6) respectively, into equation (2) one gets the following second order expression of the Lagrangian force density of acoustic fields

$$\mathbf{f}(\boldsymbol{x}, t) := \frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{2\rho_0 c^2} \boldsymbol{\nabla} p^2 - \rho_0 (\boldsymbol{\nabla} \cdot \boldsymbol{v}) \boldsymbol{v} \tag{7}$$

in precise agreement with [2, Eq. 59].

1.1 Derivation of the static acoustic tension

A very interesting property of the force density that manifests itself when $\mathbf{f}(\boldsymbol{x}, t) = \mathbf{q}_t$ is averaged over time, is that it generates a static acoustic tension field whose local spatial direction accounts for the disomogeneities of pressure-velocity fields. This tension acts along spatial directions determined by the gradient of the average potential energy density of the sound field in such a way that the action-reaction principle is satisfied (see figure 7).

This property is a simple but not obvious at all, direct consequence of the time averaging process $\langle \cdot \rangle := \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} (\cdot)\; dt$ when applied to the force density.

In fact, since $\langle h_t \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} (h_t)\, dt = \lim_{T\to\infty} \frac{1}{2T} [h(T) - h(-T)] = 0$ for any continuous function h and $\langle \mathbf{f} \rangle = \left\langle \frac{\partial \mathbf{q}}{\partial t} \right\rangle$, one has

$$\langle \mathbf{f} \rangle = \left\langle \frac{\partial \mathbf{q}}{\partial t} \right\rangle = \begin{pmatrix} \left\langle \frac{\partial \mathrm{q}_x}{\partial t} \right\rangle = 0 \\ \left\langle \frac{\partial \mathrm{q}_y}{\partial t} \right\rangle = 0 \\ \left\langle \frac{\partial \mathrm{q}_z}{\partial t} \right\rangle = 0 \end{pmatrix} = \mathbf{0}$$

that is

$$\mathbf{T}(\boldsymbol{x}) := -\rho_0 \langle (\boldsymbol{\nabla} \cdot \boldsymbol{v}) \boldsymbol{v} \rangle - \frac{1}{2\rho_0 c^2} \langle \boldsymbol{\nabla} p^2 \rangle \equiv \mathbf{0} \,. \tag{8}$$

This clearly means that the acoustic tension $\mathbf{T}(\boldsymbol{x})$ is identically equal to a null vector all over any stationary sound field and thus that the two stationary components forces

$$\begin{aligned} \langle \mathbf{f}_p \rangle &:= -\frac{1}{2\rho_0 c^2} \langle \boldsymbol{\nabla} p^2 \rangle \\ \langle \mathbf{f}_{\boldsymbol{v}} \rangle &:= -\rho_0 \langle (\boldsymbol{\nabla} \cdot \boldsymbol{v}) \boldsymbol{v} \rangle \end{aligned} \tag{9}$$

expressing the elastic and the inertial forces of the medium at second order of the field variables p and $\boldsymbol{v}$, are always equal in magnitude and with opposite spatial directions so satisfying in the average the action-reaction law in acoustics.

2. CALCULATION OF SOUND TENSION IN MONOCHROMATIC MODEL FIELDS

We provide various examples, starting from the form of the velocity scalar potential $\varphi(\boldsymbol{x}, t)$ with dimensions $[\varphi] = \mathrm{L}^2\mathrm{T}^{-1}$. We recall that

$$\boldsymbol{v} = \nabla \varphi \;\;,\;\; p = -\rho_0 \frac{\partial \varphi}{\partial t} \;\;. \tag{10}$$

2.1 Progressive Plane Wave

The simplest possible example is a monochromatic wave propagating in the positive direction along the x axis

$$\varphi(x, t) = ac \cos(k(x - ct)) = ac \cos \alpha$$

where $c = \omega/k$ is the sound speed, a is the amplitude of the velocity potential and $\alpha = kx - \omega t$ is the phase. The velocity of the air particle is given by

$$\boldsymbol{v} = \nabla \varphi = -a\omega \sin \alpha \, \hat{\boldsymbol{x}}$$

and the sound pressure is

$$p = -\rho_0 \varphi_t = -\rho_0 c \, a\omega \sin \alpha \;\;.$$

The instantaneous momentum density is

$$\mathbf{q} = \frac{p\boldsymbol{v}}{c^2} = \rho_0 a^2 k\omega \sin^2 \alpha \, \hat{\boldsymbol{x}}$$

so that the force density and its stationary value are

$$\mathbf{f}(\boldsymbol{x}, t) = \mathbf{q}_t = -\rho_0 a^2 k\omega^2 \sin(2\alpha) \, \hat{\boldsymbol{x}} \,, \langle \mathbf{f} \rangle = \mathbf{0} \;\;.$$

The two components of $\mathbf{f}(\boldsymbol{x}, t)$ turn out to be equal at all times

$$\mathbf{f}_p(\boldsymbol{x}, t) = -\frac{1}{2\rho_0 c^2} \nabla p^2 = -\frac{1}{2} \rho_0 c \, a^2 \omega \sin(2\alpha) \, \hat{\boldsymbol{x}}$$

$$\mathbf{f}_{\boldsymbol{v}}(\boldsymbol{x}, t) = -\rho_0 (\nabla \cdot \boldsymbol{v}) \boldsymbol{v} = -\frac{1}{2} \rho_0 c \, a^2 \omega \sin(2\alpha) \, \hat{\boldsymbol{x}}$$

so that when performing the time average, they are all separately zero: $\langle \mathbf{f} \rangle = \langle \mathbf{f}_p \rangle = \langle \mathbf{f}_v \rangle = \mathbf{0}$.

2.2 Quasi-stationary Plane Wave

In this case we have two terms contributing to the velocity scalar potential

$$\varphi(x,t) = ac\left(\cos\alpha + R\,\cos\beta\right) \ ,$$

where $0 \le R \le 1$ is the reflection coefficient and we have introduced the shorthand notation $\alpha = kx - \omega t$ and $\beta = kx + \omega t + \theta$ accounting for the phases of incident and reflected waves. The velocity and pressure of the air particle are respectively given by

$$\boldsymbol{v} = -a\omega(\sin\alpha + R\,\sin\beta)\,\hat{\boldsymbol{x}}$$

and

$$p = -\rho_0 c\, a\omega(\sin\alpha - R\sin\beta) \ .$$

The momentum density is

$$\mathbf{q} = \rho_0\, a^2 k\omega(\sin^2\alpha - R^2\sin^2\beta)\,\hat{\boldsymbol{x}}$$

and the force density contributions

$$\mathbf{f}_{\boldsymbol{v}}(x,t) = -\rho_0 a^2 k\omega^2\left(\cos\alpha + R\cos\beta\right)\left(\sin\alpha + R\sin\beta\right)\hat{\boldsymbol{x}}$$

and

$$\mathbf{f}_p(x,t) = -\rho_0 a^2 k\omega^2\left(\cos\alpha - R\cos\beta\right)\left(\sin\alpha - R\sin\beta\right)\hat{\boldsymbol{x}} \ .$$

are not equal at any time but, as expected, upon time average they are opposite:

$$\langle\mathbf{f}_{\boldsymbol{v}}\rangle = -\rho_0 a^2 k\omega^2\, R\sin(2kx + \theta)\,\hat{\boldsymbol{x}} = -\langle\mathbf{f}_p\rangle \ . \quad (11)$$

2.3 Outgoing Spherical Wave

In this case the sound field has a spherical symmetry and, by taking into account the boundary conditions of the air particle velocity at the surface of a small spherical source centered at the origin, the velocity scalar potential can be written as

$$\varphi(r,t) = \frac{d_0^2 b\omega}{(1 + ikd_0)\,r}\, e^{i(\omega t - k(r - d_0))} \ .$$

Here $kd_0 \ll 1$ where d_0 is the radius of the pulsing sphere at rest and $b \ll d_0$ is the displacement amplitude of its shell in harmonic motion. The velocity of the air particle at the shell surface is supposed to be

$$v(d_0) = b\omega e^{i\omega t} \ . \quad (12)$$

When the same calculations as in previous cases are performed over the field variables

$$p(r,t) = \Re\left(-\rho_0\varphi_t\right) = \Re\left(\frac{i\left(\rho_0 d_0^2 b\omega^2\right)}{r\left(1 + ikd_0\right)}\cdot e^{i(\omega t - k(r - d_0))}\right)$$

$$v(r,t) = \Re\left(\varphi_r\right) = \Re\left(\frac{d_0^2 b\omega}{r^2}\cdot\frac{1 + ikr}{1 + ikd_0}\cdot e^{i(\omega t - k(r - d_0))}\right)$$

one gets from equation (9)

$$\langle\mathbf{f}_p\rangle = \frac{1}{2}\frac{\rho_0 d_0^4 b^2 k^2\omega^2}{\left(d_0^2 k^2 + 1\right)r^3} = -\langle\mathbf{f}_{\boldsymbol{v}}\rangle \quad (13)$$

so validating in the average the action reaction law for second order inertial and elastic force densities.

2.4 Three dimensional quasi-stationary waves

A full 3-D visualization of the mechanism of action-reaction law in sound fields can be achieved by making quasi stationary waves to interfere along the three Cartesian coordinate axes $\{x, y, z\}$. Using the same notation as introduced in subsection 2.2, a general form of a 3-D velocity scalar potential can be written as

$$\varphi(x,y,z,t) = c\sum_{i=x,y,z} a_i\left(\cos\alpha_i + R_i\,\cos\beta_i\right)$$

so finding the following vector components of $\mathbf{f}_{\boldsymbol{v}}(\boldsymbol{x},t)$ and $\mathbf{f}_p(\boldsymbol{x},t)$ along the ortonormal Euclidean basis $\boldsymbol{e}_{i=x,y,z}$:

$$\left(\mathbf{f}_{\boldsymbol{v}}\right)_i = \rho_0 a_i k\left(\sin\alpha_i + R_i\sin\beta_i\right)\sum_{i=x,y,z} a_i\omega^2\left(\cos\alpha_i + R_i\cos\beta_i\right)$$

and

$$\left(\mathbf{f}_p\right)_i = -a_i k\omega\left(\cos\alpha_i - R_i\cos\beta_i\right)\sum_{i=x,y,z}\rho_0 a_i\omega\left(\sin\alpha_i - R_i\sin\beta_i\right) \ .$$

As already mentioned and expicited in the 1-D case (see (11)) it turns out that also in multidimensional cases

$$\langle\mathbf{f}_{\boldsymbol{v}}(\boldsymbol{x},t)\rangle = -\langle\mathbf{f}_p(\boldsymbol{x},t)\rangle \ ,$$

so confirming that

$$\mathbf{T}(\boldsymbol{x}) = \langle\mathbf{f}_{\boldsymbol{v}}(\boldsymbol{x},t) + \mathbf{f}_p(\boldsymbol{x},t)\rangle \equiv \mathbf{0}$$

as expected from (8).

3. VISUALIZATION OF TENSION FIELDS

With the aim of making the mechanism of opposing action of inertial and elastic forces more visible, some graphical renderings of the previous analyzed case studies will be given below.

3.1 Diverging spherical wave

Let's begin from the physically very interesting case of a progressive 3-D wave with spherical symmetry. Since in this case the sound source is located at a fixed point in space with a finite distance r from the observer, the effect of the source on the radiation is not an option and the interaction source-field must be specified with an appropriate boundary condition (see (12)). The snapshots of pressure and velocity waves fields at time intervals of $T/4$ are compared in Fig.1 with the behavior of inertial and elastic force densities at the same instant. Notice that the phase difference $\Delta\vartheta$ between the pressure and velocity waves nearly vanish in the spatial interval of a single wavelength $\lambda = 0.334\,\mathrm{m}$ while the inertial and elastic force densities act instantaneously, and react strongly against each other in the first $\lambda/2$ where a high reactive field is present, but always satisfy Eq. (13) in the average.

3.2 Quasi-stationary plane Waves

In this model field the acoustic source causing the air perturbation is located at the infinite and the boundary condition of the reflected wave is given over planes intersecting orthogonally each coordinate axes of the incident wave, as detailed respectively in subsections 2.2 and 2.4 for 1-D and 3-D waves.

3.2.1 One dimensional case

The 1-D case is illustrated in Fig. 2 for $R = 1/2$ and in Fig.4 for $R = 1$ (pure stationary waves). Differently from Fig.1 representing a standard film of many snapshots, here the film frames show a time windows T of the p and v wave fields and force densities, as measured by an array of sensors spaced $\lambda/6$ apart. Indeed, notice that in Fig. 2, the brown straigth line representing the pressure-velocity phase difference $\Delta\vartheta\,(x) = \arccos\left(\langle pv\rangle \middle/ \sqrt{\langle p^2\rangle\,\langle v^2\rangle}\right)$, is given by a brown straigth line, clearly constant in time, but appearing at different heights (in arbitrary units) during the waves motion. This means that in the case of a quasi-stationary field, the waves combine locally more similar to a progressive field or are more like a stationary one depending on the value of $0 \le \Delta\vartheta \le \Delta\vartheta_{max} \le \pi/2$. For example, in Fig. 3 where $R = 0.5$, energy reaches a maximum of coherence every $T/4$ s when $\Delta\vartheta = 0$ and drifts till to a phase difference $\Delta\vartheta_{max} = 0.927\ \mathrm{rad}$ while always travelling at a constant average speed:

$$U := \frac{\langle pv\rangle}{\langle w\rangle} = c\sqrt{\frac{1-R^2}{1+R^2}} = 0.6\,c$$

where U (see [3]) is the energy velocity modulus defined in terms of the sound intensity pv, and the energy density

$$w = w_p + w_k := \frac{1}{2}\frac{p^2}{\rho_0 c^2} + \frac{1}{2}\rho_0 v^2. \tag{14}$$

In the extreme case of a pure stationary wave ($R = 1$, $\Delta\vartheta\,(x) = \pi/2$), energy comes in the average to a perfect stop while oscillating instantaneously within cells $\lambda/4$ wide (actually a mode of vibration of the air as pictured in Fig. 4). In the other extreme case of pure progressive field ($R = 0$, $\Delta\vartheta\,(x) = 0$) the energy transport is operated at the maximum rate equal to c, by pressure and velocity waves of equal amplitude at frequency $\omega/2\pi$ and equal sinusoidal inertial and elastic forces at a double frequency, all always perfectly in phase.

3.2.2 Three dimensional case

The graphical rendering of this case, fully modelized in 2.4, is however visualized here in a 2-D reduced dimension as reported, for clarity, in Figs 5 and 7 with detailed captions. It is evident from Fig. 5 that, in agreement with equation (8), the inertial and elastic tension components act in the average in opposite directions so defining a common local line of action for a static tension field. This local tension line is compared with the direction of energy velocity at the same location. Notice that no general relationship between the two directions can be observed. Only in the case of the field visualized on the left of Fig. 5, where $a_x = a_y, a_z = 0$ and $R_x = R_y = 0$, the tension direction is always orthogonal to the energy velocity one, and moreover, the acoustic tension appears to vanish when the energy velocity reaches its maximum rate c. This behavior is confirmed in Fig. 6 where the corresponding moduli are shown.

4. CONCLUSIONS

The elastic and inertial force densities of general sound fields, respectively $\langle \mathbf{f}_p\rangle$ and $\langle \mathbf{f}_v\rangle$ in (9), have been derived on the basis of Lagrangian theory of acoustics. These second order forces have been physically identified as the two

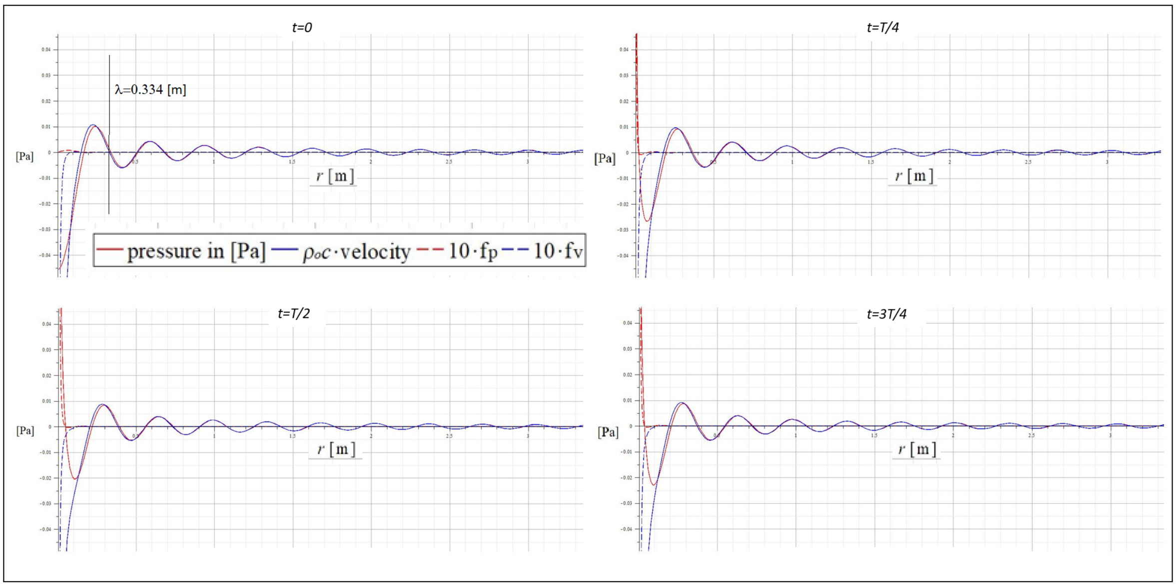

Figure 1. Snapshots of pressure and velocity wave profiles of length $10\,\lambda$, compared with elastic and inertial force densities of the air, along a ray starting from the harmonically vibrating surface of a spherical source with $d_0 = 10^{-2}\mathrm{m}$ and $b = 10^{-7}\mathrm{m}$.

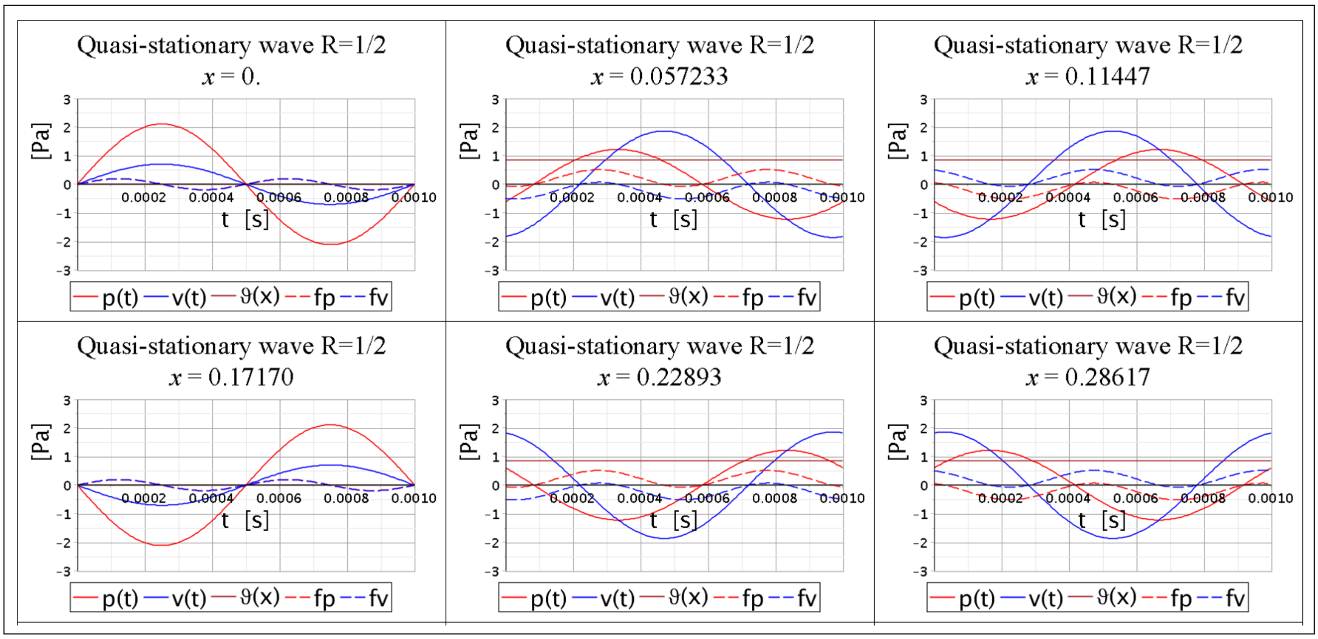

Figure 2. Pressure and $\rho_0 c$·velocity signals of a 1-D quasi-stationary wave captured during a period $T = 10^{-3}\,\mathrm{s}$ at different locations $\lambda/6$ apart, along the distance of 1 wavelegth $\lambda = 0.3434$. The underlying behaviors of elastic and inertial forces are represented by dashed lines with a 2×10^3 gain factor, while the distance of the brown straight line from the x-axis is proportional to the phase difference between $p\,(t)$ and $v\,(t)$.

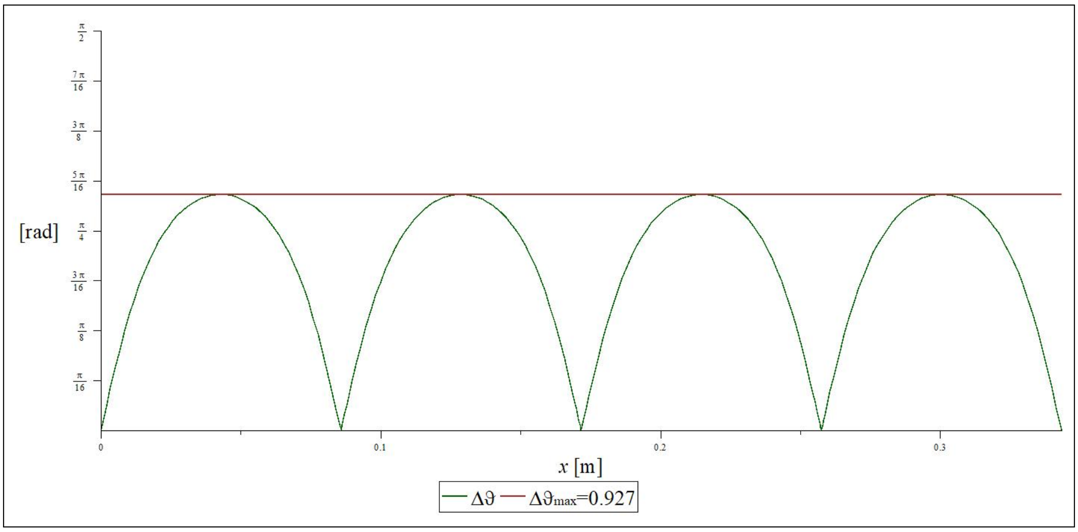

Figure 3. Phase difference $\Delta\vartheta\left(x\right)$ between pressure and velocity waves along the propagation distance of 1 wavelength $\lambda = 0.3434$ of a quasi-stationary 1-D field with reflection coefficient $R = 1/2$. Differently from $\Delta\vartheta\left(x\right)$ the energy average velocity $U\left(x\right) = 0.6 \cdot c$ $^{\mathrm{m}}/_{\mathrm{s}}$ is constant all along its trajectory (the x axis) and depends only on the boundary condition $R = 0.5$.

vector components of the always vanishing tension field of sound, expressed by their vector sum. This fundamental property of sound tension gives a quantitative formulation of the action-reaction law in acoustics. The instantaneous action mechanism of the two forces $\mathbf{f}_p$ and $\mathbf{f}_v$ has been analyzed and graphically rendered for some canonical fields modeled in one, two and three spatial dimesions. The presented case studies highlight, in particular, that the action-reaction principle holds in acoustics only in the average, but is satisfied even instantaneously uniquely in the case of 1-D pure progressive plane wave fields (see subsection 2.1) or along a ray of an outgoing spherical wave many wavelengths far from the source. Notice in figure 6 that the spatial interference of 1-D pure progressive waves produces localized reactivity fringes alternating with purely active fringes which therefore highlight the existence of trajectories where the energy flows with purely inertial free motion.

Interesting applications to room acoustics based on a theoretical scheme similar to the one given here have been recently put under experimental investigation [[4], [5]] while future works by the present authors will deepen the field-source energetic interaction to complete the field-field analysis presented in this paper.

.

ACKNOWLEDGMENTS

The authors would like to thank CNR-IMM Bologna Unit for administrative and financial support.

5. REFERENCES

[1] P. M. Morse and K. U. Ingard, *Theoretical acoustics*. Princeton university press, 1968.

[2] D. Stanzial, D. Bonsi, and G. Schiffrer, "Four-dimensional treatment of linear acoustic fields and radiation pressure," *Acta Acustica united with Acustica*, vol. 89, no. 2, pp. 213–224, 2002.

[3] D. Stanzial, G. Sacchi, and G. Schiffrer, "On the physical meaning of the power factor in acoustics," *J. Acoust. Soc. Am.*, vol. 131, no. 1, pp. 269–280, 2012.

[4] H. Dujourdy, B. Pialot, T. Toulemonde, and J.-D. Polack, "An energetic wave equation for modelling diffuse sound fields application to corridors," *Acta Acustica united with Acustica*, vol. 103, no. 3, pp. 480–491, 2017.

[5] D. Stanzial and G. Schiffrer, "On the connection between energy velocity, reverberation time and angular momentum," *Journal of Sound and Vibration*, vol. 329, no. 7, pp. 931–943, 2010.

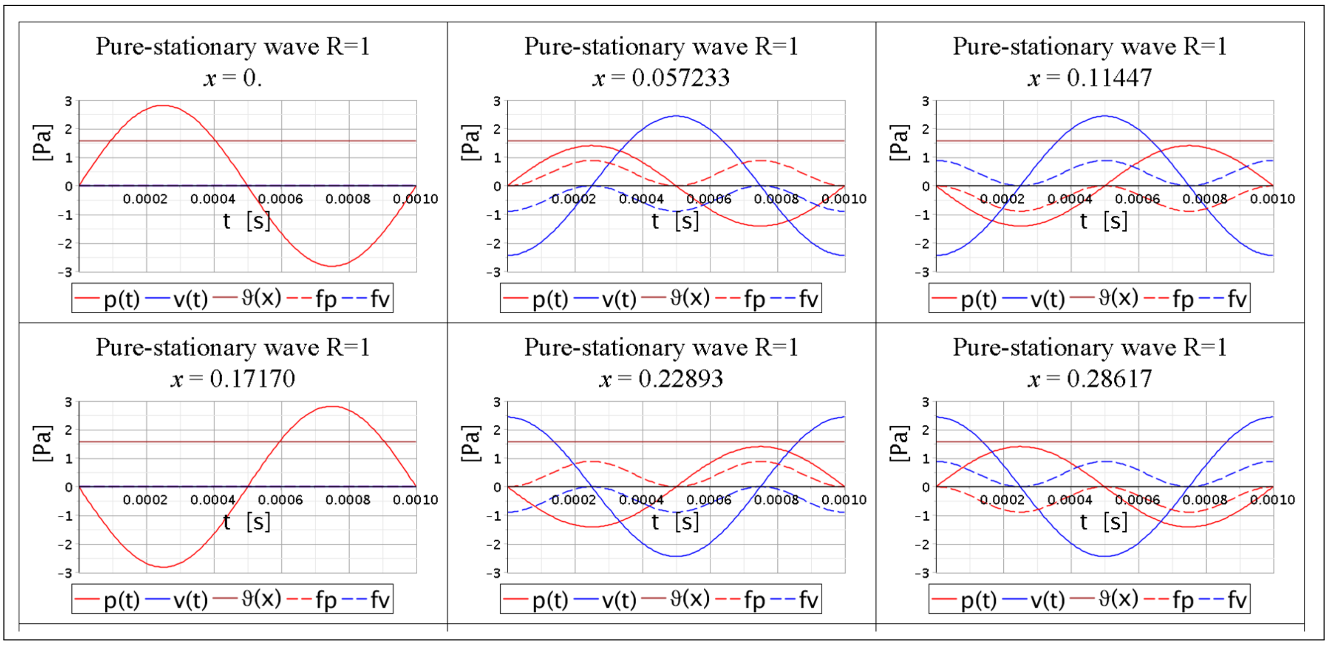

Figure 4. Pressure and $\rho_0 c$·velocity signals of a perfect stationary wave ($R = 1$) visualized as described in Fig. 2. Notice that differently from the quasi-stationary wave case here the phase between pressure and velocity waves is a constant both in time and space equal to $\pi/2$.

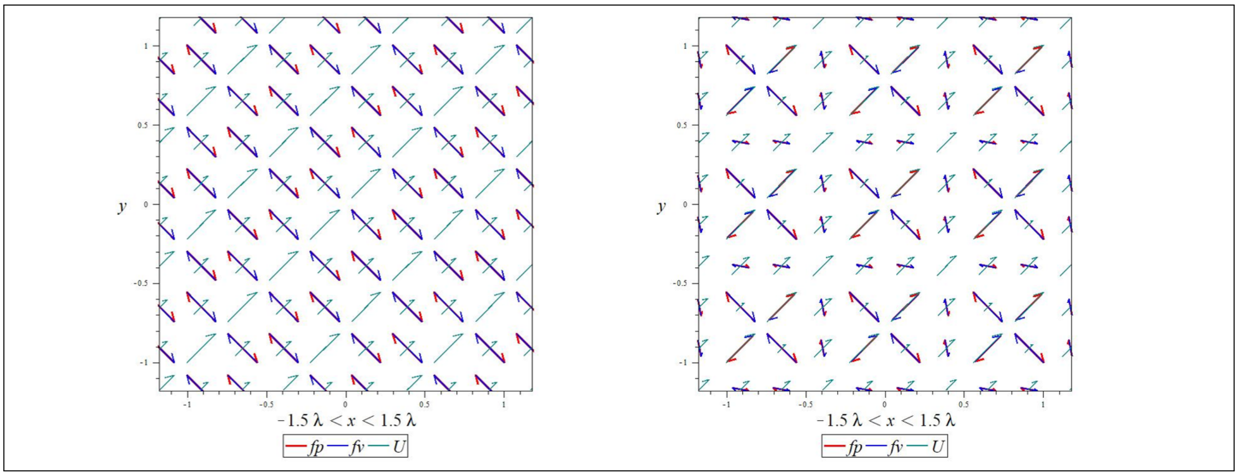

Figure 5. Two dimensional representation reduced from the 3D model of 2.4 of the acoustic tension components $\langle f_p \rangle$ and $\langle f_v \rangle$ for $a_x = a_y, a_z = 0$ and $R_x = R_y = 0$ on the left, and $R_x = R_y = 0.5$ on the right. The tension field is superposed to the energy velocity field $\boldsymbol{U}(x, y, 0)$ so to making clear that no apparently remarkable relationship exists between the tension and energy velocity fields directions.

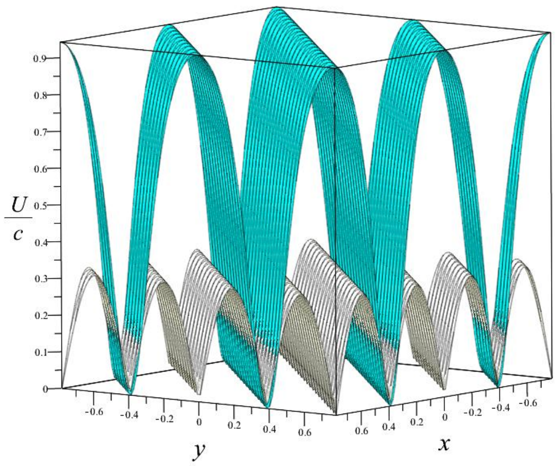

Figure 6. Comparison between the magnitudes of the c-normalized energy velocity field (in cyan) and of tension components (in arbitrary units) corresponding to the vector fields at the left side of figure 5.

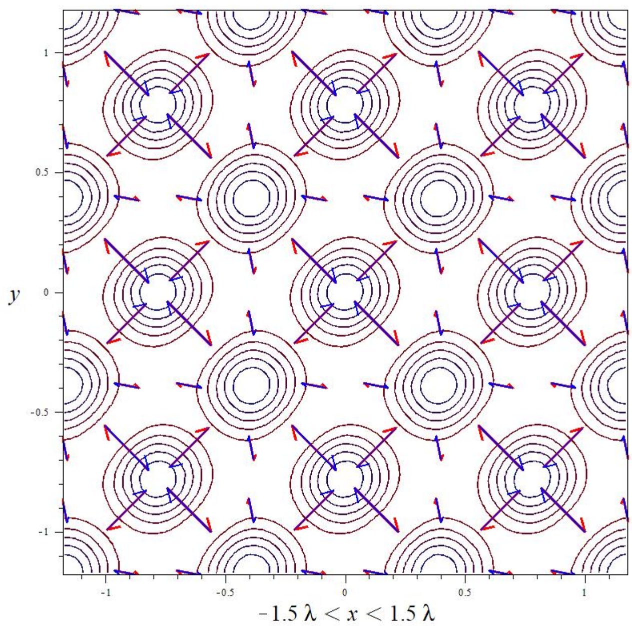

Figure 7. Directions of acoustic tension static field compared with the isolevels contour plot of the potential energy density of the quasi-stationary field pictured at the right of figure 5.