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Abstract

Within the standard framework of mixed oligopoly theory, in this paper we investigate how changes in the distribution of income affect demand and the incentives towards privatization. We show that the scope for privatization is widened when the market is poorer, and when incomes become more concentrated. These results are accounted for in terms of the way distributional shocks alter the allocative inefficiency of imperfectly competitive markets.

Keywords. Mixed oligopoly, income distribution, privatization.
JEL Codes: L13, L32, H44

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1 Introduction

The core question raised by this paper is whether and how the key features of market demand should be relevant in assessing privatization policies. We offer some insights on this issue in the perspective of the mixed oligopoly theory, which can be considered as the standard theoretical framework for the analysis of the strategic behaviour of the public sector in non-competitive markets.[1]

Indeed, the mixed oligopoly models study the role of public firms in the partial equilibrium framework of oligopoly theory. Accordingly, in these models the demand side of the market is modelled in the absence of any income effect. The main positive implication of this approach is that it allows a simple money-metric definition of the social welfare (the objective function of the public firms) as the sum of the profits and the consumers' surplus – the latter being indeed an ideal measure of consumers' welfare if income effects are ruled out. However, this analytical neatness comes at a cost: that of dismissing the role of the demand factors in such key matters as the evaluation of the welfare gains obtained by the government through the public firm's activity, and the desirability of privatizations.[2] Moreover, the markets in which public firms are typically active – health, education, transports, energy provision, just to quote some – can hardly be thought of as free from income effects.

In order to allow for demand and income effects in the analysis of mixed markets, we revisit some early contribution in the mixed oligopoly theory, by reformulating the demand side of the market as the outcome of the binary choice of a population of consumers heterogeneous with respect to income. The assumption of unit demand and binary choice, though applicable to a limited set of markets, has the advantage of establishing an immediate link between the shape of consumers' heterogeneity and that of market demand; moreover, it preserves the possibility to quantify the consumers' welfare through the consumers' surplus.

Given this general specification of the demand side, we then investigate the way in which changes in the distribution of the reservation prices – which

[1]For a comprehensive survey on the literature on mixed oligopoly and privatization, see Poyago-Theotoki (2023).

[2]The only demand factor which plays a role in the literature on privatizations is the size of the population, which has been shown to be relevant in international mixed oligopoly model; see Dadpday and Heywood (2006) and Matsumura and Matsushima (2012).

we interpret as ultimately related to changes in the distribution of incomes – affect the range of situations in which privatization turns out to be desirable.

In order to focus on the basic mechanisms of the mixed oligopoly models, we consider a simple Cournotian market with homogeneous product, where R&D, externalities, or international trade play no role. Our references are the seminal papers by De Fraja and Delbono (1989) and De Fraja (1991). In the former the firms produce under convex costs and the scope for privatization is defined in terms of number of private firms active in the market, a fully private market being more efficient than a mixed market if the number of firms is sufficiently high; in the latter, the firms' technologies are assumed to be linear and the scope for privatization is defined in terms of the public firm's average cost, which is a priori assumed to be higher than that of the private competitors.

We introduce in these models two distributional shocks, which can be thought of as very simple stylized examples of first and second order stochastic dominance, and we obtain the rather counter-intuitive result that the range of situations in which privatization is welfare enhancing widens both as the consumers become poorer, and as the consumers' incomes become more concentrated. We explain our results in terms of the size and elasticity effects of the changes in market demand, prompted by the associated changes in the distribution of income. In this way we try and extend to the mixed market case the idea that demand plays a relevant role in shaping competitiveness and market structure.[3]

The paper is organized as follows. The basic structure of the model is discussed in section 2. The effects of the distributional shocks are analysed in section 3, where we first concentrate on a generalized income increase, and then on a reduction of the variance of the incomes and reservation prices. Some final remarks and conclusions are gathered in section 4.

[3]On this point see Benassi C., A.Chirco and M.Scrimitore (2002). In the context of monopolistic competition this general idea is to be found, e.g., in the works by Osharin et al. (2014) and Bertoletti and Etro (2017).

2 A general Cournot model with consumers' binary choice

We consider a Cournotian market for a homogeneous product, where $n+1$ firms compete strategically. In this market two different ownership configurations are possible. The first is associated with a mixed oligopoly structure, in which one of the firms (indexed with 0) is publicly-owned and competes with n private firms. The latter are identical, their cost function being $C(q_i)$, with $C'(q_i) > 0$ and $C''(q_i) \geqslant 0$ for $i = 1, ..., n$, while the public firm's technology implies the cost function $C_0(q_0)$, with $C_0'(q_0) > 0$ e $C_0''(q_0) \geqslant 0$.[4] The second configuration is that of a fully privatized market, with $n+1$ identical private firms characterized by the $C(q_i)$ function, $i = 0, ..., n$, described above.

On the demand side of this market each consumer is endowed with a utility function of the following type:

$$\begin{aligned} U &= y - p \quad \text{if she purchases a unit the good} \\ U &= 0 \qquad\;\; \text{if she does not purchase} \end{aligned}$$

where p is the price of the good and y is the consumer's reservation price. Hence, the consumer enters the market and buys one unit of the good whenever her reservation price is higher than the market price. Consumers differ across their reservation prices – a heterogeneity which reflects the differences in their purchasing power, i.e. the personal distribution of incomes. We assume that y is defined over the interval $[0, \overline{y}]$, and distributed according to a continuous differentiable density function $f(y)$, so that utility maximization generates the following market demand function:

$$Q(p) = 1 - F(p)$$

where $F(p) = \int_0^p f(y)\, dy$ and the population of consumers has been normalized to 1.

Under our binary choice hypothesis, the direct market demand $Q(p)$ exhibits a straightforward link with the distribution of the reservation prices and, ultimately, of incomes. This makes it convenient to describe the Cournotian interaction between firms along the lines suggested by Kreps (1990, ch 10), i.e. by assuming that firms compete indeed with respect to prices, but under

[4] Notice that in the sequel, we shall consider both cases in which $C_0(\bullet)$ and $C(\bullet)$ are different, and cases in which they coincide.

the Cournot conjecture that each firm reacts to the rivals' move in such a way as to keep its sold quantity fixed.[5] Given this very simple description of the demand and supply sides of the market, we proceed by sketching the Cournot-Nash equilibrium solutions both in the mixed oligopoly, and in the fully private case.

The mixed oligopoly. If the market exhibits a mixed oligopoly structure, a welfare maximizing public firm interacts with n profit maximizing private firms. Given the market demand and costs functions described above, the social welfare (the sum of consumers' surplus and profits) can be written as

$$W(p) = \int_p^{\overline{y}} Q(z)\,dz + pQ(p) - C_0\left(Q(p) - \sum_{i=1}^{n} q_i\right) - \sum_{i=1}^{n} C\left(Q(p) - q_0 - \sum_{j\neq i} q_j\right) \tag{1}$$

where the first term is the net consumers' surplus, the second gives aggregate revenues, and the other two terms are the total costs of the public and private firms, respectively.

The objective function of the generic *i-th* private firm is the following profit function:

$$\pi_i(p) = p\left(Q(p) - q_0 - \sum_{j\neq i} q_j\right) - C\left(Q(p) - q_0 - \sum_{j\neq i} q_j\right) \tag{2}$$

The way in which the welfare and profit function (1) and (2) have been written highlights the key property of strategic price-setting under the Cournot conjectures: through its objective function maximization, each firm determines its desired market price, by assuming that the quantity produced by its rivals is kept constant, i.e. by assuming that price changes affect only its own quantity, and that the changes in the latter coincide with the changes in aggregate demand.

For given q_i, $i = 1, ..., n$, maximization of (1) with respect to p yields

$$p = C_0'(q_0), \tag{3}$$

[5] Within a standard private Cournot market, this solution is adopted, e.g., in Benassi C., R.Cellini and A.Chirco (2002). For a indepth analysis of the theoretical underpinning of this procedure see also Delbono and Lambertini (2018).

the standard interpretation of which is that for given quantities of the private firms, the public firm produces that quantity q_0 such that the market price equates its marginal cost, $C_0^{'}(q_0)$. Consider now the generic *i-th* private firm. For given q_0 and given q_j, $j \neq i$, profit maximization with respect to p yields

$$\left(Q(p) - q_0 - \sum_{j \neq i} q_j\right) + p\frac{dQ}{dp} - C^{'}\left(Q(p) - q_0 - \sum_{j \neq i} q_j\right)\frac{dQ}{dp} = 0$$

Recalling that $dQ/dp = -f(p)$, the above can be written as

$$q_i - \left(p - C^{'}(q_i)\right) f(p) = 0$$

Since all private firms are identical, $q_i = q$ for all i and this boils down to

$$q = \left(p - C^{'}(q)\right) f(p) \tag{4}$$

Once the shape of the density $f(\bullet)$ and the properties of the cost functions are specified, equations (3) and (4) along with $Q(p) = q_0 + nq$ explicitly determine the equilibrium values of q_0, q and p.

The fully private market. If all the $n+1$ firms are private – a situation which we interpret as the outcome of a policy of privatization of the public firm – the welfare function is

$$W(p) = \int_p^{\overline{y}} Q(z)\,dz + pQ(p) - \sum_{i=0}^{n} C\left(Q(p) - q_0 - \sum_{j \neq i} q_j\right) \tag{1'}$$

The first order condition for profit maximization evaluated under symmetry $\left(q_i = q = \frac{Q(p)}{n+1} \text{ for } i = 0, ..., n\right)$ implies

$$\frac{Q(p)}{n+1} = \left(p - C^{'}\left(\frac{Q(p)}{n+1}\right)\right) f(p) \tag{5}$$

Once the shape of the density $f(\bullet)$ and the properties of the cost function are specified, equation (5) along with $q = Q(p)/(n+1)$ explicitly determine the equilibrium values of q and p.

3 The incentive to privatization and the distribution of income

As highlighted in the early literature on mixed oligopoly and privatization, in the above basic setup the only rationale for a welfare enhancing privatization is the existence of a cost inefficiency on the public firm side.[6] This can be traced back either to differences in technology, as in De Fraja (1991), henceforth DF, or to the firms' different strategic behaviour, as in De Fraja and Delbono (1989), henceforth DFD.

In particular, the DF assumptions amount to setting the following cost functions for the public and private firms, respectively:

$$C_0(q_0) = K + c_0 q_0 \tag{6a}$$
$$C(q_i) = K + c q_i \tag{6b}$$

with $c_0 > c$. An implication of this exogenous cost differential is that there exists a threshold value of the relative inefficiency of the public firm $(c_0 - c)$, such that privatization turns out to be welfare enhancing beyond that value.

Alternatively, DFD assume a common convex (quadratic) cost function for both types of firms:

$$C(q_i) = \frac{k}{2} q_i^2 \qquad i = 0, ...m \qquad k > 0 \tag{6c}$$

In this case, it is the higher production level implied by welfare maximization that generates higher marginal and average costs for the public firm. The higher overall production observed in a mixed market is therefore associated to an inefficiently unequal distribution of costs among firms. Privatization of the public firm turns out to be beneficial, if this inefficiency outweighs the beneficial effects of expanding output – which actually occurs when the number of the rival private firms n is sufficiently high.

Our modeling the demand side of the market as strictly connected to the distribution of income, allows us to establish a link between the properties of the latter and the range of situations in which privatization is desirable. In particular, in what follows we study the way in which changes in the

[6] A discussion on this point and an alternative approach are offered in Barcena Ruiz (2012). In that model the public and private firms are assumed to be equally efficient, and privatization may occur as the outcome of a redefinition of the public firm's objective function in terms of generalized social welfare.

income distribution affect the threshold value of the cost inefficiency $(c_0 - c)$ in the DF model, and the threshold value of n in the DFD model. In order to keep tractability, we proceed with simple examples, by comparing the solutions of both models for different basic distributions of the reservation prices, which can be ranked according to a first or second order stochastic dominance criterion. To start with, in subsection 3.1 we examine the effects of a generalized income increase, formalized in terms of an increase of the upper bound $\overline{y}$ of the support of a uniform distribution. In subsection 3.2 we study the effects of income concentration, through the comparison of the solutions under a uniform and a quadratic Beta distribution. Through these analyses we try to offer a first insight on the general issue of the role and desirability of public ownership under different distributional patterns.

3.1 Generalized income increase and the incentive to privatization

We start by assuming that the distribution $f(y)$ of the reservation prices is uniform over the support $[0, \overline{y}]$. Therefore, we have

$$f(p) = \frac{1}{\overline{y}},\ F(p) = \int_0^p \frac{1}{\overline{y}} dx = \frac{p}{\overline{y}},$$

such that $\overline{y}$ is a parameter of first order stochastic dominance. The resulting market demand function is

$$Q(p) = 1 - \frac{p}{\overline{y}} \tag{7}$$

The DF model. Assuming $K = 0$ in (6a) and (6b) and normalizing $c = 0$ in (6b), the inefficiency of the public firm is fully captured by c_0. In the mixed market, the reaction functions (3) and (4) become

$$p_{DF}^M = c_0 \tag{3a}$$

$$q_{DF}^M = \frac{p}{\overline{y}} = \frac{c_0}{\overline{y}} \tag{4a}$$

where the the apex M denotes the mixed market. Using the assumptions (6a), (6b) and (7) in (1), and substituting (3a) and (4a), we obtain the equilibrium value of welfare in the mixed market:

$$W_{DF}^M = \frac{(\overline{y} - c_0)^2 + 2c_0^2 n}{\overline{y}} \tag{8}$$

If the market is fully private, denoted with the apex P, the first order condition (5) implies

$$p_{DF}^{P} = \frac{\overline{y}}{n+2} \tag{5a}$$

Under (6a), (6b) and (7), substituting (5a) into (1') yields the equilibrium value of welfare in the fully privatized case:

$$W_{DF}^{P} = \frac{\overline{y}}{2} \frac{(n+4)\,n+3}{(n+2)^2} \tag{9}$$

Our threshold value of c_0 can of course be recovered by equating (8) and (9):[7]

$$c_0^* = \frac{\overline{y}}{2n+1}\left((n+2)^2 - \sqrt{n^4+8n^3+24n^2+30n+15}\right) \tag{10}$$

Equation (10) shows that c_0^* is decreasing in n and linearly increasing in $\overline{y}$. Though analytically trivial, this result conveys a noteworthy message: the richer the market, the wider the scope for public ownership and the narrower that for privatization. As consumers become richer, market demand increases for all prices and the market welfare potential enlarges. In a mixed market the price remains unchanged; by contrast, in a fully privatized market the price increases whenever (as in this example) higher demand is associated with a lower demand elasticity. Hence, our first order stochastic dominance shock on incomes magnifies the regulatory impact of the public firm, with the related increase of the threshold value of c_0.

The DFD model. If all firms, private and public, share the same cost function (6c), then the reaction functions (3) and (4) become

$$p = k\left(1 - \frac{p}{\overline{y}} - nq\right) \tag{3b}$$

$$q = (p - kq)\frac{1}{\overline{y}} \tag{4b}$$

[7] We select the only solution which guarantees that the quantity produced by the public firm is positive.

which yield

$$p^M_{DFD} = \frac{(\overline{y}+k)\,k\overline{y}}{(\overline{y}+k)^2 + kn\overline{y}} \tag{11}$$

$$q^M_{DFD} = \frac{k\overline{y}}{(\overline{y}+k)^2 + kn\overline{y}} \tag{12}$$

Using (6c) and (7) into (1), and substituting (11) and (12) we get the equilibrium value of welfare in the mixed market

$$W^M_{DFD} = \frac{\overline{y}^2}{2}\frac{(1+n)\,k^3 + (3+4n+n^2)\,\overline{y}k^2 + (3+2n)\,\overline{y}^2k + \overline{y}^3}{\left((\overline{y}+k)^2 + kn\overline{y}\right)^2} \tag{13}$$

If the market is fully private, equation (5) becomes

$$\frac{1}{n+1}\left(1-\frac{p}{\overline{y}}\right) = \left(p - k\left(\frac{1}{n+1}\left(1-\frac{p}{\overline{y}}\right)\right)\right)\frac{1}{\overline{y}} \tag{5b}$$

solving which gives

$$p^P_{DFD} = \frac{(\overline{y}+k)\,\overline{y}}{\overline{y}\,(2+n)+k} \tag{14}$$

Substituting (14) into (1') under assumptions (6c) and (7) gives the value of welfare in the fully private market

$$W^P_{DFD} = \frac{\overline{y}^2}{2}\,(n+1)\,\frac{\overline{y}\,(3+n)+k}{(\overline{y}\,(2+n)+k)^2} \tag{15}$$

The threshold value of n above which privatization is welfare enhancing is obtained by equating (13) and (15), and is given by

$$n^* = \frac{\sqrt{4\overline{y}^3k + 13\overline{y}^2k^2 + 12\overline{y}k^3 + 4k^4} - k\overline{y}}{2k\overline{y}}$$

which is actually increasing in $\overline{y}$, so long as $k < \overline{y}/2$.

The intuition for this result is again based on the interplay between the demand and cost incentives to privatization. Indeed, in this model a generalized increase in income has a twofold effect on the desirability of privatization. On the one hand, similarly to the DF case, with higher demand and lower demand elasticity the positive impact of the public firm, in terms of exploiting the higher welfare potential and lowering the market price, is magnified. On the other hand, the shape of the cost function is such that the increase in $\overline{y}$ brings about an increase in the imbalance in the distribution of costs, which in principle strengthens the case for a fully private market. For low values of k (flat cost curves), the first effect prevails and the threshold value of n increases: the positive demand shock enlarges the range of cases in which the fully private configuration is dominated by the mixed one. As k increases, the first effect weakens, due to the equilibrium prices (11) and (14) getting closer, while the second effect is reinforced, as the cost differential increases[8] – yielding in the end a reversal of the overall effect of $\overline{y}$ on n^*, for $k > \overline{y}/2$.

To sum up, both models show that it is more likely for a market to benefit from the privatization of a public firm, if the market itself is 'poor'. Indeed, if the so-called Robinson effect is at work, i.e. if demand and demand elasticity are negatively related,[9] the lower is the consumers' willingness to pay, the closer is the fully private solution to allocative efficiency – which makes more desirable to do away with the cost inefficiency directly or indirectly associated to public ownership.

We now turn our attention to a different distributive shock, namely an increase in income concentration.

3.2 Income concentration and the incentive to privatization

In order to study the effect of changes in the concentration of income across consumers, we compare the incentives to privatization under a uniform distribution and a symmetric quadratic Beta density function. The relation between the two distributions is of second order stochastic dominance, the uniform distribution being a mean preserving spread of the symmetric Beta. Both distributions are defined over the support $[0, 1]$.

[8] Indeed, it can be proved that the average cost differential is increasing in k for $k < \overline{y}$, and decreasing beyond that value.

[9] By relying on an analogy with the individual behaviour, Joan Robinson (1969, p.70) argues that '....an increase in demand due to an increase of wealth is likely to reduce the elasticity of the demand curve, and may reduce the elasticity so much that the slope of the curve is increased.' On this point see also Benassi and Chirco (2004).

Given the symmetric quadratic Beta distribution:[10]

$$f(p) = 6p(1-p), \; F(p) = \int_0^p 6x(1-x)dx = 3p^2 - 2p^3$$

the market demand is

$$Q(p) = 1 + 2p^3 - 3p^2 \tag{7b}$$

The DF model. Under the same assumptions on (6a) and (6b) of subsection 3.1, if the market demand is given by (7b), then the equations (3a) and (4a) of the mixed market case become

$$\widetilde{p}^M_{DF} = c_0 \tag{3c}$$
$$\widetilde{q}^M_{DF} = 6c_0^2(1-c_0) \tag{4c}$$

where (3c) coincides with (3a) and is repeated here for convenience, and the $\sim$ denotes the value of the relevant variables under the Beta distribution. Therefore, the corresponding equilibrium value of welfare in the mixed market is

$$\widetilde{W}^M_{DF} = \frac{1}{2} - c_0 + (1+6n)c_0^3 - \left(\frac{1}{2} + 6n\right)c_0^4 \tag{16}$$

In the fully private case, demand being given by (7b) implies the first order condition

$$\frac{1}{n+1}\left(1 + 2p^3 - 3p^2\right) = 6p^2(1-p)$$

and the equilibrium price

$$\widetilde{p}^P_{DF} = \frac{\left(1+\sqrt{33+24n}\right)}{2(8+6n)} \tag{17}$$

The equilibrium value of welfare is therefore

$$\widetilde{W}^P_{DF} = \frac{1}{2} + \frac{3}{2}\left(\frac{\left(1+\sqrt{33+24n}\right)}{2(8+6n)}\right)^4 - 2\left(\frac{\left(1+\sqrt{33+24n}\right)}{2(8+6n)}\right)^3 \tag{18}$$

The value of $\widetilde{c}_0^*$ beyond which privatization is desirable under the Beta distribution is given by equating (16) and (18). The solution for different

[10] For an in-depth discussion of the Beta distribution and its properties, see Johnson *et al* (1995, ch.25).

values of n is obtained numerically.[11] In Table 1 we compare these values of $\widetilde{c}_0^*$ with those given by equation (10), i.e. c_0^* under the uniform distribution with $\overline{y} = 1$.

Table 1
Threshold values of the public firm inefficiency

	$\widetilde{c}_0^*$	c_0^*
$n = 1$	0.044492	0.056080
$n = 2$	0.025603	0.031404
$n = 3$	0.017046	0.020056
$n = 4$	0.012360	0.013913
$n = 5$	0.0094770	0.010216
$n = 6$	0.0075582	0.0078187
$n = 7$	0.0062067	0.0061764
$n = 8$	0.0052131	0.0050021
$n = 9$	0.0044578	0.0041336
$n = 10$	0.0038679	0.0034731

The Table shows that for $n < 7$ the higher income concentration associated with the Beta distribution implies a reduction in the threshold value of c_0: income concentration widens the range of situations in which privatization is welfare enhancing. For $n \geqslant 7$, the opposite occurs. This pattern can be explained with reference to the effects of the distributive shock on the demand side of the market, shown in Figure 1. As we move from the uniform to the Beta distribution, we observe the demand effect, according to which the demand constraint perceived by the firms is relaxed (tightened) for $p < (>) 1/2$; the size effect, according to which for any positive price under the Beta distribution the maximum possible welfare is lower;[12] and

[11] As for equation (10), we consider the positive solution which ensures that the production of the public firm is positive.

[12] This is a direct consequence of the Beta function being an inverse mean preserving spread of the uniform distribution. Accordingly, the integral of its cumulative function is lower than that of the dominated uniform distribution for all values of $p < 1$. This in turn implies that the area below the demand curve is lower under the Beta distribution for all values of $p \in (0, 1)$.

the elasticity effect, with market demand elasticity increasing (decreasing) for $p > (<) 1/4$.

The size effect unambiguously reduces the relative advantage in terms of welfare of the presence of a public firm. As far as the demand and demand elasticity effects are concerned, it can be checked that for $n \leqslant 2$, as incomes become more concentrated, firms perceive an increase in demand coupled with an increase in demand elasticity; the fully private market outcome is closer to allocative efficiency, and this reinforces the size effect in reducing the scope for public ownership. As n increases, the elasticity effect is reversed, and the increase in demand is accompanied by a reduction in elasticity. This partially counterbalances the size effect up to $n = 7$, dominating it for $n > 7$.

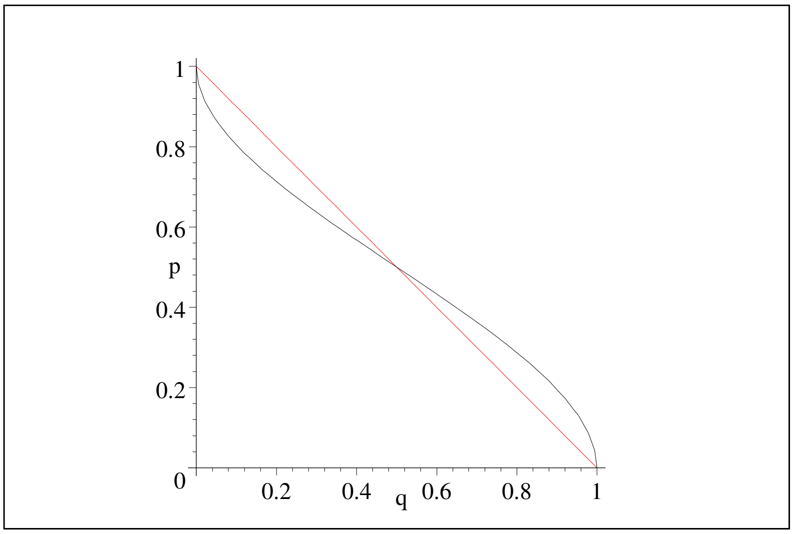

Figure 1 Demand under uniform and Beta distribution

The DFD model. If demand is given by (7b) and the supply side of the market is described by (6c) for all firms, then the reaction functions of the public and private firms are respectively

$$p = k\left(1 + 2p^3 - 3p^2 - nq\right) \tag{3d}$$

$$q = (p - kq)\, 6p(1 - p) \tag{4d}$$

The system (3d)-(4d) can be solved only by giving specific numeric values to k and n, delivering the equilibrium values $\widetilde{p}^M_{DFD}(k, n)$ and $\widetilde{q}^M_{DFD}(k, n)$. For those k and n, the corresponding equilibrium value of welfare can then be calculated as

$$\widetilde{W}^M_{DFD} = \frac{1}{2}+\frac{3}{2}\left(\widetilde{p}^M_{DFD}\right)^4-2\left(\widetilde{p}^M_{DFD}\right)^3-k\frac{\left(1+2\left(\widetilde{p}^M_{DFD}\right)^3-3\left(\widetilde{p}^M_{DFD}\right)^2-n\left(\widetilde{q}^M_{DFD}\right)\right)^2}{2}-nk\frac{\left(\widetilde{q}^M_{DFD}\right)^2}{2} \tag{19}$$

If the market is fully private, the first order condition (5) becomes

$$\frac{1}{n+1}\left(1+2p^3-3p^2\right) = \left(p-k\left(\frac{1}{n+1}\left(1+2p^3-3p^2\right)\right)\right)6p(1-p) \quad \text{(5d)}$$

Also in this case the solution for the equilibrium price, $\widetilde{p}^P_{DFD}(k,n)$, is obtained only for specific numeric values of k and n. Given the latter, the welfare in the fully private case is then given by

$$\widetilde{W}^P_{DFD} = \frac{1}{2}+\frac{3}{2}\left(\widetilde{p}^P_{DFD}\right)^4-2\left(\widetilde{p}^P_{DFD}\right)^3-k\frac{\left(1+2\left(\widetilde{p}^P_{DFD}\right)^3-3\left(\widetilde{p}^P_{DFD}\right)^2\right)^2}{2(n+1)} \tag{20}$$

For given k, the comparison of the values of $\widetilde{W}^M_{DFD}$ and $\widetilde{W}^P_{DFD}$ obtained for different numeric values of n allows to identify by approximation the threshold value $\widetilde{n}^*$ beyond which $\widetilde{W}^P_{DFD} > \widetilde{W}^M_{DFD}$, i.e. privatization becomes welfare enhancing. The first column of Table 2 lists these thresholds for different values of $k \leqslant 1$.[13] They can easily be compared with those calculated for the uniform distribution, listed in the second column.

It turns out that for all values of k the critical value of n with the Beta distribution is lower, thus signaling that the concentration of incomes widens the range of situations in which privatization is desirable.[14] The intuition behind this result relies again on the size effect and the elasticity effect of the distributional shock. The key role is played here by the size effect, i.e. the reduction in potential welfare given by the shift from the uniform to the Beta distribution. The elasticity effect may either reinforce the size effect – when the price is high enough to ensure that demand elasticity increases – or partially counterbalance it. The decrease in the maximum achievable consumers' surplus makes public ownership less attractive, and this notwithstanding a possible increase of the price over cost margin in the alternative fully private configuration.

[13] We restrict our analysis to $k \leq 1$, as in this case the difference between the average costs of the public vs the private firms is increasing in k under the uniform distribution.

[14] That the threshold numbers of firms decrease or increase in k, depends on the relative sensitivity of welfare to changes in k in the mixed vs fully private market configurations.

Some final remarks and conclusions are gathered in the next section.

Table 2
Threshold values for the number of firms

	$\widetilde{n}^*$	n^*
$k = 0.01$	9.0	9.7
$k = 0.025$	4.8	6.1
$k = 0.05$	3.0	4.3
$k = 0.075$	2.4	3.6
$k = 0.10$	2.0	3.2
$k = 0.25$	1.5	2.3
$k = 0.50$	1.6	2.1
$k = 0.75$	1.9	2.2
$k = 1.00$	2.2	2.4

4 Final remarks and conclusions

In this paper we have reconsidered the canonical models of mixed oligopoly by De Fraja (1991), and De Fraja and Delbono (1989), modelling the market demand as the outcome of the binary individual choice of a population of consumers, heterogeneous with respect to income. Within this setup we have studied how stylized changes in the distribution of the consumers' willingness to pay – which we interpret as ultimately related to changes in the distribution of income – modify the range of situations in which the privatization of the publicly owned firm is welfare enhancing.

We have focused on two types of 'distributional' changes: a generalized increase in incomes that generates the stretching of a uniform distribution of the reservation prices – an example of first order stochastic dominance – and a concentration of incomes around the mean, which implies a shift from a uniform to a quadratic Beta distribution of the reservation prices – an example of mean preserving, second order stochastic dominance. In these examples, the scope for privatization turns out to be wider, the 'poorer' is the market and the higher is income concentration. The presence of a public firm is more beneficial the richer are the consumers and the more dispersed are incomes.

These results can be traced back to the way in which the distributional shocks affect the market demand and, through the latter, the size of the allocative inefficiency under imperfect competition. Our generalized increase in incomes increases the potential welfare achievable in the market, and is accompanied by a reduction in demand elasticity, both effects widening the scope for the regulatory intervention of the public firm. On the contrary, our example of income concentration implies a reduction of the potential welfare, the effects of which on the desirability of privatization are amplified (or only partially counterbalanced) by the effects on demand elasticity.

The above relationship between income, demand and incentive to privatization can be seen as the direct consequence, in a mixed oligopoly framework, of the way in which the demand side factors typically affect market competitiveness. The scope for a public firm is wider in a richer market, because the deadweight loss is higher in that market. However, one might argue that this seems at odds with a popular view, which invokes the direct intervention of public firms in some key markets in order to protect the weakest and poorest segments of the population. But this discrepancy is not surprising when we recall that the mixed oligopoly approach is strictly of the partial equilibrium type, and that it assigns to the public firm exclusively an allocative efficiency objective. While pursuing the latter has an impact on the functional distribution of income, it disregards the personal distribution – one unit of additional income having the same weight independently of its accruing to a rich or a poor consumer. In our opinion this suggests to extend the analysis of the relation between income distribution and privatization to a general equilibrium framework, where the public firms can be assigned more comprehensive objective functions.

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