MATHEMATICAL EXPERIENCES TO CONTRAST LEARNING LOSS AND SCHOOL DROP-OUT

IX Seminar "Data from and for educational system: tools for research and teaching"

edited by
Patrizia Falzetti

FrancoAngeli

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Introduction

by Patrizia Falzetti

The seminar dedicated to INVALSI data is a valuable opportunity to explore crucial issues in education. For example, over the course of its various editions, the Mathematics sessions have provided a valuable opportunity to analyse results, recurring errors, and the cognitive processes involved in solving test items in detail, thereby fostering targeted didactic reflection.

This volume collects six contributions presented at the ninth edition of the Seminar "Data from and for the educational system: tools for research and teaching" (Rome, 17-19 October 2024). Each contribution addresses some of the main challenges in contemporary Mathematics education from different yet complementary perspectives: linguistic complexity, socio-economic inequalities, the impact of the pandemic, teacher education and the risk of school dropout.

The first chapter explores how students' linguistic repertoires can support Mathematics learning in multilingual contexts. The second chapter analyses the evolution of learning loss in Mathematics between 2020 and 2024, with particular attention to disadvantaged students. The third chapter presents the results of an online survey investigating Mathematics teachers' knowledge of the national standardised INVALSI tests. The fourth chapter examines early school leaving in Italy and Europe, focusing on lower and upper secondary education. The fifth chapter describes and discusses teaching experiences aimed at fostering problem-solving processes in disadvantaged contexts. Finally, the sixth chapter analyses school dropout and the role of inclusive education in its prevention.

Overall, these contributions provide a comprehensive overview of the factors that influence the learning of Mathematics, including language, context, motivation, teaching quality, and inclusion. The volume thus provides a

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useful framework for reconsidering Mathematics as a formative experience that can foster participation, equity and meaning.

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3. Results of INVALSI standardized assessment as a tool for professional development of Mathematics teachers

by Maria Chiara Cibien, Marta Saccoletto,
Carlotta Soldano, Camilla Spagnolo

This study investigates Mathematics teachers' knowledge regarding the INVALSI national standardised tests. The sample consists of 131 in-service and pre-service teachers of primary, lower secondary, and high secondary school. An online questionnaire was used to collect data, and the analysis of teachers' responses is presented, particularly regarding their interpretation of student mistakes. The main theoretical framework used for the analysis is the *Mathematics Teacher Specialised Knowledge* Model (Carrillo-Yañez *et al.*, 2018). Findings highlight difficulties in identifying common student mistakes at the national level, with differences emerging based on school level and teaching experience. These insights will inform the design of an innovative teacher professional development model within the national project "Mathematics standardised assessment as a tool for teachers' professional development".

Questa ricerca indaga le conoscenze degli insegnanti di Matematica in relazione alle prove standardizzate nazionali INVALSI. Il campione di riferimento consiste di 131 insegnanti in servizio e futuri insegnanti di scuola primaria, secondaria di primo grado e di secondo grado. Per la raccolta dei dati è stato somministrato un questionario online, di cui verrà presentata l'analisi delle risposte relativamente all'interpretazione degli errori degli studenti. Il principale quadro teorico di riferimento utilizzato per l'analisi è il modello Mathematics Teacher Specialized Knowledge *(Carrillo-Yañez* et al.*, 2018). I risultati evidenziano difficoltà nell'individuazione degli errori più comuni degli studenti a livello nazionale, con differenze riscontrate in base al grado scolastico considerato e all'esperienza di insegnamento. Queste evidenze contribuiranno alla progettazione di un innovativo modello di formazione professionale per gli insegnanti, nell'ambito del progetto nazio-*

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nale "Mathematics standardised assessment as a tool for teachers' professional development".

1. Introduction

The European Parliament and the Council launched a recommendation on lifelong learning in 2006 in which Member States are invited to promote key competences for all citizens in their lifelong learning strategies.

There is serious worrying data coming out from international research focusing on Mathematical competences. OECD data show that students who are fifteen years old and have reached Level 1 acquire limited mathematical knowledge on their own, which they are only able to apply in some familiar situations. Referring to INVALSI data of 2022, a similar situation occurs in Italy. Already at the end of the last century, the Curriculum and Evaluation Standards for School Mathematics (Mathematical Sciences Education Board, 1989) and the *Professional Standards for Teaching Mathematics* (NCTM, 1991) have promoted a vision for teaching and learning that has growing support from the Mathematics education community. There is increasing recognition that without carefully planned professional development programs, the chance of widespread implementation of this vision of Mathematics is small. Support is essential for practicing teachers who express an interest in teaching in a way described in the Standards, with all that such teaching implies in terms of content, pedagogy, and assessment.

The topic of teacher professional development is increasingly gaining ground in the field of educational research, also in Mathematics education.

2. The national research project

The project "Mathematics standardised assessment as a tool for teachers' professional development" (MaSt) stems from a firm and shared conviction that standardized assessments, used in a formative perspective, can become a potentially revolutionary element for the educational and didactic function of school systems. In a democratic school perspective (equity and quality together), we embraced the Bloomian belief (Bloom, 1972) that all students can learn basic skills in each topic, and we trust in the link between the development of students' skills and the teachers' professional development paths. Literature highlights that schools show an evident fragility with respect to developing an effective ability to promote good mathematical

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competences. It is well known that teachers have difficulties in adopting valid practices for individualized teaching techniques in order to be able to pursue quality and equity for the results of every and each one of the student population, but also in finding strategies to improve the learning for the fundamentals (Stipek *et al.*, 2001). The possibility for a teacher to use in his classroom the insights on mathematical learnings provided by the standardized tests – through their theoretical frameworks, the released items with the interpretation of macro-phenomena observed, the valuation of their results – in a formative way is an important opportunity to renew the praxis in teaching and assessment.

The use of standardized surveys to substantiate the formative assessment practices in the classroom is of high priority and of great importance. The impact of standardized assessment is traditionally a top-down impact: the results of the surveys affect public opinion; this moves the policy makers acting on the structure of the system at different levels (curriculum, teacher training, recruitment). Only in the final phase, the surveys and their results influence teachers' local action. The innovative idea of our Project is to provide instruments for a bottom-up approach in which theoretical frameworks/test/results are used immediately as a tool by teachers, to accomplish formative assessments and individualized teaching (from the perspective of quality and equity of students' achievement).

To make this approach fruitful some conditions should be fulfilled: first, there must be a link between assessments and curriculum (Mons, 2009; Meckes and Carrasco, 2010; Ferretti *et al.*, 2018). Second, this link should be perceived by the teachers, since assessment is a key component of teachers' identity (Hannula *et al.*, 2016; Ferretti *et al.*, 2021).

In Italy, the first of these conditions is met: as a matter of fact, the national system of standardized assessment (INVALSI) framework and tests are aligned with National Guidelines goals (INVALSI, 2018). There is a need for tools to develop educational activities to achieve the second condition as well. From this perspective, the main purpose of our Project is to design a model for teachers' professional development, consisting of theoretical and operational tools, starting from the resources provided by the INVALSI national assessment tests.

3. Theoretical framework

The research design framework considers three main components and results from the integration of three established theories.

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We assume that the model builds on the principles of Jaworski' s *Community of inquiry* (2006), encouraging participants to critically reflect on their practices while engaging in them, question what they do as they do it, and explore new aspects of practice. That means that, in our conception, inquiry is intended as a "way of being" in which teachers develop a specific attitude that defines their way of acting, thinking, learning and teaching.

We also referred to the *Mathematics Teacher Specialized Knowledge model* (MTSK) by Carrillo-Yañez (Carrillo-Yañez *et al.*, 2018), that coordinates two broad areas of knowledge, *Mathematical Knowledge* (MK) and *Pedagogical Content Knowledge* (PCK) by placing teacher beliefs at the centre (Fig. 1). MK is the knowledge possessed by a Mathematics teacher in terms of disciplinary knowledge within an educational context and consists of three subdomains:

– *Knowledge of Topics* (KoT), that includes the knowledge of mathematical definitions, properties, representations, etc.
– *Knowledge of the Structure of Mathematics* (KSM), that involves examining elementary Mathematics from a more advanced perspective, as well as understanding how to switch and connect activities across different mathematical domain's.
– *Knowledge of Practices in Mathematics* (KPM), that regards having the ability to prove, justify, make deductions and inductions, provide examples, etc.

PCK is the knowledge related to Mathematics content in terms of teaching-learning processes and is composed of:

– *Knowledge of Mathematics Teaching* (KMT), that comprehends the knowledge of the Mathematics teaching theories, resources, technologies, strategies for introducing and representing mathematical content and concepts.
– *Knowledge of Features of Learning Mathematics* (KFLM), that comprises the knowledge about how students might act and interact with Mathematics, considering also affective aspects.
– *Knowledge of Mathematics Learning Standards* (KMLS), that consists of the knowledge of national curricula and international documents for every school level.

Beliefs about Mathematics and its learning and teaching are at the centre of the model precisely to emphasize the reciprocity between beliefs and knowledge domains. The MTSK model allowed us to frame the teachers' responses to our research questionnaire.

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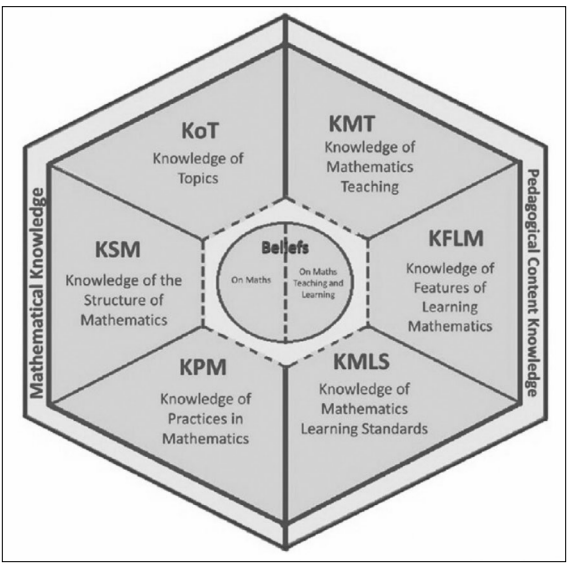


Fig. 1 – MTSK Model

Source: Carrillo-Yañez *et al.* (2018), p. 34

Finally, the third component of our theoretical framework is the *Meta-didactical Transposition Model* by Arzarello and colleagues (2014). This model focuses on the practices and dynamics that emerge throughout teacher training programs, and enables the description and interpretation of variables in teacher education processes, as well as the analysis of their mutual relations and developments. In the meta-didactic transposition process, educational practices and theoretical purposes are shared between teachers and trainers. The term "meta-didactics" emphasises that these practices involve reflections on teaching activities: indeed, within the meta-didactic transposition, practices become meta-didactic practices as they refer to the activities and reflections that characterise teacher education processes. Meta-didactic practices refer to the practices and theoretical reflections developed within a community of inquiry in the sense of Jaworski, as mentioned above.

4. Methodology

In the initial phase of the research, the aim is to identify the training needs of in- and pre-service teachers. To this purpose, a questionnaire was designed and administered. It is declined in six distinct versions, formulated accord-

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ing to the group of teachers involved (pre- and in-service teachers) and the school level in which they are teaching or they will be teaching (focusing on grade 5, 8 and 10). The questionnaire includes questions based on specific INVALSI items selected from the GESTINV database (www.gestinv.it). The items were chosen as representative examples of macro phenomena at national level: multiple-choice items with a notably low percentage of correct responses among students and whose distractors (incorrect answers) can be framed with mathematics education constructs known in literature. Additionally, the questionnaire includes "transversal" questions that are not directly related to the proposed INVALSI items. These questions are specifically designed to explore participants' perceptions of the purpose and usefulness of the INVALSI tests, as well as their impact on current or future teaching practices. Previous works by the INVALSI-Didactics Disciplinary Group, which is part of the observatory of the SIRD – Italian Society for Educational Research[1] (Arzarello and Ferretti, 2022; Ferretti *et al.*, 2022), have shown how a questionnaire using INVALSI items can provide useful information about in-service grade 5 teachers.

In this paper, we focus specifically on questions based on selected INVALSI items to explore in-service and pre-service teachers' anticipations and interpretations of the most common mistakes students make when answering these items. As we will see, Pedagogical Content Knowledge (PCK) (Carrillo-Yañez *et al.*, 2018) is involved in this activity, and the analysis provides insights into the needs of both in-service and pre-service teachers.

In the try-out phase of the questionnaire, we selected four INVALSI items for each grade (grades 5, 8, and 10), for a total of 12 INVALSI items.

5. Results

131 pre-service and in-service teachers participated in the try-out of the questionnaire, including 27 in-service and 47 pre-service teachers for grade 5, 12 in-service and 11 pre-service for grade 8, and 14 in-service and 20 pre-service for grade 10. The questionnaire was administered online, which made it possible to reach teachers from a variety of schools and universities across Italy, thus ensuring a heterogeneous set of respondents. While the sample was not designed to mirror the national distribution of Mathematics teachers, its diversity supports the exploratory aims of our study. In presenting the results, we refer in particular to the responses of secondary school

[1] https://www.sird.it/en/home-en/.

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teachers, as the overall pattern of answers was comparable between in-service and pre-service teachers; however, similar tendencies were also found among primary teachers.

We now present some of the results we have collected and analysed with regard to the interpretation of mistakes, i.e. what we have called "distractor analysis". For each INVALSI item included in the questionnaire, we asked teachers to answer the following question (without revealing the national percentage of correct answers): "The correct option is option X. What do you think was the most popular wrong option at national level? Why?". In this way, we can investigate whether teachers are aware of the mistakes students are most likely to make and, simultaneously, whether they have a clear understanding of the underlying causes of those mistakes.

An example is item 6 of the 2012 INVALSI test for grade 8 students. It asks whether it is possible to calculate the area of the triangle (in grey) knowing the area of the rectangle (Fig. 2).

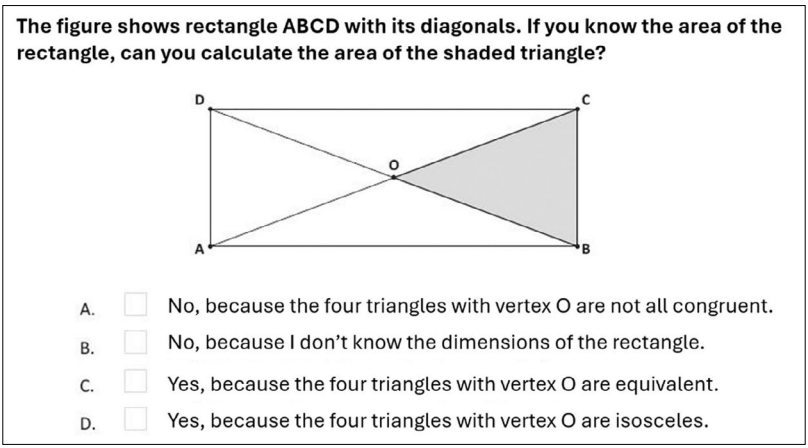


Fig. 2 – Item 6 of 2012 INVALSI test for grade 8 (translation by the authors)

The data from the GESTINV database (Fig. 3) show that only 24.1% of the sample selected the correct answer C, while the most frequently chosen distractor B was selected by 30.8% of the students. This option suggests that the area of the triangle cannot be calculated due to the unknown dimensions of the rectangle. Subsequently, 26.9% of the respondents selected A (the area cannot be known because the triangles are not equivalent) and 16.4% chose D (stating that it can be known because the triangles are isosceles). Examining the characteristic curves of the item, it is evident that distractor B is selected by students across all ability levels with a similar probability. A

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similar pattern is observed for distractor A, which is chosen by a percentage almost identical to that of distractor B. It can also be observed that the characteristic curve for the correct answer does not closely align with the ideal curve: there is an underestimation for students with lower ability in the test and an overestimation for higher ones.

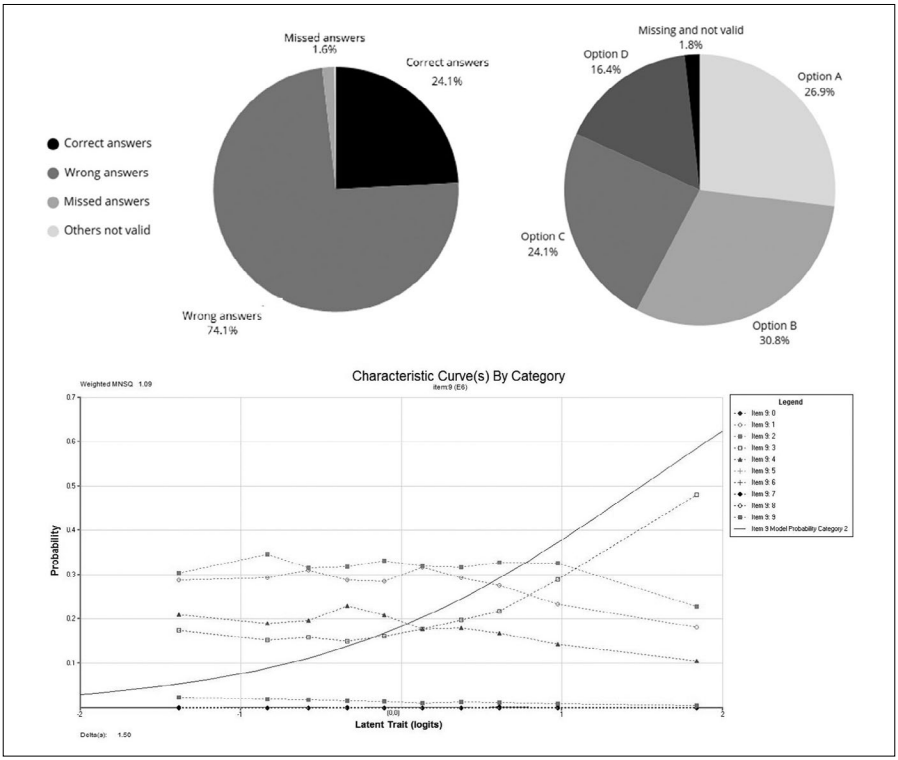


Fig. 3 – National results for item INVALSI 6 of the 2012 for grade 8 students

At the top left, the national percentages of correct, incorrect, and invalid responses; at the top right, the national percentages for each answer option; below, the characteristic curve describing the pattern of distractor choices at the national level in relation to students' ability levels across the entire test

Table 1 represents the analysis of the low secondary school teachers' answers. More than half of in-services identify as the most likely distractor the one that is actually chosen by the national sample, even though a good portion (33.3%) states that the most chosen wrong answer by students might be option A. Conversely, among pre-service teachers, almost 73% claim that the most chosen distractor is option A and only 18% align with the national results.

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Tab. 1 – The main choices and reasons of lower secondary school teachers (12 in-service and 11 pre-service) who responded to the questionnaire, about the distractor analysis of item 6 of 2012 INVALSI test for grade 8

In-service teachers (12)	*Pre-service teachers (11)*
58.3% choose option B, because *the absence of numerical data prevents calculations leading to the result* (e.g. "B because they are looking for the values of the dimensions to calculate the area but do not know them")	18% choose option B because of the *absence of numerical data* (e. g. "B because pupils expect dimensions as they are always given in their textbooks and problems")
33.3% choose option A because *visually the triangles are different* (e. g. "A because it stops at a glance")	72.2% choose option A because *visually the triangles are different* (e. g. "Maybe A because to the eye the triangles do not look equivalent") or because of the presence of the *"equal" distractor element* (e. g. "The most frequently chosen wrong option was A, because the word "equal" leads the student to the mistake")

Despite this difference between in-service and pre-service, the reasons given by both groups to explain their choice of distractors are the same. As it can be seen in Table 1, both state that the choice of distractor B could be due to the absence of numerical data in the problem text and, in a good percentage, they also give the same interpretation for the choice of option A (i.e. that students perceive the triangles to be visually different).

The awareness of pupils' mistakes can be interpreted as evidence of Knowledge of Features of Learning of Mathematics (KFLM), a Pedagogical Content Knowledge subdomain of the MTSK model. It includes knowing how students might act, their mistakes and difficulties in specific topics, and, in general, how students interact with Mathematics while also considering affective aspects. For example, comment like "B because pupils expect dimensions as they are always given in their textbooks and problems" can be considered as Knowledge of Mathematics Teaching (KMT), understood as the way a topic is typically taught.

It appears that in-service teachers' anticipation of students' mistakes is more aligned with the INVALSI results than that of pre-service teachers. In this case, it can be regarded as a deeper KFLM, which appears to be more developed in in-service teachers than in pre-service teachers. However, this does not also apply to the interpretation because, as it can be seen, teachers (both pre- and in-service) that choose respectively the two wrong options A and B determine the same reason for both distractors.

Another example is from the questionnaire for high secondary school teachers. Item 21 of the 2017 INVALSI tests for grade 10 students (Fig. 4)

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is presented to the teachers; it asks to calculate the slope of a straight line, knowing two points belonging to it.

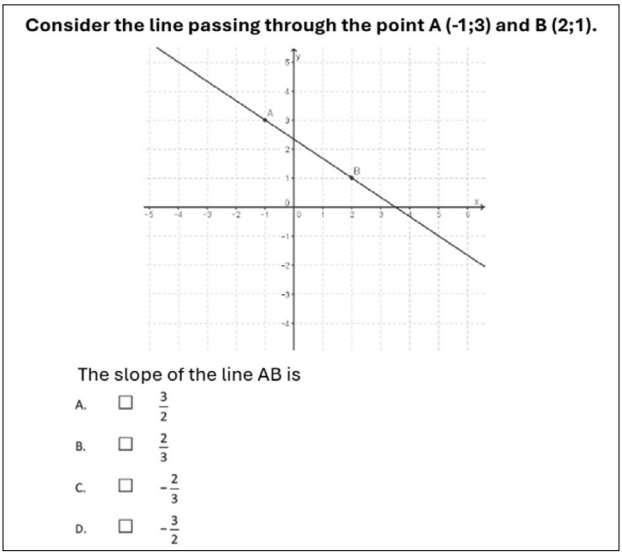


Fig. 4 – Item 21 of 2017 INVALSI test for grade 10 (translation by the authors)

The correct answer is chosen by 29.6% of the students nationwide and the percentages of distractor choices are very similar to each other, especially for the two most chosen: D is selected by 24.1% and A by 23.7%. This can also be observed from the characteristic curves, which are much "closer" to each other than the distractor curves of the item shown in the previous example. As before, however, the ideal curve underestimates students with lower ability in the test and overestimates higher ones (Fig. 5).

In this instance, most in-service teachers do not interpret mistakes in line with the national results: in fact, only 21% choose option D and even only 7% select option A. On the other hand, the percentages of pre-service teachers are more "Equi distributed", as indeed are the students' answers, even though they do not correspond to the national sample; indeed, we recall that, in order, the most chosen distractor was option D, followed by option A and then B. Pre-services selected mainly D (35% of the respondent), then B (25%) and only lastly A (20%). It is also notable that the motivations given by the pre-servicers are also very varied, compared to those of the in-service (Tab. 2). The latter, in fact, interpret the mistake, regardless of the distractor, as a problem in the use of the formula for calculating the angular coefficient; this is also mentioned by the pre-service, but other reasons (such as confu-

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sion between the angular coefficient and the ordinate at the origin, influence of the data in the request, mention of the cathexes of the right triangle, etc.) are taken into account. Hence, in this case, it seems that pre-service teachers are more conscious of possible students' mistakes, and they also give many different interpretations, which are not considered by in-service teachers.

Similarly to the previous case, the responses of high secondary school teachers can also be classified as evidence of aspects of KFLM. In particular, it is observed that, regardless of the selected distractor, most teachers attribute the error to a difficulty related to students' knowledge and/or application of the formula. These reflections can also be interpreted as elements of the subdomain of KMT. It is plausible that teachers refer to established teaching practices, which represent habitual strategies for addressing the topic in class (for example, specifically through the introduction of the formula). Consequently, they may be more inclined to attribute the error to a critical issue related to this teaching approach. In this sense, pre-service teachers may have proposed more diverse interpretations, as they have not yet internalized such practices, given that they do not have direct classroom teaching experience.

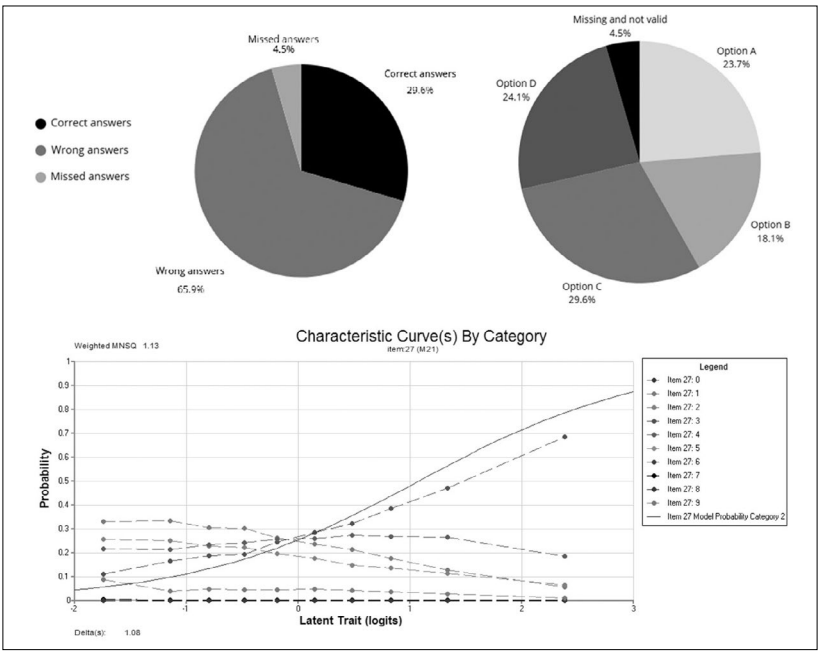


Fig. 5 – National results for item INVALSI 21 of the 2017 for grade 10 students

At the top left, the national percentages of correct, incorrect, and invalid responses; at the top right, the national percentages for each answer option; below, the characteristic curve

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Tab. 2 – The main choices and reasons of high secondary school teachers (14 in-service and 20 pre-service) who responded to the questionnaire, about the distractor analysis of item 21 of 2017 INVALSI test for grade 10

In-service teachers (14)	*Pre-service teachers (20)*
21% choose option D because of *problems with the formula* (e.g. "D, lack of knowledge of the correct formula for calculating the slope of the line")	35% choose option D because of *problems with the formula* (e.g. "D because they invert delta y and delta x") or because they *are influenced by the data* (e.g. "D because they read the coordinates of the points from the graph instead of differentiating them")
7% choose option A because of *problems with the formula* (e.g. "A because students confuse x with y and confuse the use of signs")	20% choose option A because they *are influenced by the data* (e.g. "A because 3 and 2 are also two numbers that appear in the data points (3 first and 2 second)"), or they *incorrectly identify the cathexes of the triangle* (e.g. "I assume answer A because, when considering the points on the plane, they could observe the lengths of the cathexes formed by the line with the axes from the abscissas"), or *they confuse the angular coefficient and the ordinate at the origin* (e.g. "A, because the line intersects the y-axis in 3/2").
57% choose option B because they correctly *apply the formula but do not take into account the sign* (e.g. "In my opinion, the most popular option is B. They "correctly" apply the formula for the angular coeff. but do not report the – sign (which is moreover in the denominator, something students are not used to)") or because *they do not reason on the slope* (e. g. "B, because the student does not take the negative slope into account")	25% choose option B because of the *omission of the minus sign* (e.g. "B because signs are often considered irrelevant")

5. Conclusion

The INVALSI tests serve as a valuable tool for examining the knowledge of Mathematics teachers. By leveraging data from the national sample, these tests make it possible to guide both pre-service and in-service teachers in reflecting on the potential mistakes made by students and on their possible underlying causes. The present study adopted this perspective and analysed how teachers interpreted selected INVALSI items that present frequent student errors. The examples discussed are only a subset of those included in the questionnaire and are intended as illustrative of the types of knowledge

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and beliefs that emerge when teachers engage with large-scale assessment data through the lens of the MTSK model. Similar patterns were observed in teachers' responses to other items, which could not be included for reasons of space but point in the same direction.

Overall, the findings suggest that much of the knowledge mobilised during the interpretation process can be framed within specific subdomains of Pedagogical Content Knowledge, particularly KFLM and KMT. More broadly, the study shows how the MTSK framework can serve as a useful theoretical lens for characterising teachers' reasoning when analysing students' performance in standardised assessments. At the same time, the results highlight a potential challenge in teachers' perceptions, as participants often experienced difficulties in identifying the most frequent mistakes made by students at the national level.

Differences also emerged between the interpretations offered by in-service and pre-service teachers, as had already been observed in other studies conducted within the MaSt project (for further details, see Cibien, 2025). Although the qualitative nature of the data does not allow us to determine whether these differences are statistically significant, they seem to indicate that classroom experience may influence the ways in which teachers attribute causes to students' errors and the knowledge domains they activate. Conversely, pre-service teachers sometimes proposed a broader range of possible explanations, which may suggest a certain "flexibility" in their approach. This observation, however, warrants further investigation through a more structured quantitative comparison.

On this basis, these insights contribute to identifying strengths and weaknesses in teachers' processes of recognising and interpreting students' mistakes. This study has allowed us to gather significant information to be used as a foundation for the implementation of the model for professional development for teachers addressed by the main Project MaSt. Teachers' difficulties in identifying and interpreting student errors must be taken into consideration when designing the training model, meaning that it would be beneficial to implement activities aimed at fostering this awareness.

A limitation of the study is the small size of our sample. In the new phase of the research, efforts are being made to reach a larger and more diverse sample of in-service and pre-service teachers. In this broader context, a quantitative study will be conducted to examine in greater depth the educational needs and beliefs of Italian Mathematics teachers regarding the INVALSI tests and to verify whether the tendencies observed in the present qualitative analysis are confirmed at scale.

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The seminar dedicated to INVALSI data represents a valuable opportunity to explore crucial issues in education. For example, over the course of its various editions, the mathematics sessions have provided a valuable opportunity to analyse results, recurring errors, and the cognitive processes involved in solving test items in detail, thereby fostering targeted didactic reflection.
This volume collects six contributions presented at the ninth edition of the Seminar "Data from and for the educational system: tools for research and teaching" (Rome, 17–19 October 2024). Each contribution addresses key challenges in contemporary mathematics education from different yet complementary perspectives: linguistic complexity, socio-economic inequalities, the impact of the pandemic, teacher education, and the risk of school dropout.
Overall, these contributions provide a comprehensive overview of the factors influencing the learning of mathematics, including language, context, motivation, teaching quality, and inclusion. The volume thus constitutes a useful framework for reconsidering mathematics as a formative experience that can foster participation, equity and meaning.

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