

Proceedings of the Nineteenth ERME Topic Conference

Connecting the Learning of Mathematics Teaching to Practice

Charles University, Faculty of Education
Prague (Czech Republic)
4–7 February 2026

Edited by

Reidar Mosvold
Janne Fauskanger
Federica Ferretti
Naďa Vondrová

erme
european society for research in mathematics education

FACULTY
OF EDUCATION
Charles University

Editor Committee

Fanni Biro (Hungary)
Ove Gunnar Drageset (Norway)
Janne Fauskanger (Norway)
Federica Ferretti (Italy)
Marita Friesen (Germany)
Mark Hoover (USA)
Miguel Montes (Spain)
Reidar Mosvold (Norway)
Naďa Vondrová (Czech Republic)

Organizing Committee

Libuše Samková (Czech Republic) – Lead
Miguel Montes (Spain) – Co-Lead
Mark Hoover (USA) – Co-Lead
Fanni Biro (Hungary)
Janne Fauskanger (Norway)
Federica Ferretti (Italy)
Marita Friesen (Germany)
Reidar Mosvold (Norway)
Naďa Vondrová (Czech Republic)

International Program Committee

Stefan Zehetmeier (Austria) – Lead
Miguel Montes (Spain)
Andreas Stylianides (United Kingdom) – ERME Board
Janne Fauskanger (Norway)
Libuše Samková (Czech Republic)

Local Organizing Committee

Naďa Vondrová (Czech Republic) – Lead
Maria Chiara Cibien (Italy)
Federica Ferretti (Italy)
Radka Havlíčková (Czech Republic)
David Janda (Czech Republic)
Jakub Michal (Czech Republic)
Sophia Pantelaki (Czech Republic and Greece)
Libuše Samková (Czech Republic)
Jana Slezáková (Czech Republic)

Conference webpage: https://etc2026.pedf.cuni.cz/

Proceedings of the Nineteenth ERME Topic Conference
Connecting the Learning of Mathematics Teaching to Practice
held on 4–7 February 2026 in Prague, Czech Republic

Publisher: Charles University, Faculty of Education, and ERME
Place: Prague (Czech Republic)
Year: 2026
ISBN: 978-80-7603-673-4

All contributions were peer-reviewed.
© Copyright left to authors.

Recommended citation for the proceedings

Mosvold, R., Fauskanger, J., Ferretti, F., & Vondrová, N. (Eds.) (2026). *Proceedings of the Nineteenth ERME Topic Conference: Connecting the Learning of Mathematics Teaching to Practice*. Charles University, Faculty of Education, and ERME.

Recommended citation for single entries in the proceedings

Authors (2026). Title of the contribution. In R. Mosvold, J. Fauskanger, F. Ferretti, & N. Vondrová (Eds.), *Proceedings of the Nineteenth ERME Topic Conference: Connecting the Learning of Mathematics Teaching to Practice* (pp. xx–yy). Charles University, Faculty of Education, and ERME.

Introduction to the proceedings from ETC19

In the first eight CERMEs, a single working group explored the education of mathematics teachers. Subsequently, the group split into three: *Mathematics Teacher Education and Professional Development* (TWG18); *Mathematics Teaching and Teacher Practice(s)* (TWG19); and *Mathematics Teacher Knowledge, Beliefs and Identity* (TWG20). This ERME topic conference brings these groups together again to consider what it takes to learn teaching.

As the professional education of teachers has done little to change conventional mathematics teaching, scholars have argued for greater attention to the learning of practice, learning to "do" mathematics teaching. The topic conference and the papers in these pre-conference proceedings focus on what it takes to get the learning of practice at the forefront, to infuse regard for teaching as a practice (in and outside the classroom) throughout the system of professional education (pre-service and in-service), in goals, design, activities, and assessments.

Research-based contributions related to the following three questions are included:

1. What foundational things do mathematics teachers need to learn to do as they learn to teach?
2. What is involved in learning to do mathematics teaching?
3. How can we design and carry out professional education for the learning of practice?

At the conference, we will share our growing understanding and raise cross-study questions, warranting claims in relation to teaching and the learning of practice. Considering recent developments in the three TWGs, this topic conference and all its papers encourage attention to issues of equity (in mathematics teaching and in the recruitment and education of teachers) and the design of activities and tasks for learning teaching in ways that impact practice. In addition, for studies carried out in the context of one's own practice, explicit attention is given to how multiple roles are ethically managed and how analytic clarity is maintained in design, methods, and claims.

Typically, each working group at CERME addresses a distinct topic. We could have organized the papers for this ERME topic conference as distinguished among TWG18, 19, and 20, but our goal was specifically to work across this separation. Therefore, we categorised papers into four domains foundational to teacher development. (We use the word in bold as shorthand for the domain.)

- Consideration of **curriculum** of teacher education (mathematical and pedagogical knowledge and skill) and the central endeavour of extending this curriculum to prospective and practicing teachers.
- Organisation and **enactment** of design, interaction, and discourse of teacher education.
- Understanding, relating to, and responding to **teachers** as people and learners.
- Attending to broader social, cultural, and political **issues** that matter for teacher education, including imperatives of social justice.

Research on teacher development typically attends to all of these domains, yet any individual study will likely foreground one as it manages others. Identifying and categorising our work in relation to these domains helps us put diverse studies into productive conversation with each other. These domains are adapted from ones TWG19 has used productively in recent years. Whereas those domains are on the level of mathematics teaching (mathematics, enactment, students, and issues), the domains we apply in this conference are bumped up a level, from teaching to teacher learning and development.

For this ERME topic conference, we have organised people into four working groups, with the aim of having all four groups mostly analogous. No submissions focused on the **issues** domain, so we will use only three of the domains. Each working group has a set of papers foregrounding curriculum, another set foregrounding enactment, and another foregrounding teachers. Instead of organising the proceedings by working group, we have decided to organise it by domain, to display work in each domain across the full conference.

Table of contents

CURRICULUM **1**

Towards a framework for building on students' work in whole-class discussions
Kristin Krogh Arnesen and Anita Valenta **2**

Studying curriculum designers' decisions and their rationale in defining goals and activities for learning and teaching statistics
Georgia Athanasiou, Despina Potari and Konstantinos Stouraitis **10**

A purpose-turn, not a practice-re-turn
Iben Christiansen and Lisa Österling **18**

Early algebra in the encounter between blind and non-blind students
Ana Carolina Faustino and Elielson Ribeiro de Sales **26**

From theories about playful learning to classroom practice: Directions for future research
Linn Flaten and Jørgen Sjaastad **34**

A task for teacher education: Learning for the practice of teaching fraction division
Gabriela Gibim, Laura Rifo, Nuria Climent and Miguel Ribeiro **42**

Exploring the subject-specificity of practice-based mathematics teacher education: The case of examining student work
Patricio Herbst, Daniel Chazan, Amanda Brown and Qiuyu Chen **50**

Exploring foundational practices for teaching mathematics: The case of fractions
Hui-Chuan Li **58**

Implementing a four-step lesson structure to teach problem solving
Sabine Lit, Marjolein Kool and Paul Drijvers **66**

From reading about modes of reasoning to classifying reasoning and proving tasks: How pre-service teachers select tasks and apply theoretical concepts
Jakub Michal **74**

Tasks of teaching in the context of building thinking classrooms
Reidar Mosvold, Janne Fauskanger and Raymond Bjuland **82**

The interplay between student teachers' content knowledge and teaching practices: Insights from the ERME community
Reidun Persdatter Ødegaard and Kristin Krogh Arnesen **90**

Potentials and limitations of computer-based assessment in mathematics teachers' pedagogical practice: A systematic review
Laura Pikhof, Anne-Mai Kaljuste and Tiina Kraav **98**

Teaching through representation: Fostering student autonomy through bar models in mathematics learning
Claire Poh **106**

Exploring the professional mathematics teacher's knowledge in school and university practices
Diana Isabel Quintero-Suica and Miguel Montes **114**

Initiating and establishing mathematical practices of determining and transforming numbers as a foundational skill in fostering mathematics teaching
Vivien Wiebe and Uta Häsel-Weide **122**

ENACTMENT **130**

The 5P framework: A practice-based model for learning to teach mathematics
Christiyanti Aprinastuti and Andri Anugrahana **131**

Using simulations to study prospective teachers' building on student thinking
Timothy Boerst and Meghan Shaughnessy **139**

Training strategies for teaching how to teach preservice teacher education
Nuria Climent, Rocío Plazas, Ivonne Sandoval, Luis C. Contreras and Paula García **147**

Meta-didactical praxeologies in a teacher education pathway: The dynamics of sharing between teachers and researchers
Daniele De Giorgi **155**

A transfer process centered model for effective professional development programs
Susanne Digel and Jürgen Roth **163**

Cultivating coherence in teacher education
Ove Gunnar Drageset and Kristin Isaksen **171**

The Aesthetigram as a methodological and didactic tool to explore pre-service teachers' relationship with mathematics
Sarah Dufour, Marie Luquette and Geneviève Lafrance **179**

Direction of educational change in professional development: Who holds the mandate?
Andreas Ebbelind, Hanna Palmer and Jorryt van Bommel **187**

Becoming a mathematics teacher with dual AI roles student and mentor simulations in professional identity development
Selen Galiç, Mathias Tejera, Sebastián Parodi and Zsolt Lavicza **195**

Finding what is not yet there: Student teachers' perceived learning outcomes from action research in school
Yvonne Grimeland **203**

Connecting critical thinking to the learning of mathematics teaching practice
Helena Grundén **211**

Developing pre-service teachers through STEAM competencies
Alexánder Hernández, Fernando Hitt and Matías Camacho-Machín **219**

Developing an inquiry-based teaching practice: Guided reinvention through activities and teaching strategies
Sigrun Holmedal **227**

Preparing mathematics teachers for collaborative professional reflection on classroom situations – research into PSTs' feedback cycles on analyses of PST-generated classroom cartoons
Sebastian Kuntze and Jens Krummenauer **235**

The use of lesson plays in learning to teach programming
Marit Buset Langfeldt **243**

Opportunities for preservice teachers' training arising on the design of scale 3D models and associated activities adapted to visual disabilities
Cristina Lois-Prados, Teresa F. Blanco, Antía Fernández-López and Alejandro Gorgal-Romarís **251**

Mathematics teacher noticing: A situated Taxicab lab experience connecting research and practice-based teacher development
Annamaria Miranda and Loredana Saliceto **259**

Unpacking an example of a practice-based teacher education sequence of tasks
Miguel Montes and Melissa Denisse Castillo-Medrano **267**

From approximation to real practice: Focusing on the 'unexpected' in pre-service mathematics teachers' scripts
Samet Okumus **275**

Data-based decision making in mathematics: Designing a professional development programme for teachers in challenging contexts
Kathrin Pasel and Gilbert Greefrath **283**

Connecting theory and practice in initial teacher education: Representing word problems and their teaching
Libuše Samková **291**

Investigating mathematics teacher educator practices in teaching practicum course
Sanem Sarılıcan and Fatma Aslan-Tutak **299**

Developing pre-service teachers' teaching practices through classroom experiments
Alexander Schüler-Meyer **307**

Imaginations of practice: Toward a generative pedagogy for mathematics teacher learning
Gil Schwarts and Alon Pinto **315**

Teaching simulations as formative assessments of teaching practice
Meghan Shaughnessy, Timothy Boerst and Nicole Garcia **323**

What we talk about when we talk about the mathematics laboratory: Designing professional education for inclusive practice
Livia Silvestri, Giulia Bini, Giulia Ferrari, Ornella Robutti and Donatella Merlo **331**

Facilitators' role in supporting core practices in a lesson-study context
Charlotte Krog Skott and Giorgos Psycharis **339**

Study and research paths for teacher education: Connecting theory with practice
Susana Vásquez, Marianna Bosch and Berta Barquero **347**

When the mathematical concept is the pedagogical argument: Teachers' productive pedagogical-mathematical discourse
Rima Zgair-Araidy and Nadav Ehrenfeld **355**

TEACHERS **363**

Three first-time mathematics mentor teachers' mentoring perspectives while attending professional development for area specialised mentoring
Fatma Aslan-Tutak, Arife Sahin, Derya Celik and Mustafa Guler **364**

Teachers' instructional visions and self-efficacy as meta-design principles to foster high-quality mathematics instruction
Anna Bengtsson and Therese Carlsson **372**

Supporting early career teachers in developing students' mathematical reasoning
Cory Bennett, JadaLin Perkins and Mick Morgan **380**

Turn-taking in preservice teachers' collective proportional reasoning involving multimodal full-body interaction
Morten Bjørnebye, Henrik Løvik Njølstad, Bjarte Rom and Jonas Oskarsson **388**

Teachers' conceptual orientation does not yet guarantee productive practices with content-specific conceptual focus: Vignette-based study
Sinja Bonde, Kerstin Hein and Susanne Prediger **396**

Changes in early-career mathematics teachers' noticing during a reflection-based professional development program
José Antonio Bonilla Solano, Marcela Ferrari Escolá and Magdalena Rivera Abrajan **404**

Teachers' engagement with students' errors in mathematics: Introducing hermeneutical competence
Maria Chiara Cibien **412**

Primary teachers' reflections on student errors in mathematics: Insights from standardized assessment results
Maria Chiara Cibien, Francesca Martignone, Marta Saccoletto and Camilla Spagnolo **420**

Affective domain in mathematics teacher educators' activities
Federica Ferretti, Francesca Martignone and Luis J. Rodríguez-Muñiz **428**

Tracing mathematics teacher educators' specialised knowledge through a classroom observation framework
Pablo Giadas, Laura Muñiz-Rodríguez and Luis J. Rodríguez-Muñiz **436**

Pre-service teachers engaging with a collaborative problem-solving model: Bridging cognitive and socio-emotional dimensions
Ellen Kristine Solbrekke Hansen, Margrethe Naalsund and Jørgen Sjaastad **444**

TPACK learning paths of pre-service teachers when designing digital mathematical escape games
Raja Herold-Blasius and Vera Kleinschmidt **452**

Beliefs, attitudes, and pedagogical development in Finnish primary school mathematics education
Eunji Kim **460**

Teachers' perceptions and use of talk moves following a professional development course
Marte Krohg Østby **468**

Learning to teach mathematics playfully: When infrastructural constraints and learning goals compete in the redesign of games
Vasiliki Laina and Jonas Mayrhofer **476**

Interpretative knowledge: The role of collective discussion with in-service teachers
Alice Lemmo and Gabriella Catini **484**

Developing classroom observation competency in pre-service teachers: A study of a teacher education program
Tatsuya Mizoguchi and Akiko Tomosada **492**

Bridging gender stereotypes and learning outcomes: A new perspective on teacher identity
Serena Monica and Federica Ferretti **500**

Preservice secondary mathematics teachers learning to notice noteworthy events in their practicum
José L. Morales-Reyes and Diana Zakaryan **508**

Fostering pre-service teachers' noticing of mathematical thinking through textual AI-human interactions: Opportunities and challenges
Sigal-Hava Rotem and Christos Chytas **517**

The identity development across multiple contexts of an out-of-field mathematics teacher
Soryna Sabbah and Nadav Ehrenfeld **525**

Exploring pre-service primary teachers' self-efficacy through mathematics didactics portfolios
Darina Jirotková and Jana Slezáková **533**

Looking back after the first year of teaching: What changed in teachers' perspectives on analogies
Lukáš Vízek and Libuše Samková **541**

Investigating mathematics teachers' evaluations of mathematical activities: An algebra task example
Bilge Yılmaz Aslan, Begüm Özmusul and Ali Bozkurt **549**

Trained teachers' implementation of a specific way of teaching combinatorics: Factors influencing implementation fidelity
David Zenkl and Naďa Vondrová **557**

Teachers' engagement with students' errors in mathematics: Introducing hermeneutical competence

Maria Chiara Cibien

University of Ferrara; mariachiara.cibien@unife.it

Errors play a central role in mathematics learning and recent research highlights the importance of how teachers recognize and interpret them. This study investigates mathematics teachers' competencies in identifying and understanding students' errors, building on findings from an Italian national project, which revealed widespread difficulties among teachers in interpreting common student errors. To address this, in this study is introduced the theoretical construct of hermeneutical competence, describing how teachers analyse students' errors and the reasoning behind them. Examples illustrating ways of examining this competence are reported, and findings suggest factors that may influence its development. By focusing on both error identification and interpretation, hermeneutical competence offers a framework for research and professional development, supporting the enhancement of instructional practice across all school levels.

Keywords: Students' errors, teacher education, hermeneutical competence, large scale assessments.

Introduction

The role of errors in mathematics teaching and learning has long been a central topic in mathematics education literature (Borasi, 1987). More recently, this interest has significantly extended to the field of teacher education (Brodie, 2014). This line of research aims to analyse how mathematics teachers engage with students' errors (Hoth et al., 2022): the degree of awareness they possess, the interpretations they provide, and the strategies they employ to capitalize on errors pedagogically, considering their disciplinary and pedagogical knowledge, beliefs, and prior experiences. The present study aligns with this perspective, seeking to deepen the understanding of mathematics teachers' competencies regarding the specific processes of identifying and interpreting students' errors. Interest in these aspects arises from a widely documented difficulty observed in the national research project *Mathematics standardized assessment as a tool for teachers' professional development – MaSt*, involving Italian teachers across various grade levels (from grade 5 to grade 13). The main objective of this study was to investigate the professional development needs of mathematics teachers in relation to national Large-Scale Assessments (LSAs) (Bolondi et al., 2025). In Italy, LSAs are administered by the public research institution INVALSI, the National Institute for the Evaluation of the Education and Training System (www.invalsi.it). They involve students in primary school (grades 2 and 5), lower secondary school (grade 8), and high secondary school (grades 10 and 13). The assessments are conducted annually, and the results are made publicly available each year. Schools receive detailed reports on their data, including comparisons with other educational institutions in the same geographical area, with the same characteristics (type of school, socio-economic status, etc.). In the aforementioned study, teachers were asked to reflect on the most common errors made by Italian students at the national level, based on items from past INVALSI assessments. It emerged that, regardless of grade level, most participating teachers found it challenging to identify the most frequently chosen incorrect responses and, consequently, to interpret these errors consistently with

the main findings in mathematics education research (Cibien, 2025; Viola et al., 2024). This difficulty highlighted the need to investigate which knowledge and competencies teachers mobilize when reflecting on students' possible errors and the reasoning behind them. To address this need, the necessity of delineating a new theoretical construct has emerged, specifically aimed at describing the competencies mathematics teachers activate in the processes of identifying and interpreting students' errors: hermeneutical competence, which will be detailed in the following section. The introduction of hermeneutical competence seeks to contribute to research in mathematics education by providing a new theoretical tool to study a fundamental and specific aspect of mathematics teachers' instructional practice. This construct differs from existing approaches due to its targeted focus on both the identification of errors and the interpretation of the reasoning and motivations underlying them. Within the context of teacher education, it also constitutes a central element to consider and foster as a key competence, to be developed during the learning-to-teach process and consolidated in the teaching practice of teachers at all grade levels. In this way, the proposal of hermeneutical competence contributes to the broader reflection on the foundational aspects that mathematics teachers need to develop as they learn to teach. The aim is also to contribute to the international discussion on teachers' engagement with students' errors, in line with studies that have employed LSAs data to investigate how teachers make sense of students' mathematical reasoning (Williams & Ryan, 2000). The focus of the paper will be Italian national LSAs INVALSI.

Theoretical background

The concept of hermeneutical competence arises from the need to describe the specific competence that mathematics teachers are expected to have in relation to two fundamental processes, identified by Wutte and Seifried (2017) as part of *Professional Error Competence* (PEC): the identification and interpretation of students' errors. Unlike the approach proposed by Wutte and Seifried, these processes are here regarded not merely as knowledge, but as competence, understood in the sense of Weinert (2001) as an integration of knowledge, skills, and attitudes mobilized to address diverse educational situations. It is based on this definition of competence that the theoretical construct of hermeneutical competence has been conceptualized, integrating elements from the two frameworks of *diagnostic competence* (Schrader, 2011) and *interpretative knowledge* (Ribeiro et al., 2013).

Diagnostic competence (Schrader, 2011) refers to the knowledge, skills, and beliefs necessary for teachers to carry out diagnostic activities, including the collection, recognition, and interpretation of information on learning conditions, processes, and outcomes, using tools such as tests, student work analyses, observations, or interviews. Its focus may be content-related, addressing mathematical content, or student-related, addressing motivation or behavior (Hoth et al., 2016) and this information allows teachers to plan interventions, inform students and parents, and guide resource allocation (Leuders et al., 2017). Specifically, diagnostic competence in error situations (Heinrichs & Kaiser, 2018) refers to interpreting students' errors using informal or semi-formal methods, rather than planning corrective actions.

Interpretative knowledge, as defined by Ribeiro and colleagues (2013), refers to the knowledge a mathematics teacher needs to attribute meaning (i.e., interpret) students' solutions. This concept is not limited to students' work containing errors but extends to any solution exhibiting ambiguities or

inaccuracies in reasoning, regardless of the correctness of the answer. Moreover, interpretative knowledge does not only intervene in the interpretation process itself but also manifests in its application to support students' learning, as occurs in the diagnostic activities described by Schrader (2011). This knowledge thus constitutes a central aspect of a mathematics teacher's instructional practice (Ribeiro et al., 2016).

Diagnostic competence in error situations (Heinrichs & Kaiser, 2018) and the process of attributing meaning to students' solutions within the framework of interpretative knowledge (Ribeiro et al., 2013) represent the crucial theoretical elements that have led to the conceptualization of hermeneutical competence, a construct consistently theorized with reference to error situations in mathematics education. Hermeneutical competence is thus defined as the practice-related competence required of mathematics teachers in order to identify and interpret students' errors on the basis of their work, whether verbal, graphical, or written. Its practice-related nature makes it dynamic, requiring teachers to account for the variety of classroom situations and the relation between hermeneutical objects and mathematical content. Within this framework, hermeneutical objects (in the sense proposed by Figal, 2022) are represented by INVALSI items, which are designed according to the National Curriculum Guidelines and grounded in key findings from mathematics education research, enabling the identification of macro-phenomena observable at the national level (Ferretti & Bolondi, 2019; Bolondi & Ferretti, 2021). Their characteristics reduce confusion between object and interpretative process (Figal, 2022), while instead providing a reliable statistical measure of competence, not subject to individual interpretations. Teachers' reflection on these items and on the macro-phenomena emerging from LSAs can foster the emergence and analysis of their hermeneutical competence. Moreover, using LSAs results within professional development programs allows for the modification and enhancement of this competence, enriching it in light of the very evidence produced. Strong hermeneutical competence entails examining and correctly interpreting LSAs items, recognizing their purpose, identifying potential student difficulties, and inferring and interpreting common national-level errors, grounded in consolidated knowledge rather than solely in subjective experience. LSAs items are thus particularly suitable as hermeneutical objects for analyzing and measuring mathematics teachers' hermeneutical competence.

The aim of this article is to explore the theoretical construct of hermeneutical competence through the analysis of teachers' responses to research questionnaires and semi-structured interviews. Rather than providing full psychometric validation, the study investigates how the construct manifests and how interactions with LSAs data may influence its development. In this way, the analysis offers qualitative insights that illuminate the complexity and multidimensionality of hermeneutical competence in teaching contexts.

Methods

The introduction of the new theoretical construct of hermeneutical competence raises the question of how it may be subjected to empirical investigation: how it might manifest itself in teachers' practices, how it can be measured, and under which strategies or conditions it may be enhanced. The need to introduce the concept of hermeneutical competence and to empirically explore its manifestations emerged from the analysis of a research questionnaire administered, within the framework of the

national *MaSt* project (Viola et al., 2024), to teachers representing different school levels from across Italy. The questionnaire was administered online, anonymously, through Google Forms, to a total of 473 teachers. The sample was large and diverse, including participants from different school levels and with different professional statuses. Specifically, the respondents comprised 155 in-service and 88 pre-service primary school teachers, 61 in-service and 25 pre-service lower secondary school teachers, 70 in-service and 74 pre-service high secondary school teachers. The questionnaire presented teachers with a set of multiple-choice items drawn from previous INVALSI assessments, appropriate to their respective school level, asking them to indicate what they considered to be the purpose of the item, as well as which incorrect option was most frequently selected by students nationwide, and to justify their choice by interpreting the causes of the error and providing an explanation for it. The questions were adapted from the validated questionnaire developed by the SIRD research group (for further details, see Arzarello & Ferretti, 2022) and the INVALSI items chosen were selected principally because they are emblematic examples of macro-phenomena identified at the national level. In particular, they were multiple-choice items characterized by very high percentages of incorrect responses (thus indicating a widespread difficulty on a national scale) and by incorrect options readily interpretable through constructs well established in the literature. Findings from this research questionnaire revealed that, regardless of the school level taught, the participating teachers experienced considerable difficulties in identifying and interpreting the most common errors made by Italian students at the national level in large-scale mathematics assessments (for further discussion, see Viola et al., 2024). Teachers' answers to the questionnaire were analyzed using a grounded theory approach (Charmaz, 1983): the response categories were developed inductively from the data, classified according to the incorrect option selected and the explanation provided (for a detailed example, see Cibien, 2025). Results from the analysis of responses to the item-related question *"What do you think has been the most popular wrong option nationwide? Why?"* showed a widespread difficulty among teachers, both in identifying the most frequently chosen incorrect response at the national level and in interpreting the possible reasoning underlying students' choices. This analysis made it possible, in a preliminary phase, to identify areas of relative weakness in teachers' hermeneutical competence, in particular referring to the process of errors' identification. To further investigate these aspects, a series of semi-structured qualitative interviews was conducted with secondary school teachers who had expressed willingness to continue participating in the research after completing the questionnaire, following the interview guide approach (Patton, 1980).The interview questions were designed based on the data collected from the questionnaire, particularly regarding the items to be addressed with the teachers and the selection of errors to focus on. The interviews are still in progress; preliminary findings are drawn from initial analyses, with 42 secondary school teachers interviewed so far. During the interviews, the teachers were once again presented with the same items previously included in the questionnaire and were asked to expand and elaborate on the reflections they had made earlier, regarding the choice of the most common incorrect option selected by students and their interpretation of it. In doing so, the aim was to gain deeper insight into the reasons underlying their choices, which in the questionnaire responses were often expressed very concisely. Following this initial discussion, teachers were then presented with protocols containing students' errors previously collected, representing various types of errors: for each incorrect option, concrete examples were provided to explore which errors teachers

regarded as most plausible and widespread at the national level. This approach allowed for the direct observation of teachers' interpretations of students' work. At the end of the protocol reflection, teachers were asked to estimate the difficulty of the items, indicating the expected percentage of correct responses at the national level, as well as to provide an estimate of the distribution of choices among the various incorrect options. This latter request aimed, in particular, to investigate the degree of confidence and awareness with which teachers had identified the most frequently selected incorrect option and how far their interpretations of the causes of errors were in line with the literature. Subsequently, teachers were shown the actual national response percentages provided by INVALSI. This step allowed a discussion both on the accuracy of their perceptions and on possible explanations for the observed results. Students' protocols and national LSA data were used to investigate how teachers' hermeneutical competence might be shaped and developed through engagement with these resources. The following section presents excerpts from interviews with secondary school teachers to illustrate how this competence manifests and changes over time.

Discussion

The discussion is based on excerpts from two interviews with high secondary school teachers. One of the items presented to the teachers was item 15 from the 2012 INVALSI assessments for Grade 10 students, which asked: "*Dividing a number by 0.2 is the same as multiplying it by…*", with the following response options: A) 1/5, B) 1/2, C) 2, and D) 5. At the national level, the correct option D was chosen by 24.4% of students; among the incorrect options, A was selected most frequently (46.1%), followed by B (20.2%) and C (7.7%). Missing or invalid responses accounted for 1.6%.

The item was discussed with an experienced teacher with more than 30 years of service (referred to as T1). In the preliminary questionnaire, T1 had indicated option B as the incorrect option most frequently chosen at the national level, without providing justification. During the interview, when asked to explain her choice, she argued that "The form 1/2 simultaneously conveys the idea of division and includes the number 2, which is given in the problem; the student takes these two elements and finds a solution. This results from a hasty reading." She also added that she initially had been uncertain between options B and C, since both recall the "given" number (2) and might therefore appear appealing to students. She was then shown several protocols representative of different types of errors (corresponding to each of the three incorrect options), but all attributable to intuitive models (Fischbein et al., 1985) whereby students tend to generalize properties of operations they already know to fractions. T1 reported that she would not have expected such errors and, in some cases (e.g., a student choosing A and justifying the choice by computing 1/5*2 = 2/10 = 0.2), stated that she could not reconstruct the reasoning underlying the students' responses. The discrepancy between the teacher's interpretation and what is documented in the literature regarding this type of reasoning suggests a hermeneutical competence still in development, both in identifying the error and in interpreting possible critical steps in the solution process linked to the underlying intuitive model.

The same item was discussed with T2, a first-year high secondary school teacher. She also identified option B as the most likely incorrect option at the national level, ruling out option A on the grounds that "it is unlikely that students would perform a conversion." She was shown the same protocols as T1 but, unlike the latter, regarded the errors as plausible. However, she did not interpret them in

relation to intuitive models; instead, she attributed them to hasty reading: “They do not read the entire text and simply perform a conversion; they write spontaneously.” After reflecting on the protocols and on the national data, T2 reported that she had not anticipated the observed distribution of distractors and acknowledged that she might have underestimated some competencies of Grade 10 students: “Between 1/5 and 1/2, I did not expect so many to know that 0.2 is 1/5; I had this bias.” In this case the reflection on the protocols appears to have contributed, at least partially, to a shift in her hermeneutical competence: although she did not initially identify or interpret the item as expected, the guided reflection prompted her to reconsider her interpretation of the errors discussed.

In conclusion, the interviews highlight a range of ways in which hermeneutical competence manifests among mathematics teachers. While some teachers appeared to struggle in identifying the incorrect options most frequently chosen at the national level and in interpreting students’ errors in relation to them, others, despite misidentifying the most common error, were able to offer plausible interpretations or revise their initial assumptions when engaging with the resources presented. These variations point to different manifestations of hermeneutical competence and show how the combined use of student protocols and national LSAs data helps make teachers’ processes visible and, at times, stimulates them. Overall, findings suggest that interviews constitute an effective means for exploring and potentially fostering this competence within mathematics teacher professional development.

Conclusion

This study outlined the process leading to the definition of the new theoretical construct of hermeneutical competence and the initial steps taken to investigate its manifestations. Questionnaire data and preliminary semi-structured interviews highlighted the complexity of how teachers identify and interpret students’ errors, confirming hermeneutical competence as a distinct, practice-related dimension of what mathematics teachers need to know and be able to do. Findings also point to the need for refined research tools to capture its multifaceted nature and for further studies on how it evolves, how it interacts with teachers’ beliefs, experiences, and knowledge, and which conditions best support its development.

From a teacher-education perspective, the study underscores the relevance of hermeneutical competence for promoting meaningful engagement with students’ mathematical reasoning. As widely documented (Brodie, 2014; Hoth et al., 2022), teachers’ approaches to students’ errors play a decisive role in shaping classroom discourse and learning opportunities. Findings suggest that structured activities involving the analysis of students’ protocols, supported by LSAs data, may constitute valuable tools for fostering this competence within initial teacher education and continuous professional development. Embedding such activities in teacher education programs can help teachers develop more informed, research-grounded interpretations of students’ errors.

In conclusion, hermeneutical competence offers a new theoretical lens for examining how teachers identify and interpret students’ mathematical errors, complementing frameworks such as interpretative knowledge (Ribeiro et al., 2013) and diagnostic competence in error situations (Heinrichs & Kaiser, 2018). Further empirical research will be essential not only to refine and validate the construct, but also to translate it into actionable guidelines and tools that can meaningfully support mathematics teacher education and professional development.

Acknowledgment

Study part of the project "2022CKWY77 - *Mathematics standardized assessment as a tool for teachers' professional development cup.* F53D23006210001" funded by the European Union - NextGenerationEU under the National Recovery and Resilience Plan (PNRR) - Mission 4 Education and research - Component 2 From research to business - Investment 1.1 Notice Prin 2022 - DD N. 104 of 2/2/2022.

References

Arzarello, F., & Ferretti, F. (2022). Links between the INVALSI Mathematics test and teaching practices: an exploratory study. In P. Falzetti (Ed.), *INVALSI data to investigate the characteristics of students, schools and society* (pp. 96–109). FrancoAngeli.

Bolondi, G., & Ferretti, F. (2021). Quantifying solid findings in mathematics education: loss of meaning for algebraic symbols. *International Journal of Innovation in Science and Mathematics Education*, *29*(1), 1–15. https://doi.org/10.30722/IJISME.29.01.001

Bolondi, G., Cibien, M. C., & Spagnolo, C. (2025). How teacher professional development based on Large-Scale Assessment resources can act on teachers' beliefs affecting their teaching practice. *LUMAT-B: International Journal on Math, Science and Technology Education*.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the learning of Mathematics*, *7*(3), 2–8.

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, *85*(2), 221–239. https://doi.org/10.1007/s10649-013-9507-1

Charmaz, K. (1983). The grounded theory method: An explication and interpretation. In R. M. Emerson (Ed.), *Contemporary field research: A collection of readings* (pp. 109–126). Little, Brown.

Cibien, M. C. (2025). Pre-service teachers' professional development and mathematics Large-Scale Assessment. In M. Bosch, G. Bolondi, S. Carreira, C. Spagnolo, & M. Gaidoschik (Eds.), *Proceedings of CERME14* (pp. 3377–3384). Free University of Bozen-Bolzano and ERME.

Ferretti, F., & Bolondi, G. (2019). This cannot be the result! The didactic phenomenon "the Age of the Earth". *International Journal of Mathematical Education in Science and Technology*, *52*(2), 194–207. https://doi.org/10.1080/0020739X.2019.1670366

Figal, G. (2022). The Task of Hermeneutics. *Critical Hermeneutics*, *6*(2), 17–33.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education, 16*(1), 3–17. https://doi.org/10.5951/jresematheduc.16.1.0003

Heinrichs, H., & Kaiser, G. (2018). Diagnostic competence for dealing with students' errors: Fostering diagnostic competence in error situations. In T. Leuders, K. Philipp, & J. Leuders (Eds.), *Diagnostic competence of mathematics teachers: Unpacking a complex construct in teacher*

education and teacher practice (Vol. 11, pp. 79–94). Springer. https://doi.org/10.1007/978-3-319-66327-2_4

Hoth, J., Döhrmann, M., Kaiser, G., Busse, A., König, J., & Blömeke, S. (2016). Diagnostic competence of primary school mathematics teachers during classroom situations. *ZDM – Mathematics Education, 48*(1), 41–53. https://doi.org/10.1007/s11858-016-0759-y

Hoth, J., Larrain, M., & Kaiser, G. (2022). Identifying and dealing with student errors in the mathematics classroom: Cognitive and motivational requirements. *Frontiers in psychology*, *13*, 1057730. https://doi.org/10.3389/fpsyg.2022.1057730

Leuders, T., Dörfler, T., Leuders, J., & Philipp, K. (2017). Diagnostic competence of mathematics teachers: unpacking a complex construct. In T. Leuders, T. Dörfler, J. Leuders, & K. Philipp (Eds.) *Diagnostic competence of mathematics teachers: Unpacking a complex construct in teacher education and teacher practice* (pp. 3–31). Springer International Publishing. https://doi.org/10.1007/978-3-319-66327-2

Patton, M. Q. (1980). *Qualitative Evaluation Methods*. Sage.

Ribeiro, C. M., Mellone, M., & Jakobsen, A. (2013). Characterizing prospective teachers' knowledge in/for interpreting students' solutions. In Ribeiro, C. M., Mellone, M., & Jakobsen, A. (Eds.) *Proceedings of the 37th International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 89–96). PME. https://doi.org/10.33683/ddm.18.3.3.1

Ribeiro, M., Mellone, M., & Jakobsen, A. (2016). Interpreting students' non-standard reasoning: insights for mathematics teacher education. *For the Learning of Mathematics*, *36*(2), 8–13.

Schrader, F. W. (2011). *Lehrer als Diagnostiker [Teachers as diagnosticians]*. In E. Terhart, H. Bennewitz, & M. Rothland (Eds.), *Handbuch der Forschung zum Lehrerberuf* [Handbook of research on the teaching profession] (pp. 683–698). Waxmann Verlag GmbH.

Viola, G., Ferretti, F., Gambini, A., Francesca, M., & Spagnolo, C. (2024). Teachers' professional development and mathematics LSA: first result of national project. In Evans, T., Marmur, O., Hunter, J., Leach, G., & Jhagroo, J. (Eds.), *Proceedings of the 47th International Group for the Psychology of Mathematics Education (*Vol. 1, pp. 223–224). PME

Weinert, F. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. Salganik (Eds.), *Defining and selecting key competencies* (pp. 45–66). Hogrefe and Huber.

Williams, J., & Ryan, J. (2000). National Testing and the Improvement of Classroom Teaching: Can they coexist? *British Educational Research Journal, 26*(1), 49–73. https://doi.org/10.1080/014119200109516

Wuttke, E., & Seifried, J. (2017). Competence, teacher competence and professional error competence: An introduction. In E. Wuttke & J. Seifried (Eds.), *Professional error competence of preservice teachers: Evaluation and support* (pp. 1–14). Springer International Publishing. https://doi.org/10.1007/978-3-319-52649-2_1