


# Novel measurement of the strong-phase difference between $D^0 \rightarrow K^- \pi^+$ and $\bar{D}^0 \rightarrow K^- \pi^+$ decays using $C$ -even and $C$ -odd quantum-correlated $D\bar{D}$ pairs

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A novel measurement technique of strong-phase differences between the decay amplitudes of  $D^0$  and  $\bar{D}^0$  mesons is introduced which exploits quantum-correlated  $D\bar{D}$  pairs produced by  $e^+e^-$  collisions at energies above the  $\psi(3770)$  production threshold, where  $D\bar{D}$  pairs are produced in both even and odd eigenstates of the charge-conjugation symmetry. Employing this technique, the first determination of a  $D^0$ - $\bar{D}^0$  relative strong phase is reported with such data samples. The strong-phase difference between  $D^0 \rightarrow K^- \pi^+$  and  $\bar{D}^0 \rightarrow K^- \pi^+$  decays,  $\delta_{K\pi}^D$ , is measured to be  $\delta_{K\pi}^D = (192.8^{+11.0+1.9}_{-12.4-2.4})^\circ$ , using a dataset corresponding to an integrated luminosity of  $7.13 \text{ fb}^{-1}$  collected at center-of-mass energies between 4.13–4.23 GeV by the BESIII experiment.

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## I. INTRODUCTION

It has long been recognized that quantum-correlated  $D\bar{D}$  pairs produced in  $e^+e^-$  collisions at the charm threshold can be exploited to determine decay properties that are necessary inputs for measurements of  $CP$  violation and charm mixing [1,2] (here a  $D$  denotes a neutral charm meson that is not necessarily in a flavor eigenstate). The CLEO and BESIII Collaborations have reported measurements exploiting quantum-correlated  $D\bar{D}$  pairs produced at the  $\psi(3770)$  resonance in many decay modes, for example in the  $D \rightarrow K^- \pi^+$  decay [3–5]. The  $D\bar{D}$  pairs are produced through the annihilation process,  $e^+e^- \rightarrow \gamma^* \rightarrow c\bar{c}$ , and thus are in an odd eigenstate of the charge-conjugation operator, with eigenvalue  $C = -1$ , due to the intermediate virtual photon. To satisfy  $C$  quantum-number conservation, the amplitude of the  $D\bar{D}$  pair is

$$|D\rangle|\bar{D}\rangle + C|\bar{D}\rangle|D\rangle, \quad (1)$$

and hence the decays of the two  $D$  mesons are correlated. In  $D\bar{D} \rightarrow Y_1$  vs  $Y_2$  decays, where  $Y_1$  and  $Y_2$  are modes accessible to both the  $D^0$  and  $\bar{D}^0$  mesons, interference occurs between the two final state paths  $D^0 \rightarrow Y_1, \bar{D}^0 \rightarrow Y_2$  and  $D^0 \rightarrow Y_2, \bar{D}^0 \rightarrow Y_1$ . The effects on the interference

can be exploited to determine the hadronic parameters associated with the decay amplitudes of  $Y_1$  and  $Y_2$ .

The presence of quantum correlations in  $D\bar{D}$  pairs produced at energies above the charm threshold, where additional particles may be produced, has been previously proposed [6–10]. However, until now, correlations in this regime have never been studied experimentally. In this paper, the  $D\bar{D}$  pairs produced in the  $e^+e^- \rightarrow D\bar{D}$ ,  $e^+e^- \rightarrow D^*\bar{D}$ , and  $e^+e^- \rightarrow D^*\bar{D}^*$  processes, where each  $D^*$  meson decays into a  $D$  meson and a photon or neutral pion, are demonstrated to be quantum correlated using a dataset corresponding to an integrated luminosity of  $7.13 \text{ fb}^{-1}$  collected at center-of-mass energies between 4.13 and 4.23 GeV by the BESIII experiment. Charged conjugation is implied here and throughout the paper.

At higher energies the  $C$ -eigenvalues of the additional particles produced in the  $D^*$  decays dictate that of the  $D\bar{D}$  pair. It naturally follows that the  $D\bar{D}$  pair can be in a  $C$ -even or odd eigenstate. Table I displays the six possible  $D\bar{D}$  production mechanisms for the processes examined in this paper. The  $\gamma$  and  $\pi^0$  particles possess the eigenvalues  $C_\gamma = -1$  and  $C_{\pi^0} = +1$ , respectively, and so that of the  $D\bar{D}$  pair produced in the process,  $e^+e^- \rightarrow D\bar{D} + n\gamma + m\pi^0$ , is given by

$$C = (C_\gamma)^{n+1} \times (C_{\pi^0})^m = (-1)^{n+1}. \quad (2)$$

The quantum-correlated  $D\bar{D}$  pairs reconstructed in the dataset are exploited to measure the strong-phase difference between  $D^0 \rightarrow K^- \pi^+$  and  $\bar{D}^0 \rightarrow K^- \pi^+$  decays, denoted

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TABLE I. Mechanisms by which quantum-correlated  $D\bar{D}$  pairs with eigenvalue  $C$  are produced through the  $e^+e^- \rightarrow D\bar{D}, D^*\bar{D},$  and  $D^*\bar{D}^*$  processes.

Production mechanism	$C$
$e^+e^- \rightarrow D\bar{D}$	-1
$e^+e^- \rightarrow D^*\bar{D} \rightarrow D\bar{D}\gamma$	+1
$e^+e^- \rightarrow D^*\bar{D} \rightarrow D\bar{D}\pi^0$	-1
$e^+e^- \rightarrow D^*\bar{D}^* \rightarrow D\bar{D}\gamma\gamma$	-1
$e^+e^- \rightarrow D^*\bar{D}^* \rightarrow D\bar{D}\pi^0\gamma$	+1
$e^+e^- \rightarrow D^*\bar{D}^* \rightarrow D\bar{D}\pi^0\pi^0$	-1

$\delta_{K\pi}^D$ , and defined through the amplitude ratio,

$$r_{K\pi}^D e^{-i\delta_{K\pi}^D} = \frac{A(\bar{D}^0 \rightarrow K^-\pi^+)}{A(D^0 \rightarrow K^-\pi^+)}, \quad (3)$$

where  $r_{K\pi}^D$  is the magnitude of that ratio. The strong-phase difference  $\delta_{K\pi}^D$  is of particular interest, as it is an auxiliary parameter both in measurements of charm mixing and  $CP$  violation [11,12], and measurements of  $CP$  violation in  $B^\pm \rightarrow DK^\pm$  decays where the  $D$  subsequently decays to the  $K\pi$  final state [13]. The deviation of  $\delta_{K\pi}^D$  from  $\pi$  also provides information on the breaking of  $U$ -spin symmetry in the decays of charmed hadrons [14–17]. The most precise single measurement of this parameter with quantum-correlated  $D\bar{D}$  pairs is  $\delta_{K\pi}^D = (187.6_{-9.7-6.4}^{+8.9+5.4})^\circ$ , where the first uncertainty is statistical and the second is systematic, performed by BESIII at the  $\psi(3770)$  resonance using a dataset corresponding to an integrated luminosity of  $2.93 \text{ fb}^{-1}$  [5]. This measurement, however, is less precise than the indirect determination of  $\delta_{K\pi}^D = (190.2 \pm 2.8)^\circ$ , obtained by the LHCb Collaboration from a fit to an ensemble of charm mixing and  $b$ -hadron decay data [18]. Improved measurements with quantum-correlated  $D\bar{D}$  pairs are desirable to test the consistency of both determinations, and to maximize the precision with which  $CP$ -violating and mixing parameters can be determined from external measurements of beauty and charm decays.

The decay rate for the process  $D\bar{D} \rightarrow Y_1$  vs  $Y_2$  is central to the discussion throughout the paper, and it is given by

$$\begin{aligned} & \frac{\Gamma(D\bar{D} \rightarrow Y_1 \text{ vs } Y_2)}{A_1^2 A_2^2} \\ &= [r_1^2 + r_2^2 + 2CR_1R_2r_1r_2 \cos(\delta_1 - \delta_2)] \\ & \quad - (1+C)y[R_1r_1 \cos \delta_1(1+r_2^2) + R_2r_2 \cos \delta_2(1+r_1^2)] \\ & \quad - (1+C)x[R_1r_1 \sin \delta_1(1-r_2^2) + R_2r_2 \sin \delta_2(1-r_1^2)] \\ & \quad + \mathcal{O}(x^2, y^2), \end{aligned} \quad (4)$$

where  $x$  and  $y$  are the charm-mixing parameters defined in Ref. [18], and  $A_i, r_i, R_i,$  and  $\delta_i$  are defined as

$$A_i^2 = \int |A(D^0 \rightarrow Y_i)|^2 d\mathbf{p}, \quad (5)$$

$$\bar{A}_i^2 = \int |A(\bar{D}^0 \rightarrow Y_i)|^2 d\mathbf{p}, \quad (6)$$

$$r_i^2 = \frac{\bar{A}_i^2}{A_i^2}, \quad (7)$$

$$R_i e^{i\delta_i} = \frac{\int A(D^0 \rightarrow Y_i)A(\bar{D}^0 \rightarrow Y_i)^* d\mathbf{p}}{\sqrt{A_i^2 \bar{A}_i^2}}, \quad (8)$$

and  $d\mathbf{p}$  is an element of the phase space of the decay. One important difference between the decay rates of  $C$ -even and -odd  $D\bar{D}$  pairs is that the former are affected by charm mixing at first order, whilst the dependence is quadratic for those in a  $C$ -odd eigenstate.

The remainder of the paper is organized as follows. First, the BESIII detector is described in Sec. II. Then, the selection requirements are outlined in Sec. III, with a focus on those which are used to isolate the  $D\bar{D}$  pairs produced through each production mechanism in Table I. Following this, Sec. IV presents a demonstration of the quantum correlations in each process using final states which are forbidden when  $C = 1$  and enhanced when  $C = -1$ , and vice versa. Finally, the strong-phase difference  $\delta_{K\pi}^D$  is measured in Secs. V–VII, before a combination with the results of Ref. [5] is detailed in Sec. VIII.

## II. BESIII DETECTOR, SIMULATION, AND DATA SAMPLES

The BESIII detector [19] records symmetric  $e^+e^-$  collisions provided by the BEPCII storage ring [20] in the center-of-mass energy range from 1.84 to 4.95 GeV, with a peak luminosity of  $1.1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  achieved at  $\sqrt{s} = 3.773 \text{ GeV}$ . BESIII has collected large data samples in this energy region [21]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/ $c$  is 0.5%, and the  $dE/dx$  resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution in the plastic scintillator TOF barrel region is 68 ps, while that in the end-cap region was 110 ps. The end cap TOF system was upgraded in 2015 using multigap resistive plate chamber technology,

providing a time resolution of 60 ps, which benefits  $\sim 85\%$  of the data used in this analysis [22–24].

Monte Carlo (MC) simulated data samples produced with a GEANT4-based [25] software package, which includes the geometric description of the BESIII detector and the detector response, are used to determine detection efficiencies and to estimate backgrounds. The simulation models the beam-energy spread and initial-state radiation (ISR) in the  $e^+e^-$  annihilations with the generator KKMC [26]. The generator does not include the quantum correlations. Corrections are applied for this where required for background estimates. The inclusive MC sample includes the production of open-charm processes, the ISR production of vector charmonium(-like) states, and the continuum processes incorporated in KKMC [26]. All particle decays are modeled with EVTGEN [27] using branching fractions either taken from the Particle Data Group (PDG) [28], when available, or otherwise estimated with LUNDCHARM [29,30]. Final-state radiation from charged final-state particles is incorporated using the PHOTOS package [31].

### III. SELECTION REQUIREMENTS

The selection of candidates is divided into two stages. First, the  $D$ -meson decays are reconstructed in six final states,  $K^\pm\pi^\mp$ ,  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $\pi^+\pi^-\pi^0$ ,  $K_S^0\pi^0$ , and  $K_S^0\pi^+\pi^-$ . The  $D$ -decay candidates are combined to form  $D\bar{D}$  pairs, from which a single best candidate is selected per event through the criteria described later in this section. Then, requirements are placed on the pairs to distinguish between the various  $D\bar{D}$  production mechanisms.

Charged tracks detected in the MDC are required to be within a polar angle ( $\theta$ ) range of  $|\cos\theta| < 0.93$ , where  $\theta$  is defined with respect to the  $z$  axis, which is the symmetry axis of the MDC. Furthermore, the distance of closest approach to the interaction point (IP) must be less than 10 cm along the  $z$  axis,  $|V_z|$ , and less than 1 cm in the transverse plane,  $|V_{xy}|$ . Particle identification (PID) for charged tracks combines measurements of the energy deposited in the MDC ( $dE/dx$ ) and the flight time in the TOF to form likelihoods  $\mathcal{L}(h)$  ( $h = p, K, \pi$ ) for each hadron  $h$  hypothesis. Tracks are identified as kaons and pions by comparing the likelihoods for the kaon and pion hypotheses,  $\mathcal{L}(K) > \mathcal{L}(\pi)$  and  $\mathcal{L}(\pi) > \mathcal{L}(K)$ , respectively.

The  $\pi^0$  mesons are reconstructed through the  $\pi^0 \rightarrow \gamma\gamma$  decay. Photon candidates are identified using isolated showers in the EMC. The deposited energy of each shower must be more than 25 MeV in the barrel region ( $|\cos\theta| < 0.80$ ) and more than 50 MeV in the end cap region ( $0.86 < |\cos\theta| < 0.92$ ). To exclude showers that originate from charged tracks, the angle subtended by the EMC shower and the position of the closest charged track at the EMC must be greater than ten degrees as measured from the IP. To suppress electronic noise and showers unrelated to the event, the difference between the EMC

time and the event start time is required to be within [0, 700] ns. The  $\pi^0$  candidates are reconstructed from two photons, where at least one produces a shower in the barrel of the EMC to provide the necessary energy resolution. The reconstructed invariant mass of the  $\gamma\gamma$  pair must be  $M_{\gamma\gamma} \in [115, 150]$  MeV/ $c^2$ . Finally, a kinematic fit constraining the two photons to the known  $\pi^0$  mass [28] is performed which is required to converge with a loose requirement on the  $\chi^2$  of the fit.

Each  $K_S^0$  candidate is reconstructed from two oppositely charged tracks satisfying  $|V_z| < 20$  cm. The two charged tracks are assigned as  $\pi^+\pi^-$  without imposing further PID criteria. They are constrained to originate from a common vertex and are required to have an invariant mass  $M_{\pi^+\pi^-}$  satisfying  $|M_{\pi^+\pi^-} - m_{K_S^0}| < 12$  MeV/ $c^2$ , where  $m_{K_S^0}$  is the known  $K_S^0$  mass [28]. A secondary-vertex fit is performed to the  $K_S^0$  children tracks which must converge with a loose requirement on the  $\chi^2$  of the fit.

The  $K_S^0$  mesons used to form  $D \rightarrow K_S^0\pi^0$  candidates are subject to flight-distance significance requirements to reject background from  $D \rightarrow \pi^+\pi^-\pi^0$  decays. Specifically, the decay length of the  $K_S^0$  candidate from the IP is required to be greater than twice the vertex resolution. The opposite requirement is placed on the  $\pi^+\pi^-$  pair in the selection of  $D \rightarrow \pi^+\pi^-\pi^0$  candidates.

In each event the  $D$ -decay candidates are reconstructed using the above criteria. From these, a single best  $D\bar{D}$  candidate is chosen to be that with an average reconstructed invariant mass closest to the known  $D^0$  mass [28], which is subsequently used to discriminate between the different production mechanisms. The strategy is first to isolate the  $e^+e^- \rightarrow D\bar{D}$ ,  $e^+e^- \rightarrow D^*\bar{D}$ , and  $e^+e^- \rightarrow D^*\bar{D}^*$  processes, and then the possible  $D^*$  decays.

The isolation begins by applying a requirement on the missing energy,  $E_{\text{miss}}$ , defined as the difference between the center-of-mass energy and that of the  $D\bar{D}$  pair,

$$E_{\text{miss}} \equiv E_{\text{cm}} - E_{D\bar{D}}, \quad (9)$$

where  $E_{\text{cm}}$  is the collision energy in the center-of-mass frame and  $E_{D\bar{D}}$  is the energy of the  $D\bar{D}$  pair calculated using the four-momentum,  $p$ , of each particle determined in an event-wide kinematic fit where all the hypothesized intermediate particles ( $K_S^0, \pi^0, D^0, D^{*0}$ ), are constrained to their known masses,  $m$ , from the PDG [28]. Figure 1(a) shows the  $E_{\text{miss}}$  distribution of  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  candidates in simulation. This final state is used throughout the paper as a control channel due to the large branching fraction and reconstruction efficiency of the  $D^0 \rightarrow K^-\pi^+$  decay. Furthermore, both decays in the  $D^0 \rightarrow K^+\pi^-, \bar{D}^0 \rightarrow K^-\pi^+$  path to the final state are doubly Cabibbo suppressed compared to the alternative,  $D^0 \rightarrow K^-\pi^+, \bar{D}^0 \rightarrow K^+\pi^-$ , so the quantum correlations have little impact on the decay rate. In Fig. 1(a), the missing energy peaks at zero for

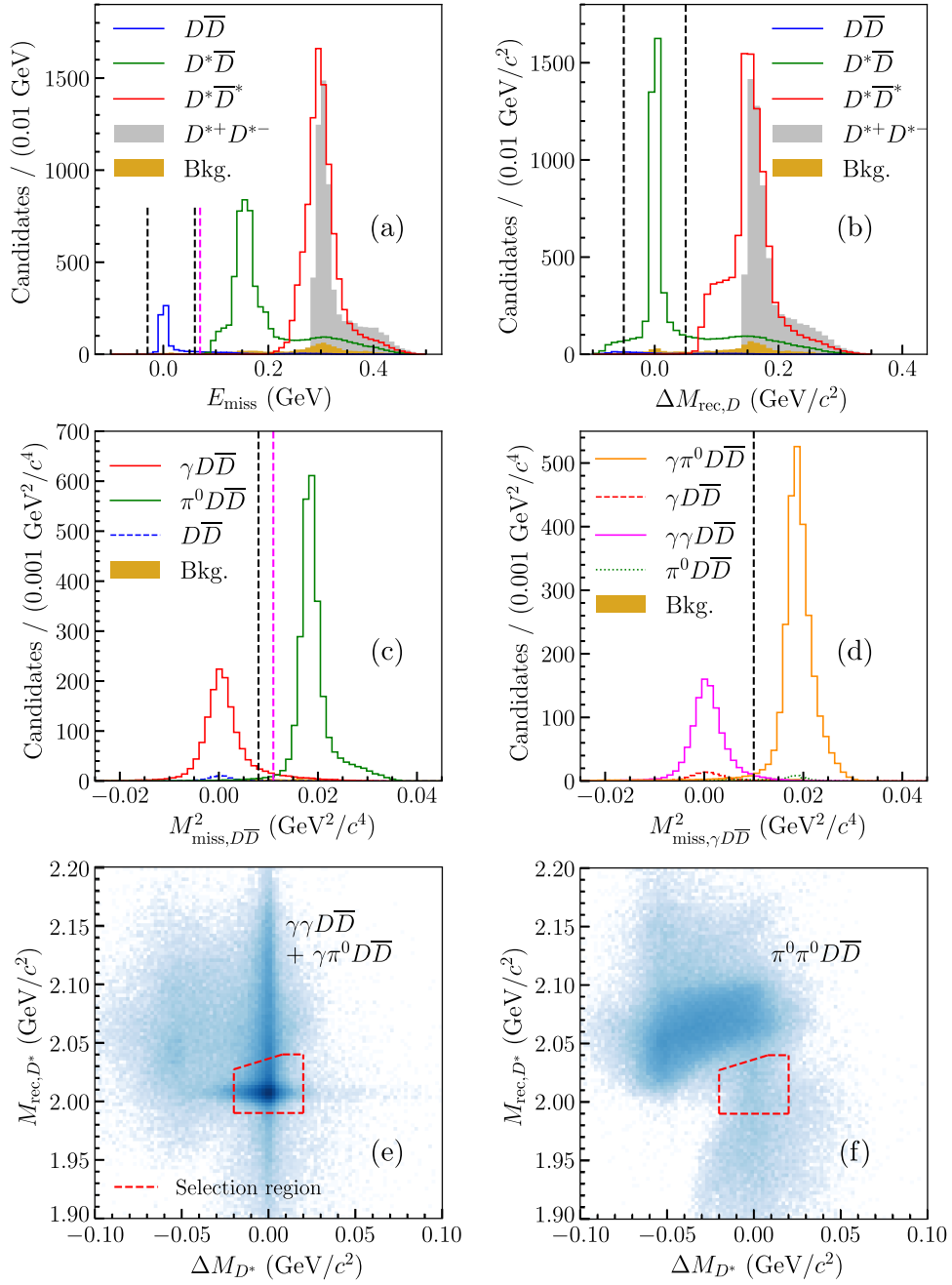


FIG. 1. The (a)  $E_{\text{miss}}$ , (b)  $M_{\text{rec},D}$ , (c)  $M^2_{\text{miss},D\bar{D}}$ , (d)  $M^2_{\text{miss},\gamma D\bar{D}}$  distributions of  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  candidates in MC simulation samples. The  $\Delta M_{D^*}$  vs  $M_{\text{rec},D^*}$  distribution of simulated  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  decays which originate from the (e)  $D^*\bar{D}^* \rightarrow \gamma\gamma D\bar{D}$  and  $D^*\bar{D}^* \rightarrow \gamma\pi^0 D\bar{D}$  and (f)  $D^*\bar{D}^* \rightarrow \pi^0\pi^0 D\bar{D}$  processes are also shown. The vertical dashed lines are used to distinguish between production mechanisms and correspond to the requirements in Table II.

candidates that originate from the  $e^+e^- \rightarrow D\bar{D}$  process, but is larger for the  $e^+e^- \rightarrow D^*\bar{D}$  and  $e^+e^- \rightarrow D^*\bar{D}^*$  processes due to the missing light neutral particles from the  $D^*$  decays.

The minimum difference between the recoil mass of each  $D$  meson and the known mass of the  $D^{*0}$  meson,

$$\Delta M_{\text{rec},D} \equiv \min(|M_{\text{rec},D_1} - m_{D^{*0}}|, |M_{\text{rec},D_2} - m_{D^{*0}}|), \quad (10)$$

is used to distinguish between the  $e^+e^- \rightarrow D^*\bar{D}$  and  $e^+e^- \rightarrow D^*\bar{D}^*$  processes. The simulated  $\Delta M_{\text{rec},D}$  distribution for  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  candidates that pass the  $E_{\text{miss}} > 70$  MeV requirement, to remove the contribution from the  $e^+e^- \rightarrow D\bar{D}$  process, can be seen in Fig. 1(b). The recoil mass,  $M_{\text{rec},D}$ , is the invariant mass of the other particles in the event after one of the  $D$  meson decays is reconstructed with momentum  $\vec{p}_D$ ,

$$c^2 M_{\text{rec},D} = \left[ \left( E_{\text{cm}}^2 - \sqrt{c^2 |\vec{p}_D|^2 + m_{D^0}^2} \right)^2 - |c^2 \vec{p}_D|^2 \right]^{\frac{1}{2}}. \quad (11)$$

It is possible to use  $E_{\text{miss}}$  to separate the  $D^* \bar{D}$  processes from  $D^* \bar{D}^*$ , however,  $M_{\text{rec},D}$  displays a better resolution because it only relies on the kinematic information of a single  $D$  meson from each event.

It is evident from Fig. 1 that the requirements placed do not perfectly isolate each production mechanism. The primary cause is events with a reduced center-of-mass energy due to ISR, which produce the long tails that are clearly visible in the  $E_{\text{miss}}$  and  $M_{\text{rec},D}$  distributions. The cross feed between production mechanisms is accounted for in the later steps of the analysis as discussed in Sec. IV.

Two sources of backgrounds pass the  $E_{\text{miss}}$  and  $M_{\text{rec},D}$  requirements. Firstly, there are contributions from incorrectly reconstructed  $D$  decays, which peak underneath the signal due to the  $D$ -meson mass constraints applied when calculating  $E_{\text{miss}}$  and  $M_{\text{rec},D}$ . These are subtracted from the samples through fits to the reconstructed invariant masses of both  $D$ -decays as discussed in Sec. IV. Secondly,  $D^0 \bar{D}^0$  pairs which decay to the correct final state and are produced in the  $e^+ e^- \rightarrow D^{*+} D^{*-}$  process, where  $D^{*+} \rightarrow \pi^+ D^0$ , pass the selection requirements to isolate the  $D \bar{D}$  pairs from  $D^* \bar{D}^*$  production with large rates. These are indistinguishable from signal in the aforementioned fits. However, they are significantly suppressed by requiring no charged tracks be found in addition to those used to reconstruct the  $D \bar{D}$  pair and by requiring all pion candidates in the event have a tracked momentum greater than 100 MeV/ $c$ .

The selection efficiency of the neutral particle from the  $D^*$  decay is low. Therefore, to maximize the signal yields, the  $D \bar{D}$  production mechanisms that originate from the  $D^* \bar{D}$  and  $D^* \bar{D}^*$  processes are distinguished by partial reconstruction. For example, to discriminate between the  $D^{*0} \rightarrow \gamma D^0$  and  $D^{*0} \rightarrow \pi^0 D^0$  decays that originate from the  $e^+ e^- \rightarrow D^* \bar{D}$  process, a missing mass variable, defined as

$$c^4 M_{\text{miss},D\bar{D}}^2 \equiv E_{\text{miss}}^2 - c^2 |\vec{p}_{D_1} + \vec{p}_{D_2}|^2, \quad (12)$$

is used. Figure 1(c) shows the  $M_{\text{miss},D\bar{D}}^2$  distribution in simulation.

Up to this point, separation between the  $D \bar{D}$ ,  $D^* \bar{D} \rightarrow \gamma D \bar{D}$ , and  $D^* \bar{D} \rightarrow \pi^0 D \bar{D}$  production mechanisms has been achieved using three discriminating variables which only require reconstruction of the  $D \bar{D}$  pair. However, the same strategy is much less effective in isolating the three production mechanisms that arise from  $D^* \bar{D}^*$  decays due to the two additional neutral particles. Again, to maintain a high efficiency a partial reconstruction technique is employed. A single  $D^* \rightarrow \gamma D$  candidate from each event, chosen to be that which has a reconstructed invariant mass closest to the  $D^{*0}$  from all combinations of

the two selected  $D$  mesons and all photon candidates, is used to distinguish the  $D^* \bar{D}^* \rightarrow \gamma \gamma D \bar{D}$  and  $D^* \bar{D}^* \rightarrow \pi^0 \gamma D \bar{D}$  production mechanisms from  $D^* \bar{D}^* \rightarrow \pi^0 \pi^0 D \bar{D}$ . This is achieved by selecting a region in the two-dimensional phase space defined by the difference between the reconstructed invariant mass of the  $D^*$  candidate and the PDG value [28], denoted  $\Delta M_{D^*}$ , and the recoil mass of the  $D^*$  candidate,  $M_{\text{rec},D^*}$ , which is defined similarly to Eq. (11).

For  $D \bar{D}$  pairs produced in  $D^* \bar{D}^* \rightarrow \gamma \gamma D \bar{D}$  and  $D^* \bar{D}^* \rightarrow \pi^0 \gamma D \bar{D}$  decays, the corresponding distributions will peak around  $\Delta M_{D^*} \simeq 0$  GeV/ $c^2$  and  $M_{\text{rec},D^*} \simeq m_{D^*}$ , as shown in Fig. 1(e). The long tail at  $\Delta M_{D^*} \simeq 0$  and high  $M_{\text{rec},D^*}$  occurs due to ISR, whilst the other, shorter tails are caused by the detector resolution. For the  $D^* \bar{D}^* \rightarrow \pi^0 \pi^0 D \bar{D}$  production mechanism, the  $D^*$  meson is reconstructed by combining a  $D$  meson with a random photon, and thus the corresponding distribution will peak in a displaced region dictated by the momentum distribution of the photon, as can be seen in Fig. 1(f). Due to the correlation of the  $\Delta M_{D^*}$  and  $M_{\text{rec},D^*}$  variables for  $D^* \bar{D}^* \rightarrow \pi^0 \pi^0 D \bar{D}$  events, nonrectangular selections are applied to remove misidentified  $D^* \bar{D}^* \rightarrow \pi^0 \pi^0 D \bar{D}$  events. The  $D^0 \bar{D}^0$  pairs produced in the  $D^{*+} D^{*-}$  process show the same displacement, and therefore they are effectively removed by the same selections from the  $\gamma \gamma D \bar{D}$  and  $\gamma \pi^0 D \bar{D}$  samples.

The  $D^* \bar{D}^* \rightarrow \gamma \gamma D \bar{D}$  production mechanism is distinguished from  $D^* \bar{D}^* \rightarrow \pi^0 \gamma D \bar{D}$  using the missing mass after reconstructing the  $D$  and  $\bar{D}^*$  mesons,

$$c^4 M_{\text{miss},\gamma D \bar{D}}^2 \equiv (E_{\text{cm}} - E_{D \bar{D}^*})^2 - c^2 |\vec{p}_D + \vec{p}_{D^*}|^2. \quad (13)$$

Finally, two techniques can be applied to reconstruct  $D \bar{D}$  pairs produced in  $D^* \bar{D}^* \rightarrow \pi^0 \pi^0 D \bar{D}$  decays. The first proceeds by reconstructing a single  $D^{*0} \rightarrow \pi^0 D^0$  candidate. However, due to the low momenta of the photons produced in the  $\pi^0 \rightarrow \gamma \gamma$  decay, the reconstruction efficiency is too low. The second attempt examines the candidates that do not pass the requirements placed to reconstruct the other production mechanisms originating from the  $D^* \bar{D}^*$  process. However, the irreducible background from the  $D^{*+} D^{*-}$  process is deemed to be too large. Therefore, in this analysis, the  $e^+ e^- \rightarrow \pi^0 \pi^0 D \bar{D}$  production mechanism is not explicitly reconstructed. The small amount that remains in the sample are incorporated as signal alongside the  $e^+ e^- \rightarrow \gamma \gamma D \bar{D}$  process where the  $D \bar{D}$  pair has the same  $C$ -eigenvalue. Together they are referred to as  $D^* \bar{D}^* \rightarrow \gamma \gamma / \pi^0 \pi^0 D \bar{D}$ .

Table II summarizes the selection requirements applied to isolate the  $D \bar{D}$  pairs. As mentioned above, and shown in Fig. 1, the criteria imposed do not isolate pure samples, and there is non-negligible cross feed between the production mechanisms. Table III displays the efficiencies determined

TABLE II. The  $D\bar{D}$  pair production mechanisms of interest, their expected  $C$  eigenvalues, and the criteria applied to isolate each sample. A ‘✓’ in the  $\Delta M_{D^*}$  vs  $M_{\text{rec},D^*}$  column means that candidates are required to be in the red-dashed region displayed in Fig. 1(e).

Production mechanism [C]	$E_{\text{miss}}$ (GeV)	$\Delta M_{\text{rec},D}$ (GeV/ $c^2$ )	$M_{\text{miss},D\bar{D}}^2$ (GeV/ $c^2$ ) <sup>2</sup>	$\Delta M_{D^*}$ vs $M_{\text{rec},D^*}$ region	$M_{\text{miss},\gamma D\bar{D}}^2$ (GeV/ $c^2$ ) <sup>2</sup>	$P_{\pi,\text{slow}}$ (MeV/ $c$ )
$D\bar{D}$ [-1]	[-0.030, 0.060]	...	...	✗	...	...
$D^*\bar{D} \rightarrow \gamma D\bar{D}$ [+1]	> 0.070	[-0.050, 0.050]	< 0.008	✗	...	...
$D^*\bar{D} \rightarrow \pi^0 D\bar{D}$ [-1]	> 0.070	[-0.050, 0.050]	> 0.011	✗	...	...
$D^*\bar{D}^* \rightarrow \gamma \pi^0 D\bar{D}$ [+1]	> 0.070	> 0.070	...	✓	< 0.01	> 100
$D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0 \pi^0 D\bar{D}$ [-1]	> 0.070	> 0.070	...	✓	> 0.01	> 100

TABLE III. Efficiency, in percent, of reconstructing the  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  final state in simulation. An element with row  $i$  and column  $j$  gives the efficiency of reconstructing the production mechanism,  $i$ , after applying the selection requirements devised to isolate  $D\bar{D}$  pairs produced by process  $j$ .

		Identified as				
		$D\bar{D}$	$D^*\bar{D} \rightarrow \gamma D\bar{D}$	$D^*\bar{D} \rightarrow \pi^0 D\bar{D}$	$D^*\bar{D}^* \rightarrow \gamma \pi^0 D\bar{D}$	$D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0 \pi^0 D\bar{D}$
True	$D\bar{D}$	32.45	2.98	0	0	0.07
	$D^*\bar{D} \rightarrow \gamma D\bar{D}$	0	25.44	1.57	0	1.94
	$D^*\bar{D} \rightarrow \pi^0 D\bar{D}$	0	0.40	30.06	0.55	0.50
	$D^*\bar{D}^* \rightarrow \gamma \pi^0 D\bar{D}$	0	0	0	25.23	1.04
	$D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0 \pi^0 D\bar{D}$	0	0	0	0.25	8.83

in simulation of reconstructing a particular production mechanism given the selection requirements applied to isolate another. It should be noted that the efficiency of correctly reconstructing  $D\bar{D}$  pairs from the  $e^+e^- \rightarrow \gamma\gamma D\bar{D}$  production mechanism with the designed selection requirements is relatively high ( $\sim 25\%$ ), but it is reduced in Table III because the definition of the  $D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0 \pi^0 D\bar{D}$  production mechanism includes the  $e^+e^- \rightarrow \pi^0 \pi^0 D^*\bar{D}^*$  process.

Good agreement is found between the distributions of the selection variables in data and MC simulation samples, as shown in Fig. 2. Minor differences are observed due to the imperfect modeling of the resolution and the limited knowledge of the input cross sections and branching fractions. The consequences of these differences are probed as systematic uncertainties, as discussed in Sec. VI, but are found to be small because the choice of observables leads to weak sensitivity to the related assumptions.

#### IV. DEMONSTRATION OF QUANTUM COHERENCE

To demonstrate the quantum correlations in  $D\bar{D}$  pairs for the different production mechanisms, final states that are expected to be enhanced or forbidden are analyzed. Specifically, a coherence parameter,  $\kappa$ , is determined for each production mechanism of interest by comparing the observed signal yields, after applying the relevant selection requirements, with the predictions assuming the decays of the  $D$  and  $\bar{D}$  mesons are independent. If the observed yields

are consistent with containing contributions solely from quantum-correlated  $D$  meson pairs, then  $\kappa = 1$ , and  $\kappa = 0.5$  if the  $D\bar{D}$  pairs are completely uncorrelated.

Expressing the amplitude of the quantum-correlated  $D\bar{D}$  pair from Eq. (1) in terms of the  $D$  meson  $CP$ -even and -odd eigenstates, denoted  $D_E$  and  $D_O$ , respectively, gives

$$|D_O\rangle|D_E\rangle - |D_E\rangle|D_O\rangle \quad \text{when } C = -1, \quad (14)$$

$$|D_E\rangle|D_E\rangle - |D_O\rangle|D_O\rangle \quad \text{when } C = +1, \quad (15)$$

from which it is evident that the decay into two final states with the same  $CP$  is forbidden (enhanced) when  $C = -1$  (+1). The opposite is true for decays into a mixed  $CP$  final state.

To demonstrate the quantum correlations, the  $D\bar{D}$  pairs are reconstructed in combinations of decays into the two  $CP$ -even final states,  $D \rightarrow K^+K^-$  and  $D \rightarrow \pi^+\pi^-$ , the  $CP$ -odd  $D \rightarrow K_S^0\pi^0$  decay, and  $D \rightarrow \pi^+\pi^-\pi^0$ . The last mode is not a  $CP$  eigenstate but has a  $CP$ -even fraction of  $F_+^{\pi\pi\pi^0} = 0.9406 \pm 0.0042$  [32], and can therefore be used as a quasi  $CP$ -even eigenstate after including a small correction to account for the  $CP$ -odd content. Five final states consisting of two  $CP$  eigenstates are examined, as displayed in Table IV. The other combinations of  $D$  decays are not used, either because the purity of the sample is deemed to be too low or it provides limited additional sensitivity. The effects of the quantum correlations in each double  $CP$ -eigenstate final state are already apparent from Fig. 3

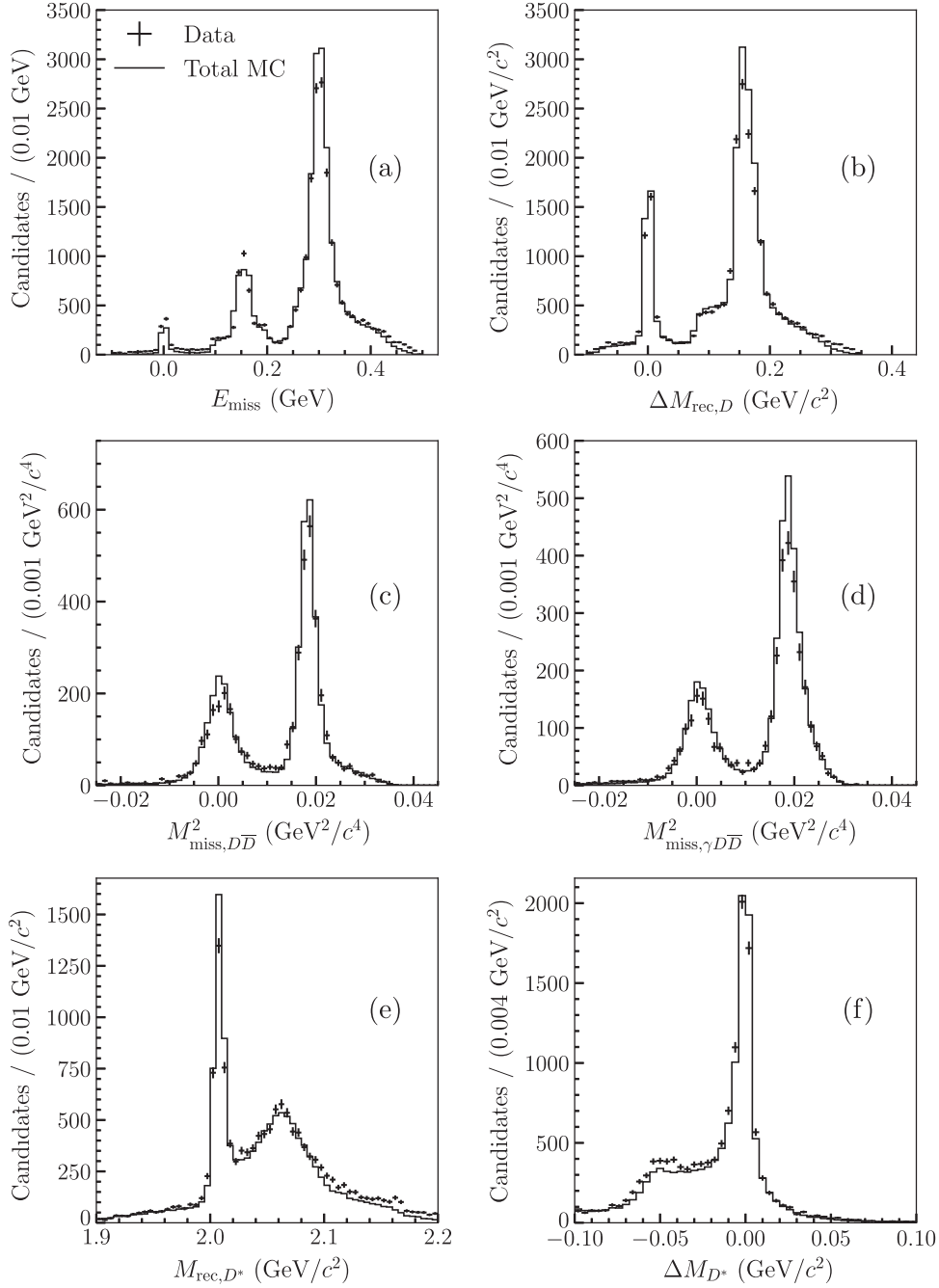


FIG. 2. Comparisons between the (a)  $E_{\text{miss}}$ , (b)  $M_{\text{rec},D}$ , (c)  $M_{\text{miss},D\bar{D}}^2$ , (d)  $M_{\text{miss},\gamma D\bar{D}}^2$ , (e)  $M_{\text{rec},D^*}$ , and (f)  $\Delta M_{D^*}$  distributions of  $D\bar{D} \rightarrow K^-\pi^+ \text{ vs } K^+\pi^-$  candidates in data and MC simulation samples that contain signal and background contributions.

without a quantitative analysis. The plots compare the  $M_{\text{miss},D\bar{D}}^2$  distributions of the candidates in data with the prediction from simulation assuming the decays of the  $D$  and  $\bar{D}$  mesons are independent. For the  $CP$ -even final states, such as  $K_S^0\pi^0 \text{ vs } K_S^0\pi^0$ , the data are clearly suppressed relative to the prediction from simulation in the region  $M_{\text{miss},D\bar{D}}^2 > 0.01 \text{ GeV}^2/c^4$  where the  $C$ -odd  $D\bar{D}$  pairs produced through the  $D^*\bar{D} \rightarrow \pi^0 D\bar{D}$  production mechanism lie. The inverse behavior, where the data are enhanced

TABLE IV. The double  $CP$ -eigenstate decays used throughout the measurement.

$D\bar{D} \rightarrow$	$CP$ eigenvalue
$\pi^+\pi^-\pi^0 \text{ vs } K_S^0\pi^0$	-1
$\pi^+\pi^-\pi^0 \text{ vs } K^+K^-$	+1
$K_S^0\pi^0 \text{ vs } K_S^0\pi^0$	+1
$K_S^0\pi^0 \text{ vs } K^+K^-$	-1
$K_S^0\pi^0 \text{ vs } \pi^+\pi^-$	-1

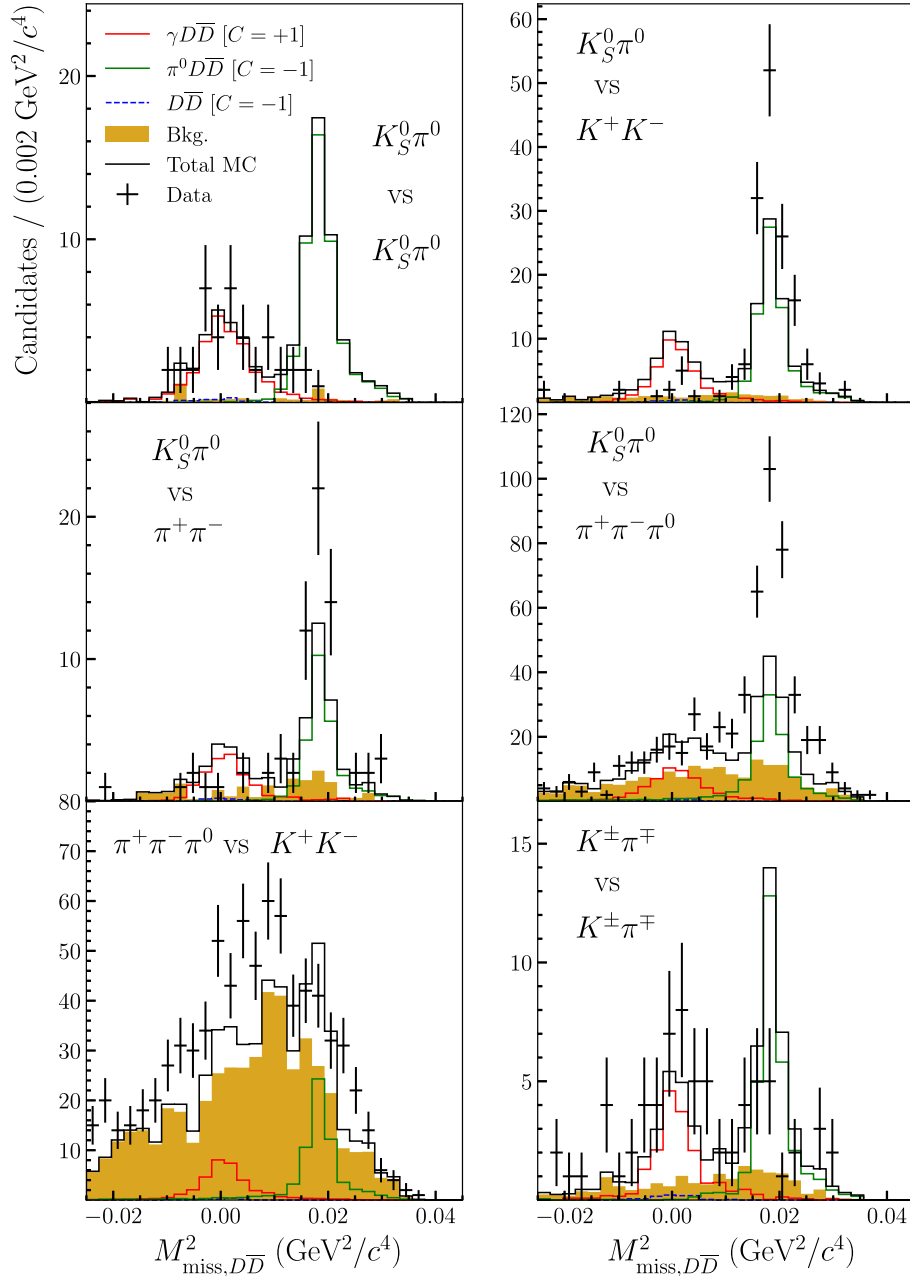


FIG. 3. Comparisons between the  $M_{\text{miss}, D\bar{D}}^2$  distributions in data and MC simulation samples for each of the final states used to demonstrate the  $C$ -coherence of the  $D\bar{D}$  pairs. The data are enhanced or suppressed relative to the MC samples, which does not account for quantum correlations, depending on the final state and the expected  $C$ -eigenvalue of the  $D\bar{D}$  pairs that can be found in a particular region.

relative to the simulation, is seen in the region  $M_{\text{miss}, D\bar{D}}^2 < 0.01 \text{ GeV}^2/c^4$  where the  $C$ -even  $D\bar{D}$  pairs originating from the  $D^* \bar{D} \rightarrow \gamma D\bar{D}$  process are found. The opposite regions are enhanced and suppressed in data for the  $CP$ -odd final states, such as  $K_S^0 \pi^0$  vs  $K^+ K^-$ . Further examples of enhancement and suppression in the data can be seen in the  $M_{\text{miss}, \gamma D\bar{D}}^2$  distributions used to distinguish between the  $D^* \bar{D}^* \rightarrow \gamma \pi^0 D\bar{D}$  and  $D^* \bar{D}^* \rightarrow \gamma \gamma / \pi^0 \pi^0 D\bar{D}$  processes, which are displayed in Fig. 4.

The  $D\bar{D} \rightarrow K^\pm \pi^\mp$  vs  $K^\pm \pi^\mp$  decay channel also provides a test of the coherence of each production mechanism. For this final state,  $r_1 = r_2 = r_{K\pi}^D$ ,  $\delta_1 = \delta_2 = \delta_{K\pi}^D$ , and  $R_1 = R_2 = 1$ . Therefore with  $C = +1$  Eq. (4) becomes,

$$4(r_{K\pi}^D)^2 \left( 1 - \frac{(x \sin \delta_{K\pi}^D + y \cos \delta_{K\pi}^D)}{r_{K\pi}^D} \right), \quad (16)$$

whilst it is suppressed to  $\mathcal{O}(x^2, y^2)$  when  $C = -1$ , and hence it has never been observed in quantum-correlated

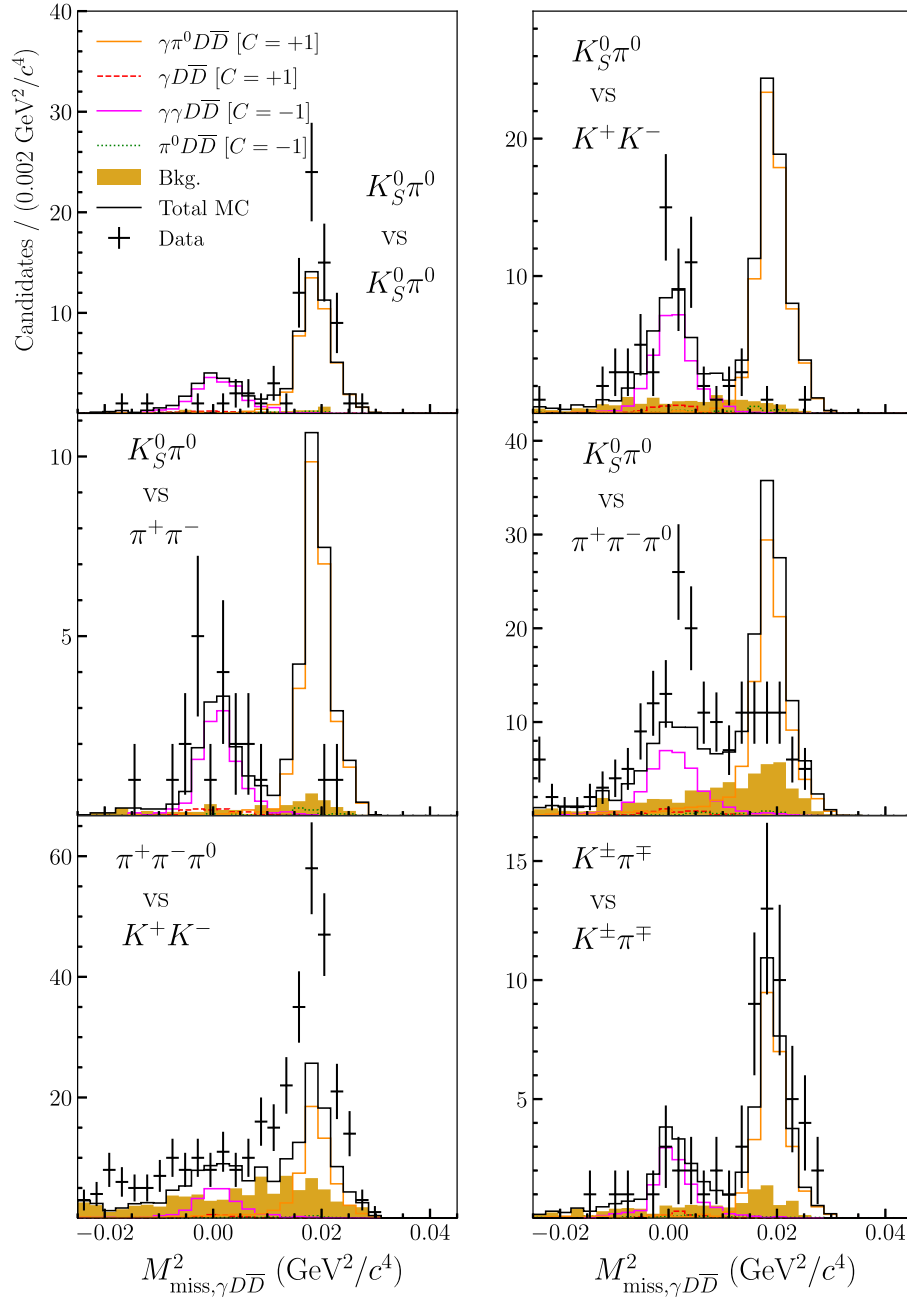


FIG. 4. Comparisons between the  $M^2_{\text{miss}, \gamma D\bar{D}}$  distributions in data and MC simulation samples for each of the final states used to demonstrate the  $C$ -coherence of the  $D\bar{D}$  pairs. The data are enhanced or suppressed relative to the MC samples, which does not account for quantum correlations, depending on the final state and the expected  $C$ -eigenvalue of the  $D\bar{D}$  pairs that can be found in a particular region.

$D\bar{D}$  pairs at the charm threshold. The enhancement from the mixing terms in the above equation is  $\mathcal{O}(10\%)$  making the  $D\bar{D} \rightarrow K^\mp \pi^\pm$  vs  $K^\pm \pi^\mp$  decays from  $C$ -even  $D\bar{D}$  pairs a promising channel for future measurements of  $x$  and  $y$  [33].

The signal yields for each final state are determined after applying the selection requirements for each production mechanism through a two-dimensional fit to the reconstructed invariant mass distributions of both  $D$  candidates.

Each mass distribution contains contributions from correctly reconstructed  $D$  decays and combinatorial background. Therefore, each fit has at least four components given by the various combinations of contributions to the mass distributions for the  $D$  decays. In the fit, the yield of each of these components is a floating parameter. Furthermore, correctly reconstructed  $D$  decays are modeled by a kernel density estimation (KDE) [34] of the distribution in simulation, convolved with a Gaussian with floating

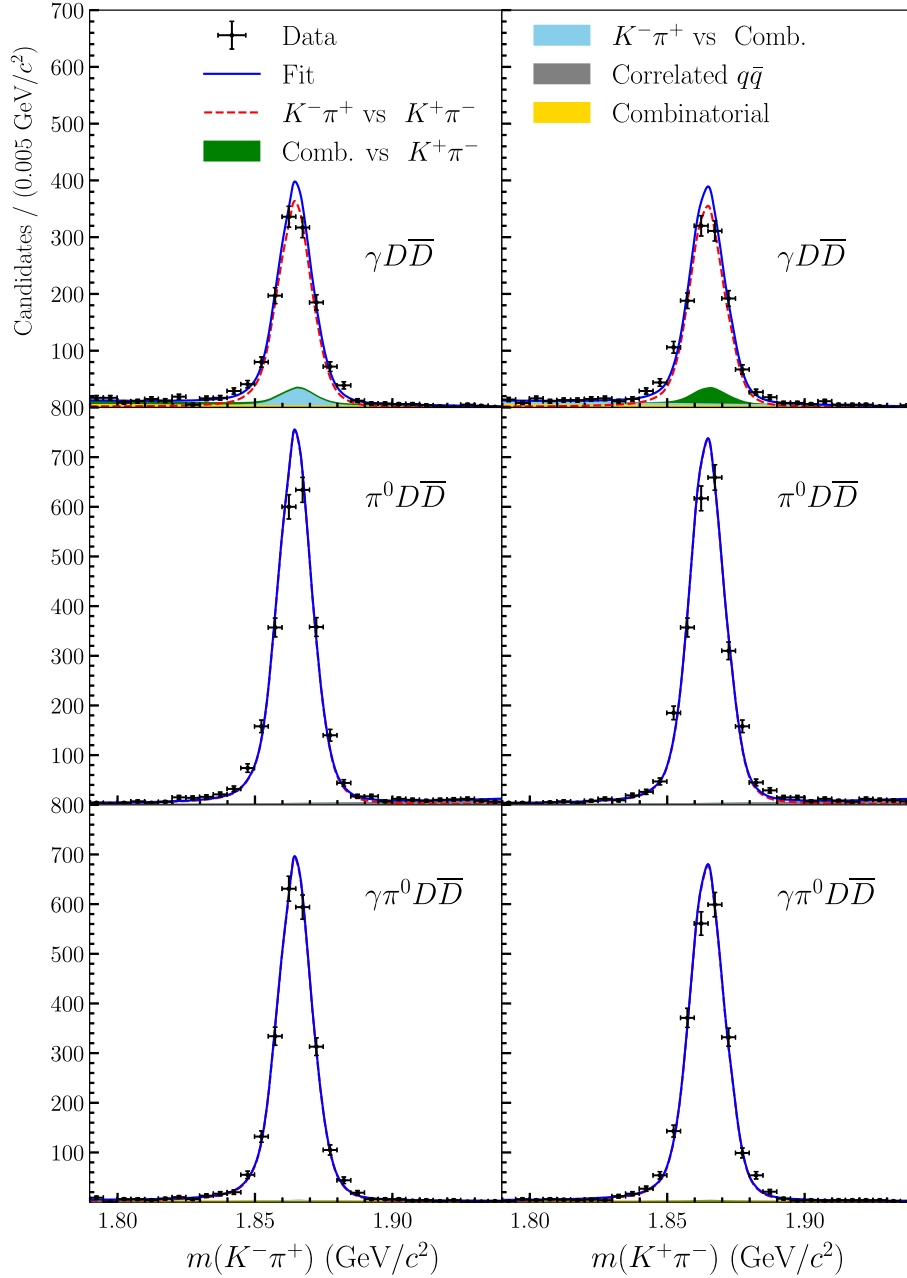


FIG. 5. Projections of the fits to the  $DD \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  decays passing the requirements which isolate the (top)  $D^* \bar{D} \rightarrow \gamma D \bar{D}$ , (center)  $D^* \bar{D} \rightarrow \pi^0 D \bar{D}$ , and (bottom)  $D^* \bar{D}^* \rightarrow \gamma \pi^0 D \bar{D}$  production mechanisms.

mean and width parameters. Combinatorial background components are modeled by linear polynomials with floating slope parameters.

Some fit categories include backgrounds from  $e^+e^-$  scattering to identical final states without proceeding through an intermediate  $e^+e^- \rightarrow XD \bar{D}$  process which primarily arise from continuum production of light hadrons. These are modeled by a Gaussian function in the difference of the reconstructed invariant masses with floating mean and width parameters, as the reconstructed invariant masses of these backgrounds are anticorrelated

due to conservation of energy. As shorthand, this background source is referred to as “correlated  $q\bar{q}$ ”.

Figure 5 displays the fits to  $DD \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  candidates that pass the selection requirements to isolate the  $D^* \bar{D} \rightarrow \gamma D \bar{D}$ ,  $D^* \bar{D} \rightarrow \pi^0 D \bar{D}$ , and  $D^* \bar{D}^* \rightarrow \gamma \pi^0 D \bar{D}$  production mechanisms. The latter two display the largest yields, whilst the first is chosen to show the small background contributions.

Finally, for the final states which include a  $D \rightarrow \pi^+ \pi^- \pi^0$  decay, there is a small contribution from  $D \rightarrow K_S^0 \pi^0$  decays which peak around the known  $D^0$  mass. They are not

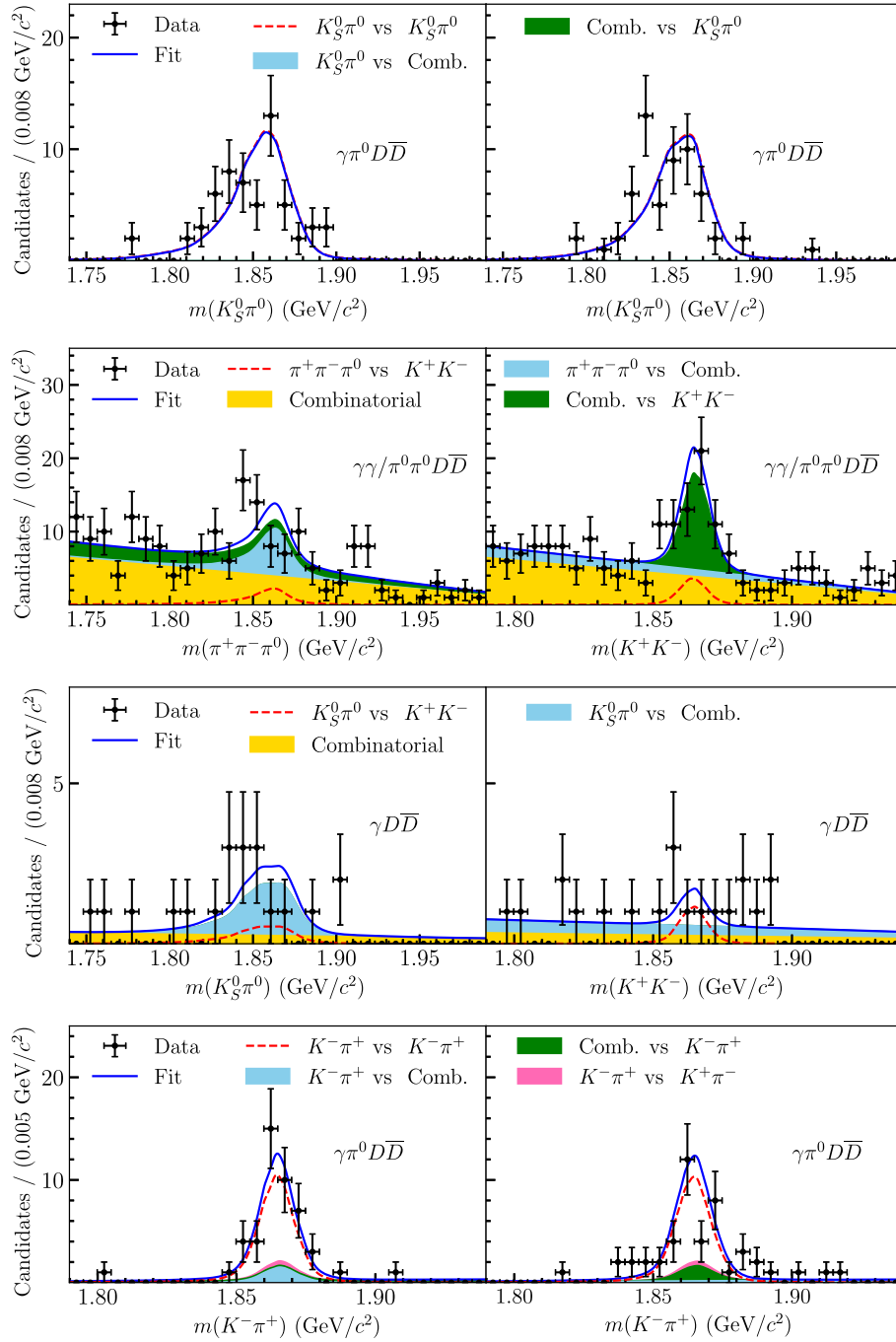


FIG. 6. Projections of the fits to various final states used to demonstrate the quantum coherence of each production mechanism. The fits to the (top)  $D\bar{D} \rightarrow K_S^0\pi^0$  vs  $K_S^0\pi^0$  candidates passing the requirements for the  $D^*\bar{D}^* \rightarrow \gamma\pi^0 D\bar{D}$  decays, (second from top)  $D\bar{D} \rightarrow \pi^+\pi^-\pi^0$  vs  $K^+K^-$  candidates passing the requirements for the  $D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0\pi^0 D\bar{D}$  decays, (second from bottom)  $D\bar{D} \rightarrow K_S^0\pi^0$  vs  $K^+K^-$  candidates passing the requirements for the  $D^*\bar{D}^* \rightarrow \gamma D\bar{D}$  decays, and (bottom)  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^-\pi^+$  candidates passing the requirements for the  $D^*\bar{D}^* \rightarrow \gamma\pi^0 D\bar{D}$  decays are shown. In each case, the left and right plots correspond to the projections of the two  $D$  decays.

explicitly modeled in the fit and are therefore absorbed into the signal component. The fractional contribution to the signal yield from this background,  $R_{K_S^0}$ , is determined in simulation to be around  $R_{K_S^0} \simeq 3\%$ . Given the  $D \rightarrow K_S^0\pi^0$  decay is  $CP$  odd and the  $D \rightarrow \pi^+\pi^-\pi^0$  decay is mostly  $CP$  even, it is

important to account for this contamination, as discussed below. The rate of the inverse scenario, where  $D \rightarrow \pi^+\pi^-\pi^0$  decays pass the selection criteria for the  $D \rightarrow K_S^0\pi^0$  decays, is reduced to a negligible rate by the  $K_S^0$  invariant mass and flight-distance significance requirements. Figure 6 displays a sample of the fits to various final states.

TABLE V. Summary of the observed yields and statistical uncertainties for all double  $CP$ -eigenstate decays in each production mechanism.

Final state		$D\bar{D}$	$D^*\bar{D} \rightarrow \gamma D\bar{D}$	$D^*\bar{D} \rightarrow \pi^0 D\bar{D}$	$D^*\bar{D}^* \rightarrow \gamma \pi^0 D\bar{D}$	$D^*\bar{D}^* \rightarrow \gamma \gamma / \pi^0 \pi^0 D\bar{D}$
$K^-\pi^+$	vs $K^+\pi^-$	$681.5 \pm 28.8$	$1198.0 \pm 38.8$	$2477.0 \pm 50.6$	$2316.7 \pm 51.5$	$1146.6 \pm 37.4$
$\pi^+\pi^-\pi^0$	vs $K_S^0\pi^0$	$80.2^{+11.0}_{-10.3}$	$23.0^{+8.0}_{-7.2}$	$272.6^{+19.2}_{-18.6}$	$29.6^{+8.2}_{-7.3}$	$92.9^{+11.4}_{-10.8}$
$\pi^+\pi^-\pi^0$	vs $K^+K^-$	$15.4^{+9.5}_{-7.7}$	$85.1^{+14.0}_{-12.9}$	$21.9^{+7.7}_{-6.3}$	$177.5^{+16.8}_{-16.0}$	$8.9^{+6.5}_{-5.7}$
$K_S^0\pi^0$	vs $K_S^0\pi^0$	$1.0^{+2.0}_{-1.0}$	$30.0^{+6.2}_{-6.2}$	$4.8^{+2.6}_{-2.6}$	$57.7^{+8.4}_{-7.8}$	$6.2^{+4.0}_{-3.4}$
$K_S^0\pi^0$	vs $K^+K^-$	$28.7^{+6.0}_{-5.4}$	$2.9^{+3.1}_{-2.7}$	$100.9^{+11.3}_{-10.3}$	$1.7^{+2.0}_{-1.3}$	$32.5^{+6.5}_{-5.8}$
$K_S^0\pi^0$	vs $\pi^+\pi^-$	$10.0^{+3.8}_{-3.2}$	$-1.6^{+2.5}_{-2.7}$	$54.0^{+7.8}_{-7.1}$	$-0.3^{+1.4}_{-1.4}$	$17.6^{+4.7}_{-4.1}$
$K^\pm\pi^\mp$	vs $K^\pm\pi^\mp$	$-0.2^{+2.1}_{-1.4}$	$18 \pm 6$	$2 \pm 3$	$35^{+7}_{-6}$	$5 \pm 3$

The cross feed discussed in Sec. III means each fitted signal yield contains contributions from  $D\bar{D}$  pairs produced through multiple processes. Corrections for the cross feed and selection efficiencies are simultaneously applied through the matrix equation,

$$\vec{n} = \mathbf{A}^{-1}\vec{N}, \quad (17)$$

where  $\vec{n}$  gives the efficiency corrected yields for each process,  $\vec{N}$  contains the observed yields after applying the selection requirements to isolate each process, as determined by the fits, and the entries of the matrix  $\mathbf{A}_{ij}$  give the probabilities of a  $D\bar{D}$  event from production mechanism  $i$  being identified with production mechanism  $j$ , as determined in simulation. An example of such a matrix can be found in Table III.

The aforementioned coherence parameters describing each production mechanism are determined in a fit which compares the observed signal yields, summarized in Table V, with the predictions assuming the  $D$  and  $\bar{D}$  decays are independent. The predicted yields of the  $D\bar{D}$  decays to two  $CP$  eigenstates, denoted  $D\bar{D} \rightarrow Y_1$  vs  $Y_2$ , are found by comparing to those measured in the  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  decay control channel,

$$\vec{N}_{Y_1, Y_2} = (2 - \delta_{Y_1}^{Y_2}) \frac{\mathcal{B}_{D^0 \rightarrow Y_1} \mathcal{B}_{D^0 \rightarrow Y_2}}{\mathcal{B}_{D^0 \rightarrow K^-\pi^+}^2} \mathbf{A}_{Y_1, Y_2} \kappa \mathbf{A}_{K\pi, K\pi}^{-1} \vec{N}_{K\pi, K\pi}, \quad (18)$$

where  $(2 - \delta_{Y_1}^{Y_2})$  is a combinatoric correction in terms of the Kronecker delta function and  $\mathcal{B}_{D^0 \rightarrow Y}$  is the  $D^0$  branching fraction to a final state  $Y$ . The matrix  $\kappa$  is defined as

$$\kappa_{ij} = \begin{cases} \kappa_i & \text{when } i = j \text{ and } i \text{ is } C\text{-enhanced,} \\ 1 - \kappa_i & \text{when } i = j \text{ and } i \text{ is } C\text{-forbidden,} \\ 0 & \text{when } i \neq j, \end{cases} \quad (19)$$

such that it is diagonal with a single coherence parameter to describe each process. For final states with a  $D \rightarrow \pi^+\pi^-\pi^0$  decay the coherence parameters are corrected by a factor,  $F_+^{\pi\pi\pi^0} / (1 + R_{K_S^0})$ , to account for the  $CP$ -odd contribution and background from  $D \rightarrow K_S^0\pi^0$  decays.

The  $D\bar{D} \rightarrow K^\pm\pi^\mp$  vs  $K^\pm\pi^\mp$  decays are included in the fit by examining the ratio of efficiency-corrected yields with respect to those determined in  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  decays. The predicted ratio is given by

$$4(r_{K\pi}^D)^2 f_\kappa \left( 1 - \frac{(x \sin \delta_{K\pi}^D + y \cos \delta_{K\pi}^D)}{r_{K\pi}^D} \right), \quad (20)$$

where  $f_\kappa = \kappa_i$  for processes that result in  $C$ -even  $D\bar{D}$  pairs and  $(1 - \kappa_i)$  in the  $C$ -odd scenario.

Two sources of systematic uncertainty are included in the fit. First, the knowledge of the  $D$  decay branching fractions is accounted for by a Gaussian constraining each to the measured value from the PDG [28]. The second systematic uncertainty is due to efficiency differences between data and MC simulation samples in the charged-track PID,  $K_S^0$ , and  $\pi^0$  reconstruction. In the fit, the selection efficiencies for each final state are multiplied by a factor which accounts for differences from data due to the three aforementioned effects. For example, in  $D\bar{D} \rightarrow K_S^0\pi^0$  vs  $K^+K^-$  decays the efficiencies are multiplied by  $(1 + \Delta\epsilon_{\text{PID}})^2 (1 + \Delta\epsilon_{K_S^0}) (1 + \Delta\epsilon_{\pi^0})$ , since there are two charged tracks subject to PID, one  $K_S^0$  meson, and one  $\pi^0$  meson in the final state. The PID and  $\pi^0$  reconstruction efficiency differences, denoted by  $\Delta\epsilon_{\text{PID}}$  and  $\Delta\epsilon_{\pi^0}$  respectively, are Gaussian constrained to  $(0 \pm 1)\%$  in the fit. The  $K_S^0$  reconstruction efficiency difference, denoted by  $\Delta\epsilon_{K_S^0}$ , is Gaussian constrained to  $(0 \pm 2)\%$  in the fit. The external parameters  $x$ ,  $y$ ,  $r_{K\pi}^D$ ,  $\delta_{K\pi}^D$ , and  $F_+^{\pi\pi\pi^0}$  are precisely known and hence they are fixed to the values determined in Refs. [18,32]. The measured coherence parameters, which are displayed in Table VI, are all found to be consistent with one with  $\chi^2/\text{n.d.f.} = 19.9/25$ , confirming the expectations.

TABLE VI. Measured coherence parameters for each production mechanism.

Production mechanism [C]	$\kappa$
$D\bar{D} [-1]$	$1.015 \pm 0.066$
$D^*\bar{D} \rightarrow \gamma D\bar{D} [+1]$	$1.044 \pm 0.044$
$D^*\bar{D} \rightarrow \pi^0 D\bar{D} [-1]$	$1.028 \pm 0.024$
$D^*\bar{D}^* \rightarrow \gamma \pi^0 D\bar{D} [+1]$	$1.027 \pm 0.017$
$D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0\pi^0 D\bar{D} [-1]$	$0.963 \pm 0.060$

The systematic uncertainties are around 35% of the total uncertainties shown in Table VI.

The effects of the quantum correlations are further evident in Fig. 7, which shows the ratio of the efficiency-corrected yield to the prediction assuming the  $D\bar{D}$  pair is not quantum correlated for each final state and production mechanism. Once again the uncorrelated predictions are determined using the  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+\pi^-$  decay yields. The measured ratios are found to be in good agreement with the predictions assuming each production mechanism is in a pure  $C$  eigenstate as computed using Eq. (4), which are zero for all suppressed decays and  $2 + \mathcal{O}(x^2, y^2)$  for enhanced decays from  $C$ -odd  $D\bar{D}$  pairs. Charm-mixing corrections are considered in the predictions for the enhanced decays of  $C$ -even  $D\bar{D}$  pairs. In the decays to two  $CP$  eigenstates the predicted ratio is  $2(1 \pm 4y)$  where the preceding sign on  $y$  is negative (positive) for two  $CP$ -even (odd) decays, and it is  $2(1 - (y \cos \delta_{K\pi}^D + x \sin \delta_{K\pi}^D)/r_{K\pi}^D)$  for the  $D\bar{D} \rightarrow K^\mp\pi^\pm$  vs  $K^\mp\pi^\pm$  decays. For the final states that include a  $D \rightarrow \pi^+\pi^-\pi^0$  decay, a small correction is

applied to the predictions to account for the  $CP$ -odd content and the background from  $D \rightarrow K_S^0\pi^0$  decays.

The results obtained constitute the first observation of  $C$ -even  $D\bar{D}$  pairs. Moreover, they are the first demonstration of the quantum correlations in  $D\bar{D}$  pairs produced above the charm threshold, and opens the way for data collected at these energies to be used for measurements of the strong phases of  $D$  mesons.

## V. MEASUREMENT OF $\delta_{K\pi}^D$

### A. Overview

A measurement of the strong-phase difference between  $D^0 \rightarrow K^-\pi^+$  and  $\bar{D}^0 \rightarrow K^-\pi^+$  decays,  $\delta_{K\pi}^D$ , is performed. This determination assumes the expected quantum-correlated behavior of the  $D\bar{D}$  pairs from each production mechanism that is validated in Sec. IV. The strong-phase difference  $\delta_{K\pi}^D$  is measured using  $D\bar{D} \rightarrow K^-\pi^+$  vs  $Y$  decays, where  $Y$  is one of the four  $CP$  eigenstates presented in Table IV or  $K_S^0\pi^+\pi^-$ , and is referred to as the ‘tag’ throughout.

From Eq. (4), it can be shown that for the  $CP$  tags, the quantum correlations enhance or suppress the decay rate relative to the scenario where the decays of the  $D$  and  $\bar{D}$  mesons are independent of each other by the factor

$$1 + \frac{2C\lambda r_{K\pi}^D \cos \delta_{K\pi}^D - (1+C)y}{1 + (r_{K\pi}^D)^2}, \quad (21)$$

where terms smaller than  $\mathcal{O}(r_{K\pi}^D y)$  are neglected, and the parameter  $\lambda = +1, -1$ , and  $2F_{\pi\pi\pi^0}^D - 1$  for the  $CP$ -even,

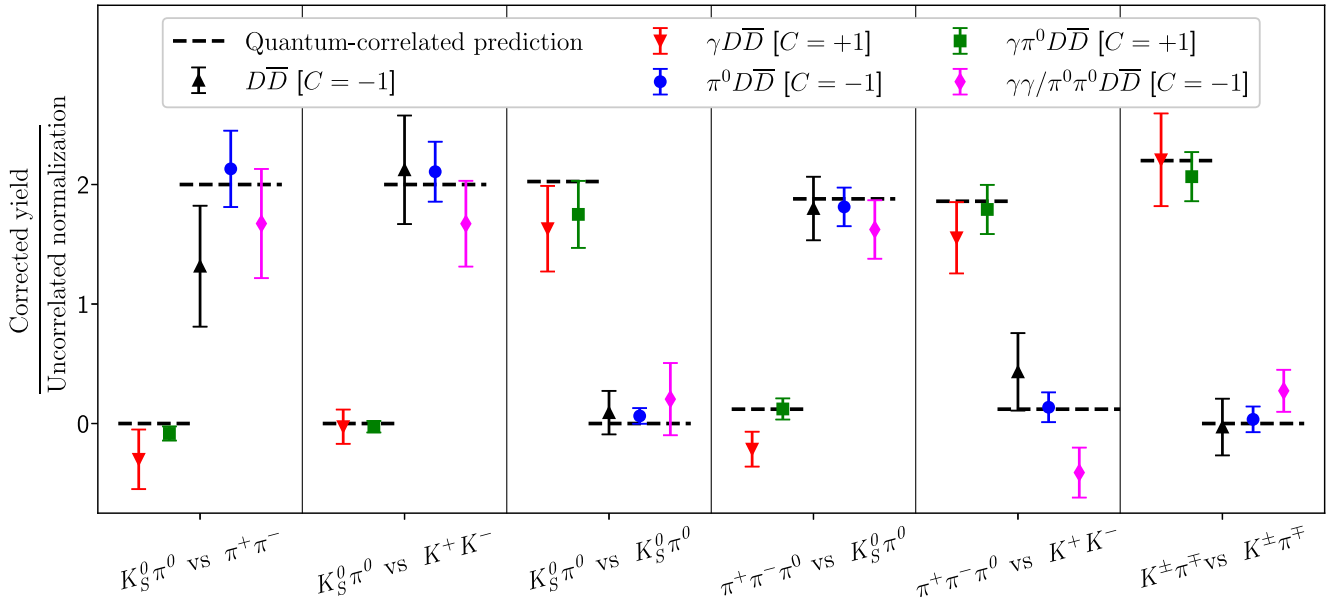


FIG. 7. The ratios of efficiency-corrected yields observed in data to those expected in the absence of correlations for each  $D\bar{D}$  final state originating from each production mechanism. The combined statistical and systematic uncertainties are shown.

-odd,  $\pi^+\pi^-\pi^0$  modes, respectively. The relative enhancement is predicted to be around  $\pm 12\%$ , where the preceding sign depends on the values of  $C$  and  $\lambda$ .

The  $CP$  tags are used to measure the observable  $r_{K\pi}^D \cos \delta_{K\pi}^D$ . However, this alone does not allow for a single solution of  $\delta_{K\pi}^D$ . Input is required from  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K_S^0\pi^+\pi^-$  decays, which provide sensitivity to the observables  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  through the variation of the  $D \rightarrow K_S^0\pi^+\pi^-$  decay strong-phase difference across phase space. In the previous measurement of  $\delta_{K\pi}^D$  performed at the  $\psi(3770)$  resonance [5],  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K_L^0\pi^+\pi^-$  decays were also used. However,  $K_L^0$  mesons are rarely reconstructed in the BESIII detector due to their long lifetime, and so the final state was partially reconstructed. They are not used in this analysis because the strategy employed to isolate the production mechanism in Sec. III also relies on partial reconstruction.

In the measurement of  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  described in Sec. VC, the  $D \rightarrow K_S^0\pi^+\pi^-$  decay phase space is divided into regions according to the ‘‘equal- $\Delta\delta_D$ ’’ binning scheme described in Ref. [35]. The phase space is characterized by the squared reconstructed invariant masses  $m_-^2 \equiv m^2(K_S^0\pi^-)$  and  $m_+^2 \equiv m^2(K_S^0\pi^+)$ . It is divided into 16 regions which are symmetric around the line  $m_-^2 = m_+^2$ . A region with index  $j$  ( $-j$ ) has  $m_+^2 > m_-^2$  ( $m_+^2 < m_-^2$ ) and therefore the indices range from  $-8$  to  $8$ , excluding  $0$ .

Inputs for the hadronic parameters of the  $D \rightarrow K_S^0\pi^+\pi^-$  decay are necessary to extract the observables. They are the

fraction of  $D^0 \rightarrow K_S^0\pi^+\pi^-$  decays that are in region  $i$ , denoted  $K_i$ , and the amplitude weighted averaged cosine (sine) difference of the strong-phase difference between  $D^0 \rightarrow K_S^0\pi^+\pi^-$  and  $\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-$  decays, denoted  $c_i$  ( $s_i$ ). They are defined as

$$K_i = \int_i |A(D^0 \rightarrow K_S^0\pi^+\pi^-)|^2 dm_-^2 dm_+^2, \quad (22)$$

$$K_{-i} = \int_i |A(\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-)|^2 dm_-^2 dm_+^2, \quad (23)$$

$$c_i = \frac{1}{\sqrt{K_i K_{-i}}} \int_i |A(D^0 \rightarrow K_S^0\pi^+\pi^-)| |A(\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-)| \times \cos(\Delta\delta_D) dm_-^2 dm_+^2, \quad (24)$$

$$s_i = \frac{1}{\sqrt{K_i K_{-i}}} \int_i |A(D^0 \rightarrow K_S^0\pi^+\pi^-)| |A(\bar{D}^0 \rightarrow K_S^0\pi^+\pi^-)| \times \sin(\Delta\delta_D) dm_-^2 dm_+^2, \quad (25)$$

where  $\Delta\delta_D$  is the strong-phase difference between  $D^0$  and  $\bar{D}^0$  decays to the  $K_S^0\pi^+\pi^-$  final state. The values used in this measurement for each are taken from Ref. [35]. With these definitions, the parameters in Eq. (4) become  $r_1 \rightarrow r_{K\pi}^D$ ,  $\delta_1 \rightarrow \delta_{K\pi}^D$ ,  $R_1 \rightarrow R_{K\pi} = 1$ ,  $A_2^2 \rightarrow K_i$ ,  $r_2^2 A_2^2 \rightarrow K_{-i}$ ,  $R_2 \cos \delta_2 \rightarrow c_i$ , and  $R_2 \sin \delta_2 \rightarrow s_i$ , such that the fraction of  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K_S^0\pi^+\pi^-$  decays in a region,  $i$ , is given by

$$Y_i = [(r_{K\pi}^D)^2 K_i + K_{-i} + 2Cr_{K\pi}^D \sqrt{K_i K_{-i}} (c_i \cos \delta_{K\pi}^D + s_i \sin \delta_{K\pi}^D)] - (1+C)y[r_{K\pi}^D \cos \delta_{K\pi}^D (K_i + K_{-i}) + c_i \sqrt{K_i K_{-i}} (1 + (r_{K\pi}^D)^2)] - (1+C)x[r_{K\pi}^D \sin \delta_{K\pi}^D (K_i - K_{-i}) + s_i \sqrt{K_i K_{-i}} (1 - (r_{K\pi}^D)^2)]. \quad (26)$$

In the above equation, the region with index  $i$  combines  $D \rightarrow K^+\pi^-$  decays in region  $j$  and  $D \rightarrow K^-\pi^+$  decays in region  $-j$ , where  $j = i$ .

### B. Measuring $r_{K\pi}^D \cos \delta_{K\pi}^D$ using $D$ decays to $CP$ -eigenstates

The signal yields are determined for each  $CP$ -eigenstate tag and production mechanism using fits similar to those performed in the previous section. Alongside the five components first described in Sec. IV, additional backgrounds from  $D\bar{D}$  decays are present, which can leak into the signal regions. Therefore, fit ranges wider than those in Sec. IV are used. Consequently, an exponential function, instead of a linear polynomial, is used to describe the combinatorial background in the majority of fits, where it is found to peak towards lower masses. In the fits to

$D\bar{D} \rightarrow K^-\pi^+$  vs  $\pi^+\pi^-\pi^0$  candidates which pass the selection requirements for the  $D^*\bar{D} \rightarrow \pi^0 D\bar{D}$  and  $D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0\pi^0 D\bar{D}$  production mechanisms, it is found that an exponential is not a good model for the combinatorial background in the reconstructed  $D \rightarrow K\pi$  invariant mass, and therefore a second-order polynomial is used. The choice of background shape is considered as a systematic uncertainty.

The aforementioned backgrounds from  $D\bar{D}$  decays, the specific details of which follow, are modeled by a KDE of the distribution in simulation. Firstly,  $D \rightarrow K^-\pi^+\pi^0$  decays can be wrongly reconstructed as  $D \rightarrow K^+K^-$ , where the  $\pi^+$  is misidentified as a  $K^+$  and the  $\pi^0$  is missed in the reconstruction. In the fits to candidates passing the selections to isolate  $D\bar{D}$  pairs from the  $e^+e^- \rightarrow D\bar{D}$ ,  $D^*\bar{D} \rightarrow \gamma D\bar{D}$ , and  $D^*\bar{D}^* \rightarrow \gamma\gamma/\pi^0\pi^0 D\bar{D}$  processes the

yield of this background is a floating parameter. In the others, the number passing the selections is significantly lower and therefore the yield is fixed relative to signal using the rate in simulation with a small correction factor to account for the effects of quantum correlations. In both cases the contamination is less than 10% of the signal yield.

Background from  $D \rightarrow K^- \pi^+ \pi^0$  decays is also present in the fits to candidates reconstructed as the  $D \rightarrow \pi^+ \pi^- \pi^0$  decay, where the  $K^-$  is misidentified as a  $\pi^-$ . In each fit the yield of this contamination is a floating parameter.

Finally, an additional background is present from  $D \rightarrow K^+ \pi^- + \pi_D^0$  decays, where the  $K^+$  is misidentified as a  $\pi^+$  and the  $\pi^0$  originates from the  $D^* \rightarrow D\pi^0$  decay, in the fits to the  $D \rightarrow \pi^+ \pi^- \pi^0$  candidates passing the selection criteria to isolate the  $D^* \bar{D} \rightarrow \pi^0 D \bar{D}$  and  $D^* \bar{D}^* \rightarrow \gamma \pi^0 D \bar{D}$  production mechanisms. Again, in each fit the yield of this background is a floating parameter.

Figures 8 and 9 display the fits for each  $CP$  tag of the candidates that pass the selection criteria to isolate the  $D^* \bar{D} \rightarrow \pi^0 D \bar{D}$  and  $D^* \bar{D}^* \rightarrow \gamma \pi^0 D \bar{D}$  production mechanisms, which have the largest yields. The fitted signal yields are corrected for residual backgrounds from  $D \bar{D}$  decays to the correct final state, where the  $D \bar{D}$  pair originates from the  $e^+ e^- \rightarrow D^{*+} D^{*-}$  process, by subtracting the yield in simulation. The corrections are only applied to  $D \bar{D}$  pairs originating from the  $e^+ e^- \rightarrow D^* \bar{D}^*$  processes and in all cases are less than 7% of the fitted yields. The signal yields after the corrections are displayed in Table VII.

The parameter  $r_{K\pi}^D \cos \delta_{K\pi}^D$  is extracted in a  $\chi^2$  fit which compares the ratios of observed and predicted signal yields in the  $C$ -even and -odd processes,

$$\chi^2 = \sum_Y \left( \frac{R_Y^{\text{obs}} - R_Y^{\text{pred}}}{\delta R_Y^{\text{obs}}} \right)^2, \quad (27)$$

where the sum runs over the  $CP$ -eigenstate tags. The observed ratios are determined summing over the yields measured in the fits which isolate  $D \bar{D}$  pairs with  $C$ -even, denoted E, and  $C$ -odd, denoted O, eigenvalues,

$$R_Y^{\text{obs}} = \frac{\sum_E N_E^{\text{obs}}}{\sum_O N_O^{\text{obs}}}. \quad (28)$$

The ratio observables are chosen to cancel all systematic uncertainties associated with the reconstruction of the final state and the total number of produced  $D \bar{D}$  pairs. The predicted yields are determined by comparing to those measured in  $D \bar{D} \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  decays,

$$\vec{N}_{K\pi,Y} = \frac{\mathcal{B}_{D^0 \rightarrow K^- \pi^+}}{\mathcal{B}_{D^0 \rightarrow Y}} \mathbf{A}_{K\pi,Y} \mathbf{f}_C^\lambda \mathbf{A}_{K\pi,K\pi}^{-1} \vec{N}_{K\pi,K\pi}, \quad (29)$$

where  $\mathbf{f}_C^\lambda$  is a diagonal matrix with elements given by

$$\frac{1 + (r_{K\pi}^D)^2 + 2\lambda C_\alpha r_{K\pi}^D \cos \delta_{K\pi}^D - \lambda(1 + C_\alpha)y}{1 + 2C_\alpha(2(r_{K\pi}^D \cos \delta_{K\pi}^D)^2 - (r_{K\pi}^D)^2)}, \quad (30)$$

for a row and column of index  $\alpha$ . The matrix gives the predicted enhancement or suppression for each production mechanism. Each diagonal element is similar to Eq. (21), but the denominator differs to account for the small effect of quantum correlations on the  $D \bar{D} \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  decay rate. It should be noted that in practice the branching fractions are not required because they cancel in the ratio.

Each efficiency matrix can be approximately divided into the contributions from the reconstruction of the final state and the isolation of each production mechanism. Hence, the predicted yields are approximately  $\propto \alpha A_{K\pi,K\pi} A_{K\pi,K\pi}^{-1} = \alpha$ , where  $\alpha$  is a constant. Therefore, using  $D \bar{D} \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  decays as a control channel is particularly beneficial because it results in the partial cancellation of systematic uncertainties associated with the efficiency of isolating each production mechanism. The cancellation is not exact due to final-state differences, for example, the resolution may vary between the  $CP$  eigenstates with and without a  $\pi^0$ .

In the fit, the observable  $r_{K\pi}^D \cos \delta_{K\pi}^D$  is a floating parameter, and the  $D \bar{D} \rightarrow K^- \pi^+ \text{ vs } K^+ \pi^-$  yields are Gaussian constrained to those in Table V. The remaining parameters  $(r_{K\pi}^D)^2$ ,  $y$  and  $F_+^{\pi\pi\pi^0}$  are fixed to the values in Refs. [18,32]. The fit determines  $r_{K\pi}^D \cos \delta_{K\pi}^D = -0.070 \pm 0.008$  with  $\chi^2/\text{n.d.f.} = 2.06/3$  which corresponds to a  $p$ -value of 56%.

A comparison between the observed and fitted ratios can be found in Fig. 10. The prediction in the scenario where the  $D$  and  $\bar{D}$  mesons decay independently is also shown, which is calculated using Eq. (29) without the enhancement matrix  $\mathbf{f}_C^\lambda$ . The differences in the cross-feed efficiencies are small between the final states, and therefore the uncorrelated prediction is approximately independent of the  $CP$  tag. From Eq. (21), the final states with  $CP$ -even tags are suppressed (enhanced) by around 12% when they originate from  $D \bar{D}$  pairs which are  $C$ -odd (even). This leads to the naive expectation that the ratios for the  $CP$ -even tags will be reduced by a factor of  $(1 - 0.12)/(1 + 0.12) \simeq 0.79$  relative to the uncorrelated prediction, which is consistent with the observations. The inverse is true for the  $CP$ -odd  $D \rightarrow K_S^0 \pi^0$  tag, hence the enhancement in the ratio relative to the uncorrelated prediction.

### C. Measuring $r_{K\pi}^D \cos \delta_{K\pi}^D$ and $r_{K\pi}^D \sin \delta_{K\pi}^D$ using $D \rightarrow K_S^0 \pi^+ \pi^-$ decays

The parameters  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  are determined by simultaneously fitting the reconstructed invariant-mass distribution of both  $D$  mesons in 80 categories given by the region of the  $D \rightarrow K_S^0 \pi^+ \pi^-$  phase space and the hypothesized production mechanism. In addition to the five core components discussed previously, two additional

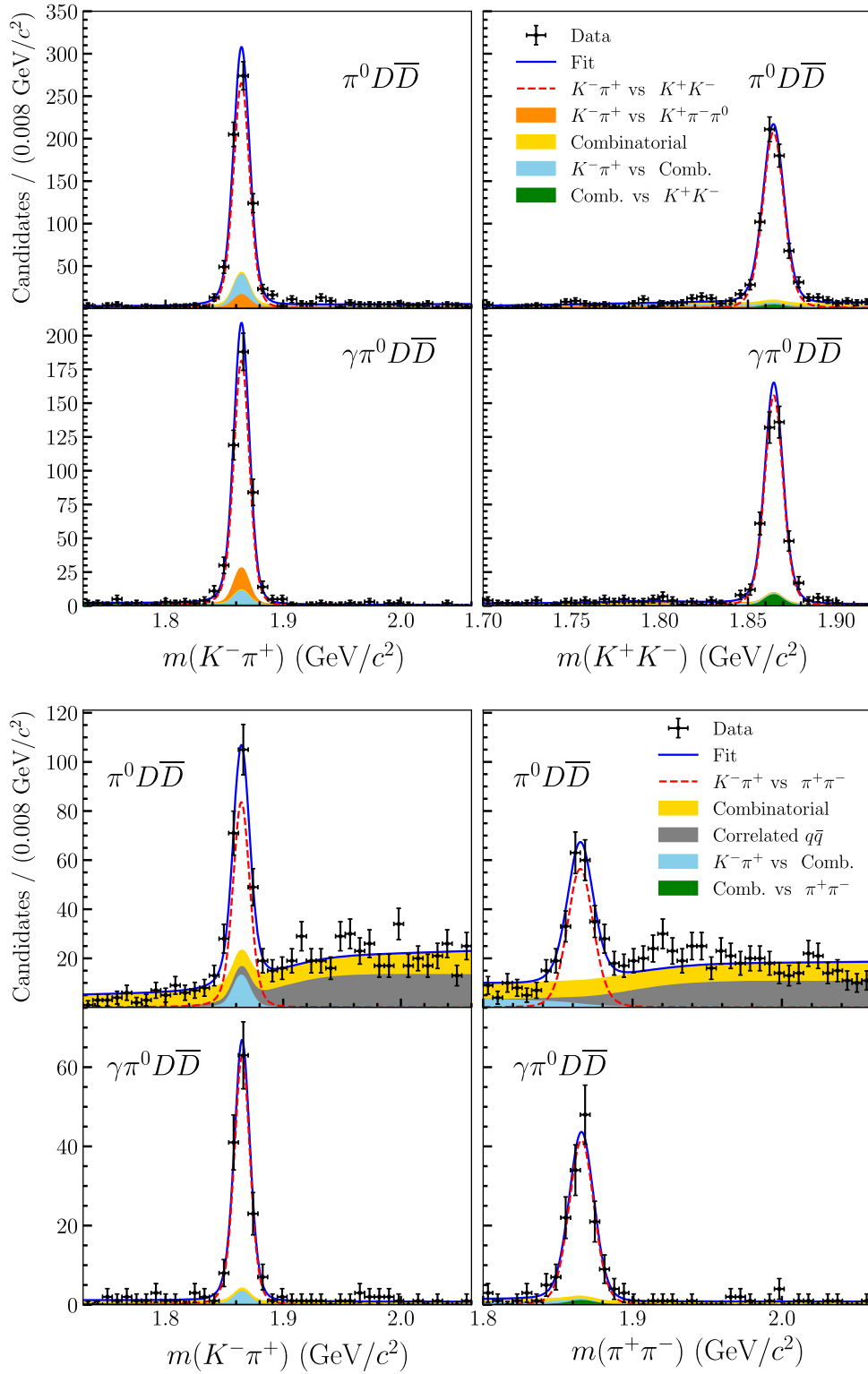


FIG. 8. Projections of the fits to (top two)  $D\bar{D} \rightarrow K^-\pi^+$  vs  $K^+K^-$  (bottom two)  $D\bar{D} \rightarrow K^-\pi^+$  vs  $\pi^+\pi^-$  candidates passing the selection requirements which isolated the  $D^*\bar{D} \rightarrow \pi^0 D\bar{D}$  and  $D^*\bar{D}^* \rightarrow \gamma\pi^0 D\bar{D}$  production mechanisms. The left and right columns display the invariant-mass projections for the  $K^-\pi^+$  and  $CP$  tag final states, respectively.

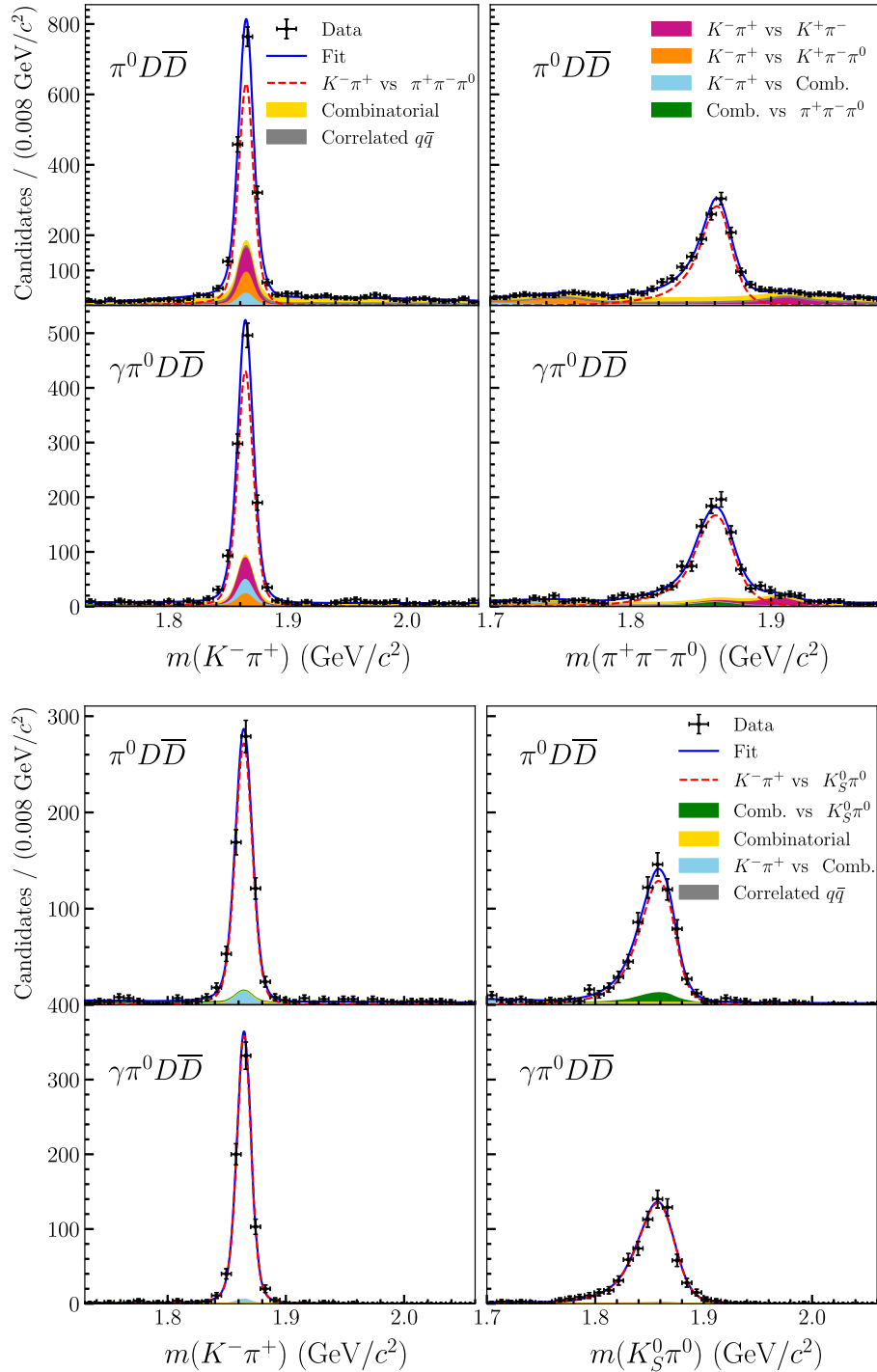


FIG. 9. Projections of the fits to (top two)  $D\bar{D} \rightarrow K^-\pi^+ \text{ vs } \pi^+\pi^-\pi^0$  (bottom two)  $D\bar{D} \rightarrow K^-\pi^+ \text{ vs } K_S^0\pi^0$  candidates passing the selection requirements which isolated the  $D^*D^- \rightarrow \pi^0 D\bar{D}$  and  $D^*\bar{D}^* \rightarrow \gamma\pi^0 D\bar{D}$  production mechanisms. The left and right columns display the invariant-mass projections for the  $K^-\pi^+$  and  $CP$  tag final states, respectively.

background components are included in the fit. The first is from  $D^0D^- \rightarrow [K^-\pi^+]_{D^0}, [K_S^0\pi^-\pi^0]_{D^-}$  decays, where the  $D^0D^-$  pair originates from  $e^+e^- \rightarrow D^{*+}D^- \rightarrow D^0\pi^+D^-$  production. These decays are only present in the categories which isolate  $D\bar{D}$  pairs originating from the  $e^+e^- \rightarrow D^*\bar{D}$

process. The selection requirements effectively remove this background in the other categories. The shape and yield of this component are fixed to those determined in simulation. The second background is from  $D\bar{D} \rightarrow K^-\pi^+ \text{ vs } K_S^0\pi^+\pi^-$  decays, where the  $D\bar{D}$  pair originates



## VI. SYSTEMATIC UNCERTAINTIES

A number of systematic uncertainties associated with assumptions in the various fits are evaluated. Firstly, the uncertainty due to the knowledge of the  $c_i$  and  $s_i$  inputs is determined by repeating the fit many times using an alternative set of  $c_i$  and  $s_i$  generated according to their uncertainties and correlations. The standard deviations of the subsequent distributions of  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  are assigned as the systematic uncertainties and are determined to be approximately  $8 \times 10^{-4}$  and  $27 \times 10^{-4}$ , respectively. The uncertainty on  $r_{K\pi}^D \sin \delta_{K\pi}^D$  is larger than that of  $r_{K\pi}^D \cos \delta_{K\pi}^D$  because the  $s_i$  inputs are less precise than the  $c_i$ . However, both are similar in magnitude to those reported in the previous BESIII measurement of  $\delta_{K\pi}^D$  [5]. The correlation between the observables due to this systematic uncertainty is calculated to be 8%.

Similarly, a systematic uncertainty is included to account for the statistical uncertainties on the  $K_i$  inputs, which are fixed in the fit, and it is estimated in a similar way to that of the  $c_i$  and  $s_i$  inputs. The systematic uncertainty is found to be  $11 \times 10^{-4}$  for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $10 \times 10^{-4}$  for  $r_{K\pi}^D \sin \delta_{K\pi}^D$  with a correlation of 3%. This is significantly reduced compared to Ref. [5], where this systematic uncertainty dominated. The decrease is investigated further by repeating the systematic study using predicted signal yields assuming that the  $D\bar{D}$  pairs produced through all production mechanisms are  $C$ -even or  $C$ -odd. The values of the observables determined in the two scenarios are found to be  $> 97\%$  anticorrelated. So, varying the  $K_i$  affects  $C$ -even and -odd  $D\bar{D}$  pairs in opposite ways, which leads to a partial cancellation and thus a reduction in the systematic uncertainty.

The cross feed between the production mechanisms is primarily caused by missing energy due to ISR. In events with ISR the effective center-of-mass energy of the  $e^+e^-$  pair is reduced. To model such events the simulation requires knowledge of the cross sections of the  $e^+e^- \rightarrow D\bar{D}$ ,  $e^+e^- \rightarrow D^*\bar{D}$ , and  $e^+e^- \rightarrow D^*\bar{D}^*$  processes at the reduced center-of-mass energies, where the relative uncertainties are at most around 20% [38]. The associated systematic uncertainty is determined by repeating the fits which determine the observables using alternate efficiency matrices which are calculated in simulation where the events with ISR are reweighted by  $\pm 20\%$ . Despite this conservative estimate the shifts in the observables are relatively small, for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  measured with the  $CP$  tags ( $D \rightarrow K_S^0 \pi^+ \pi^-$ ) it is around 10% (4%) of the statistical uncertainty, whilst for  $r_{K\pi}^D \sin \delta_{K\pi}^D$  it is found to be negligible. This systematic uncertainty is fully correlated across all observables.

A number of uncertainties are associated with the probability distribution functions used to model different processes in the fits to determine the signal yields. As mentioned above, a choice is made for the model used to

describe combinatorial background based on the fit quality. For  $r_{K\pi}^D \cos \delta_{K\pi}^D$  measured using the  $CP$  tags, the associated systematic uncertainty is evaluated by performing the  $\chi^2$  fit using signal yields from fits with alternative background shapes. A similar procedure is employed for the observables determined using the  $D \rightarrow K_S^0 \pi^+ \pi^-$  decay. In particular, a second-order polynomial is used for fits with combinatorial backgrounds that peak towards low masses, and a first-order polynomial is used to describe backgrounds with a relatively flat distribution. The systematic uncertainty is found to be around 9% of the statistical uncertainty for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  measured using the  $CP$  tags, and approximately 2% and 7% for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  from the  $K_S^0 \pi^+ \pi^-$  tag, respectively.

The shape used to model the anticorrelated background is chosen somewhat arbitrarily, but provides a good-quality fit to the data. To estimate the associated uncertainty with the choice of shape, a similar procedure to that described in the previous paragraph is performed using three alternative models which allow for additional freedom. They are an asymmetric Gaussian function with different left- and right-sided widths, a Crystal Ball function with an alternative description of the tails [39], and a Gaussian multiplied by an exponential to allow for a nonuniform distribution in  $m_{D_1} - m_{D_2}$ . The systematic uncertainty is determined to be less than 5% of the statistical uncertainty for all observables.

In general, good agreement is found between the resolution of the selection variables in data and MC simulation samples, as seen in Fig. 2. Furthermore, any mismodeling will be reduced because the variables are calculated with constraints on the masses of the  $D$  mesons. Regardless, the effect is probed using alternative efficiency matrices which are found by smearing the simulated distributions of the variables used in the selection of candidates discussed in Sec. III. For the  $CP$  tags, a Gaussian with a width of 4.5 MeV ( $4.5 \text{ MeV}/c^2$ ) for energy (mass) variables, conservatively chosen to show clear disagreement between the distributions in data and MC simulation samples, is used for each final state. This alternative choice of function leads to a negligible systematic uncertainty on  $r_{K\pi}^D \cos \delta_{K\pi}^D$ . For the  $D \rightarrow K_S^0 \pi^+ \pi^-$  tag, any efficiency differences between data and MC simulation samples are absorbed by the floating normalization parameters in the fit, assuming the differences are constant across the phase space. In general, this assumption is good. However, it may not hold for resolution differences between data and MC simulation samples. The associated systematic uncertainty is conservatively estimated by performing the fit to data many times using alternative efficiency matrices where the discriminating variables discussed in Sec. III are smeared by a random number between 0 and 3.5 MeV in each phase-space region. The uncertainty is determined to be around 5% of the statistical uncertainty for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and negligible for  $r_{K\pi}^D \sin \delta_{K\pi}^D$ .

TABLE VIII. Summary of the systematic uncertainties ( $\times 10^{-4}$ ), on the observables  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  determined using  $D$  decays to the  $CP$  eigenstates and  $K_S^0 \pi^+ \pi^-$ . Entries denoted with ‘-’ are negligible or do not apply to the relevant observable.

Systematic	$r_{K\pi}^D \cos \delta_{K\pi}^D$ $D \rightarrow CP$	$r_{K\pi}^D \cos \delta_{K\pi}^D$ $D \rightarrow K_S^0 \pi^+ \pi^-$	$r_{K\pi}^D \sin \delta_{K\pi}^D$ $D \rightarrow K_S^0 \pi^+ \pi^-$
$c_i, s_i$		8	27
$K_i$		11	10
ISR	8	6	
$D \rightarrow K_S^0 \pi^0$ bkg	9		
Comb. bkg	7	3	12
Anti-corr. bkg	3	6	1
MC resolution		7	
Bias	4		
Total	15	18	31
Statistical	80	140	170

The ratio of  $D \rightarrow K_S^0 \pi^0$  decays passing the  $D \rightarrow \pi^+ \pi^- \pi^0$  selection criteria,  $R_{K_S^0}$ , is determined using simulation. However, the ratio could be affected by mismodeling and quantum correlations. Therefore, the associated systematic uncertainty is evaluated by repeating the  $\chi^2$  fit using a ratio determined in data. Specifically, the number of  $D \rightarrow K_S^0 \pi^0$  decays in each sample is estimated through a two-dimensional fit to the reconstructed invariant mass of the  $D \rightarrow K^- \pi^+$  decay and the  $\pi^+ \pi^-$  pair, using candidates which satisfy  $m_{\pi^+ \pi^-} \in [0.35, 0.6]$  GeV/ $c^2$  and  $m_{\pi^+ \pi^- \pi^0} \in [1.8, 1.92]$  GeV/ $c^2$ . The ratios determined for the five production mechanisms are in good agreement and the average is determined to be  $(3.7 \pm 0.4)\%$ . Using this ratio, the parameter  $r_{K\pi}^D \cos \delta_{K\pi}^D$  shifts by 5% of the statistical uncertainty, which is assigned as the systematic uncertainty.

A closure check of the  $\chi^2$  fit to extract  $r_{K\pi}^D \cos \delta_{K\pi}^D$  from the  $CP$  tags is performed using signal yields which are generated using the values of  $r_{K\pi}^D$  and  $\delta_{K\pi}^D$  from Ref. [18]. The fit returns the expected value of  $r_{K\pi}^D \cos \delta_{K\pi}^D$  with a bias of around  $4 \times 10^{-4}$ . Given the bias is less than the least significant figure on the reported value, no correction is applied to  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and it is included as a systematic uncertainty.

Additional studies are performed to estimate the effects of the uncertainty in the fixed ratios parametrizing the  $D\bar{D} \rightarrow K^\pm \pi^\mp$  vs  $K^\mp \pi^\pm \pi^0$  decay background, knowledge of the  $e^+ e^- \rightarrow D^*+ D^{*-}$  and  $e^+ e^- \rightarrow D^*+ D^-$  cross sections, and the uncertainties of the fixed parameters in the fits to determine the observables, each of which is determined to be negligible. A summary of the systematic uncertainties is displayed in Table VIII. The total systematic uncertainty on  $r_{K\pi}^D \cos \delta_{K\pi}^D$  from the  $CP$  tags is approximately 19% of the statistical uncertainty, whilst it

TABLE IX. Values of the measured observables. The first uncertainties are statistical and the second are systematic.

Source	Observable	Value
$D \rightarrow CP$	$r_{K\pi}^D \cos \delta_{K\pi}^D$	$-0.070 \pm 0.008 \pm 0.0015$
$D \rightarrow K_S^0 \pi^+ \pi^-$	$r_{K\pi}^D \cos \delta_{K\pi}^D$	$-0.044 \pm 0.014 \pm 0.0018$
$D \rightarrow K_S^0 \pi^+ \pi^-$	$r_{K\pi}^D \sin \delta_{K\pi}^D$	$-0.022 \pm 0.017 \pm 0.0031$

TABLE X. The correlations between the systematic uncertainties on the observables. The observable  $r_{K\pi}^D \cos \delta_{K\pi}^D$  with the  $\dagger$  ( $\ddagger$ ) symbol is determined using the  $CP$ -eigenstate ( $K_S^0 \pi^+ \pi^-$ ) tags.

	$(r_{K\pi}^D \cos \delta_{K\pi}^D)^\dagger$	$(r_{K\pi}^D \cos \delta_{K\pi}^D)^\ddagger$	$r_{K\pi}^D \sin \delta_{K\pi}^D$
$(r_{K\pi}^D \cos \delta_{K\pi}^D)^\dagger$	1	0.18	0
$(r_{K\pi}^D \cos \delta_{K\pi}^D)^\ddagger$		1	0.19
$r_{K\pi}^D \sin \delta_{K\pi}^D$			1

is around 13% and 18% for  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  measured using the  $D \rightarrow K_S^0 \pi^+ \pi^-$  decay, respectively.

## VII. EXTRACTING $\delta_{K\pi}^D$

The three measured  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  observables, summarized in Table IX, are combined in a  $\chi^2$  fit to determine  $\delta_{K\pi}^D$ , which uses a covariance matrix that combines statistical and systematic uncertainties. The correlations between the observables due to the latter are shown in Table X. Those for each individual systematic uncertainty are assumed to be zero unless explicitly mentioned in the previous section.

The fit yields  $\delta_{K\pi}^D = (192.8_{-12.4-2.4}^{+11.0+1.9})^\circ$ , where the first uncertainty is statistical and the second is systematic, when  $r_{K\pi}^D$  is fixed to the value determined in Ref. [18]. This result is in good agreement with the global average  $\delta_{K\pi}^D = (190.2 \pm 2.8)^\circ$  from Ref. [18], and is competitive with the previous BESIII measurement  $\delta_{K\pi}^D = (187.6_{-9.7-6.4}^{+8.9+5.4})^\circ$  [5], made at the  $\psi(3770)$  resonance.

## VIII. BESIII COMBINATION

A maximum likelihood fit is performed to find the combined value of  $\delta_{K\pi}^D$  from BESIII measurements using the observables presented in this paper and the study using data collected at the  $\psi(3770)$  resonance [5]. Two of the observables in Ref. [5] are  $r_{K\pi}^D \cos \delta_{K\pi}^D$  and  $r_{K\pi}^D \sin \delta_{K\pi}^D$  determined by examining the phase space of the  $D \rightarrow K_{S,L}^0 \pi^+ \pi^-$  decays. As such, the systematic uncertainty due to fixing the  $c_i$  and  $s_i$  inputs is assumed to be completely correlated with the equivalent observables in this analysis. The systematic associated with the  $K_i$  inputs is also shared between the analyses. However,

the uncertainty is found to be significantly lower here, and the  $K_L^0\pi^+\pi^-$  tags use different  $K_i$  inputs, so the correlation is assumed to be zero. All other systematic uncertainties are uncorrelated. The full correlation matrix is displayed in the Appendix.

In the fit, the external inputs  $r_{K\pi}^D$ ,  $y$ , and  $F_{+\pi\pi^0}^D$  are Gaussian constrained to the values in Refs. [18,32]. The result is  $\delta_{K\pi}^D = (189.2_{-7.4-3.8}^{+6.9+3.4})^\circ$ , where the first uncertainty is statistical and the second is systematic.

## IX. OUTLOOK AND SUMMARY

The measurement presented in this paper is the first to use quantum-correlated  $D\bar{D}$  pairs produced above the charm threshold, and enables opportunities for similar studies in the future. The  $D\bar{D}$  pairs are examined through  $e^+e^- \rightarrow X D\bar{D}$  production, where  $X$  is a combination of photons and neutral pions. These processes allow for  $D\bar{D}$  pairs in a  $C$ -even eigenstate which, until now, have never been observed. All previous strong-phase measurements with quantum-correlated  $D\bar{D}$  pairs have been performed using data collected at the  $\psi(3770)$  resonance where they are in a  $C$ -odd eigenstate.

The  $C$ -even  $D\bar{D}$  pairs have prospects beyond strong-phase studies. These events can be used for time-integrated measurements of the charm-mixing parameters, which impact decay rates at first order compared to a quadratic dependence in the  $C$ -odd scenario. Such studies would provide important, independent tests of the results obtained by the LHCb Collaboration using time-dependent techniques. The prospects for future measurements are discussed further in Ref. [33].

The use of  $C$ -even and -odd  $D\bar{D}$  pairs in the measurement has allowed for the deployment of novel techniques with systematic benefits compared to those used at the  $\psi(3770)$  resonance. The strong-phase difference  $\delta_{K\pi}^D$  is found to be  $\delta_{K\pi}^D = (192.8_{-12.4-2.4}^{+11.0+1.9})^\circ$ , which is competitive with the previous BESIII determination [5]. A combination is performed with the results of Ref. [5] which finds a value of  $\delta_{K\pi}^D = (189.2_{-7.4-3.8}^{+6.9+3.4})^\circ$ .

In summary, the quantum coherence in  $D\bar{D}$  pairs produced above the charm threshold has been demonstrated. A method to exploit these data without lowering reconstruction efficiency due to the presence of neutral particles has been developed. Methods where the  $C$ -even and  $C$ -odd states are simultaneously measured are shown to have significant advantages over corresponding methods using only a single state of quantum-correlation. These results open up a new avenue of future studies, making use of  $D\bar{D}$  pairs produced in this energy

regime, that will complement those made at the charm threshold.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

## APPENDIX: BESIII CORRELATION MATRIX

Table XI displays the correlations between the observables in this analysis and in Ref. [5] due to the systematic uncertainties.

TABLE XI. The correlations between the systematic uncertainties on the observables in this measurement (denoted by an asterisk) and Ref. [5]. The observables  $r_{K\pi}^D \cos \delta_{K\pi}^D$  with the  $\dagger$  ( $\ddagger$ ) symbol is determined using the  $CP$ -eigenstate ( $K_S^0 \pi^+ \pi^-$ ) tags.

	$(r_{K\pi}^D \cos \delta_{K\pi}^D)^{*\dagger}$	$(r_{K\pi}^D \cos \delta_{K\pi}^D)^{*\ddagger}$	$(r_{K\pi}^D \sin \delta_{K\pi}^D)^*$	$\mathcal{A}_{K\pi}$	$\mathcal{A}_{K\pi}^{\pi\pi^0}$	$r_{K\pi}^D \cos \delta_{K\pi}^D$	$r_{K\pi}^D \sin \delta_{K\pi}^D$
$(r_{K\pi}^D \cos \delta_{K\pi}^D)^{*\dagger}$	1	0.18	0	0	0	0	0
$(r_{K\pi}^D \cos \delta_{K\pi}^D)^{*\ddagger}$		1	0.19	0	0	0.09	0
$(r_{K\pi}^D \sin \delta_{K\pi}^D)^*$			1	0	0	0.34	0
$\mathcal{A}_{K\pi}$				1	0.16	0	0
$\mathcal{A}_{K\pi}^{\pi\pi^0}$					1	0	0
$r_{K\pi}^D \cos \delta_{K\pi}^D$						1	0
$r_{K\pi}^D \sin \delta_{K\pi}^D$							1

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