# CONSTITUTIVE RELATION FOR DENSE GRANULAR FLOW

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ABSTRACT

We propose an experimentally derived constitutive equation for dense granular-fluid mixture (e.g. debris-flow). It is based on frictional (quasi-static) and collisional (dynamic) stress component.

The model has been compared with experiments carried out in an annular rheometer (Angeloni, Lamberti, Schippa 1997[1]) using glass spheres and natural sand shearing in air and in newtonian viscous fluids (i.e. water and water-glycerol solution) having different viscosity. The measures of the velocity distribution inside the channel leads to the a characterization of the actual shearing layer and to the evaluation of the related parameters (e.g. concentration, shear rate, etc).

Dynamic stress component is consistent with that originally proposed by Bagnold (1954) in the asymptotic grain-inertia regime, but in the macro-viscous regime the value of the constant scatter from the original one, because of the presence of the quasi static stress component. The weighted function related to the quasi-static and dynamic stress seems to be slightly dependent on the grain characteristics (i.e. mainly the shape) in the range of the experiments.

The constitutive model herein proposed compare satisfactorily with our experiments.

Keywords: dense granular flow, debris-flow, rheology, experiments

## INTRODUCTION

The behaviour of mixtures of large solids in air or viscous fluid is interesting in relation to a broad variety of natural phenomena such as snow avalanches, rock falls, debris-flows. In industry as well we may refer to processing, transporting and handling of mixtures of granular-fluid materials (e.g. cereals, sand, coal in air or liquid). In spite of the large interest in this topic, and of a relative large efforts in developing theoretical model, so far there is a lack of experimental works focused on the rheology of dense granular-flow (Bagnold 1954, Savage & McKeown 1983, Savage & Sayed 1984, Hanes & Inman 1985).

We carried out experiments in an annular cell involving several grains (glass and plastic spheres, natural sand, irregular plastic grains) which are different in shape and density, immersed in air and in newtonian viscous fluid (water and water glycerol solution). We also reproduced typical flow conditions of natural debris-flows (i.e. large granular concentration, relatively weak shear rate).

In fact in most natural occurrences (e.g. debris-flow) the compaction of granular shearing material under its own weight ensures that stress related to frictional contacts is relevant if not dominant. Therefore in analysing the experimental results we proposed a stress-strain relation accounting for both quasi-static (i.e. frictional) and dynamic (i.e. kinetic and collisional) components. Herein we present results related to glass spheres and natural sand.

## EXPERIMENTS

The rheometer (Angeloni et al. 1997a) consists of two concentric aluminium circular disk assemblies mounted on the same vertical shaft which is fixed on the basement of the apparatus. The lower disk assembly (no vertical motion is allowed) is restrained from rotating by a torque arm connected to a load cell. This lower part has an annular trough 100 mm wide, 35 mm deep, having a mean radius of 200 mm; it is therefore called channel. The channel is closed above by the heavy rotating upper disk assembly (rotation and vertical movements are allowed) avoiding any contacts with the side walls. Clearance is 0.4 mm and prevents the used grains, having mean diameter greater than 1 mm, from entering the meatus between the vertical walls of the shear cell and the outer edge of the upper plate, but allows fluid passing trough the gap. On the outer part of the top disk assembly there is a cogged wheel coupled to the gear of a d.c. variable speed servomotor that can induce any angular velocity of the upper disk in the range 0-100 rpm.

Before starting a run the channel is filled up with a known amount of material and liquid (if any). The upper heavy plate is then lowered into the gap. Tests are performed according to the following procedure: one or more velocities of the upper disk are slowly reached step by step, the velocity of the upper disk is continually increased till a preassigned vertical displacement of the disk is reached, the velocity of the upper disk is increased and then decreased step by step.

The stress transmitted to the lower disk, the velocity ($V$) and the vertical displacement of the upper disk are monitored during a test, while the preassigned vertical pressure (which is obtained via a system of counterweights balancing the upper disk) remains constant. Granular mean concentration ($n_m$) inside the channel is derived accounting for the mass of material, for the surface of the channel, for the displacement of the upper disk and for the residual volume related to the rough surface of the ceiling and of the channel bottom (consisting of. glued sand paper).

The channel is also provided with glass windows in the external sidewall, in order to allow visual observations, video recording and velocity measurements of the shearing layer, by a Laser Doppler Anemometer (LDA) equipment. The optic unit was driven manually in order to perform measurements at different point inside the shearing channel. Measures along different verticals located at different distances from the wall were repeated as many time as possible depending on the optical characteristics of the mixture (usually we measured 3-4 verticals). The most remote measuring point was usually located 3÷4 mm from the sidewall.

In the following table 1 the main characteristics of granular fluid materials are reported ( where $d_s$ is granular dianeter, $r$ is density; $n_{max}$ is the maximum granular concentration under natural deposition, $m$ is fluid viscosity).

Tab.1 Materials used in the experiments

| Material | $d_s$ [mm] | $r$ [kg/m$^3$] | Shape | $n_{max}$ | $j_s$ [degree] | $m \times 10^{-2}$ [Pa*s] |
|---|---|---|---|---|---|---|
| Air | -- | 1 | -- | -- | -- | 18.0 |
| Water | -- | 1000 | -- | -- | -- | 0.1 |
| Glycerine | -- | 1083<br>1095<br>1202 | -- | -- | -- | 8.4<br>39.4<br>59.8 |
| Glass | 2.00 | 2520 | Spherical | 0.63 | 23.0 | -- |
| Sand | 1.40-1.68<br>1.40-2.00 | 2670<br>2622 | Irregular | 0.52 | 38.0 | -- |

## THE QUASI STATIC AND THE DYNAMIC REGIME

Because of the relative granular density normal stress increases downward, whereas shear stress remains almost constant, and stratification occurs. This leads to a strongly concave velocity profile, and to an almost locked layer beneath an actually shearing upper layer (see fig.1).

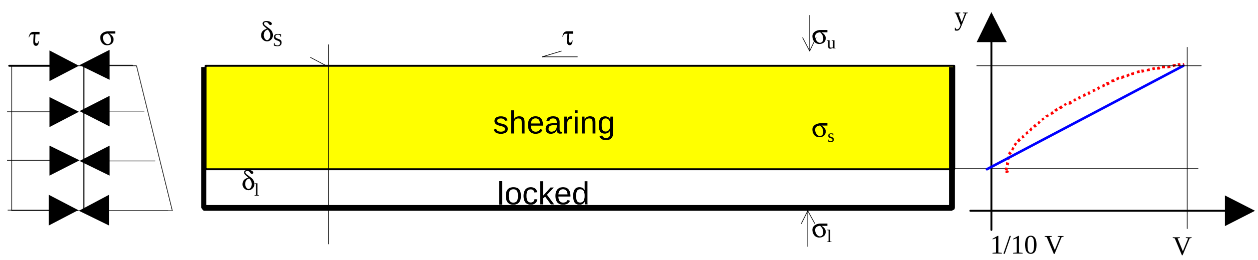


Fig. 1: Sketch of the shearing channel. (- - measured velocity; __ assumed velocity)

Assuming no slip condition between the grains and the upper rotating plate (i.e. the ceiling of the channel) we extrapolated the measured velocity profile and we assumed conventionally an almost locked layer in correspondence of grain velocity less than *V/10* (being V the linear velocity of the upper disk). The ratio $k_s$ between the shearing depth ($d_s$) and the total height of the channel ($d_t$) depends on the fluid and the grain characteristics and on the ratio ($R_s$) between the intercollisional stress and the effective grain pressure:

$$R_s = \frac{r_s \cdot d_s^2 \cdot sr^2}{s}$$

where *sr* is the shear rate:

$$sr = V / d_s$$

Accounting for the actual shearing layer we derived any significant parameters and in particular the Bagnold number and the linear concentration:

$$Ba = \frac{r_s \cdot d_s^2 \cdot l^{1/2} \cdot sr}{m}\ ; \qquad l = \left[ \left( \frac{n_0}{n} \right)^{1/3} - 1 \right]^{-1}$$

where $r_s$ is granular density, $n_0$ is the reference concentration. It was assumed equal to 0.74 for spheres (Bagnold 1954) and for the sand we conventionally assumed $n_0$=(0.74/0.63)0.52 being 0.52 the maximum measured concentration by volume for the sand (see tab.1). The shear rate was assumed to be constant inside the shearing layer and the grain concentration in the locked layer was set equal to the maximum concentration $n_{max}$.

The weaker particle fluctuation is, the higher concentration results and the lower $R_s$ value is. According to our experimental results (Angeloni et al.1997a) and to other authors (e.g. Savage , Hutter 1989), fully dynamic regime corresponds to $R_s$>0.1 and $l$<14 (and interparticle collision are dominant in these condition), whereas for $R_s$<0.05 and $l$>14 frictional effects are dominant and quasi-static regime prevails (see fig.2).

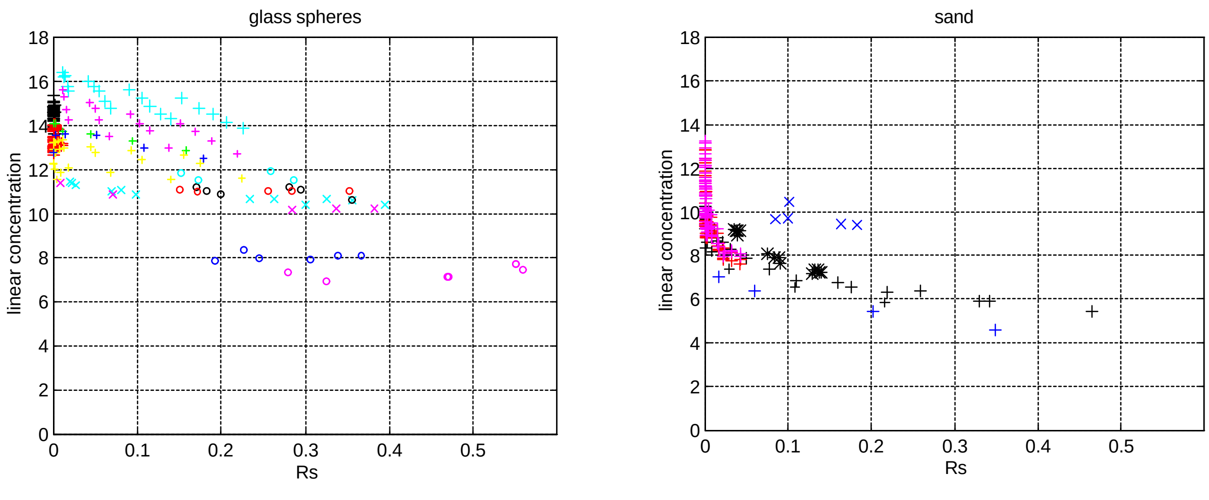


Fig.2 Linear concentration (λ) - vs - $R_s$

STRESS-STRAIN RELATION FOR DENSE GRANULAR FLOW

According to the most recent suggestion (e.g. Johnson & Jackson 1987, Hutter et al. 1996) we propose the total stress being a weighted contribution of quasi-static and dynamic components:

(1) $$t = t_{qs} \cdot [1 - F(l)] + t_d \cdot F(l)$$

For the dynamic component we propose a general relation ,valid for any Ba value, which is consistent with those originally proposed by Bagnold for the asymptotic macro-viscous and grain-inertia regime (Bagnold 1954):

(2) $$T_d = k_1 + k_2 \cdot Ba \qquad \text{where} \qquad T_d = \frac{\tau_d}{\mu \cdot sr \cdot \lambda^{3/2}}$$

where $k_1$ and $k_2$ are experimental fitting parameters (see fig.3 and tab.2)

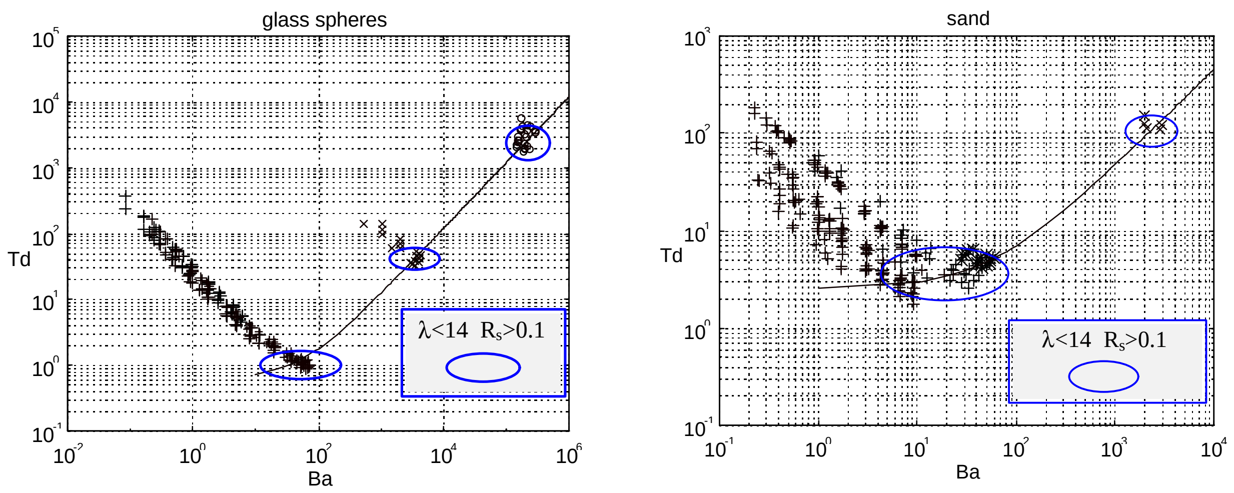


Fig. 3. Non dimensional shear stress ($T_d$ ) -vs- (Ba) (o) air (+) glycerine (x) water

In the case of mixtures at rest, we may assume the ratio between shear and normal stress being equal to the static friction coefficient; therefore the quasi static component of the shear stress results:

(3) $$\tau_{qs} = \sigma_s \cdot tg(\varphi_s)$$

where $\sigma_s$ is the effective grain pressure.

The weighted function $F(\lambda)$ should account for a limiting value of linear concentration $\lambda_0$=18, corresponding to the real maximum concentration ($n_{max}$) for the mixture at rest. The behaviour of the function $F$ is derived from the experimental results (Angeloni et al. 1997b):

(4) $$F(\lambda) = tgh[(\lambda_0 - \lambda) \cdot k_\lambda]$$

where $k_\lambda$ is a fitting experimental parameter. The resulting stress strain relation, obtained via eqs.1-4, is:

(5) $$\tau = \sigma_s tg(\varphi_s) \cdot [1 - tgh((\lambda_0 - \lambda)k_\lambda)] + tgh((\lambda_0 - \lambda)k_\lambda) \cdot \left(k_1 \mu \lambda^{3/2} \cdot sr + k_2 \rho_s d_s^2 \lambda^2 \cdot sr^2\right)$$

Fitting the proposed model with our experiments, we obtained (see figs. 4) the following parameter value:

Tab.2: the fitting parameters of the stress-strain relation eq.6

| | $k_1$ | $K_2$ | $k_l$ | $l_0$ | $j_s$ [°] |
|---|---|---|---|---|---|
| **Glass spheres** | 0.7 | 0.013 | 0.10 | 18 | 23 |
| **Sand** | 2.5 | 0.045 | 0.08 | 18 | 38 |
| **Bagnold wax beads** | 2.1 | 0.013 | --- | --- | --- |

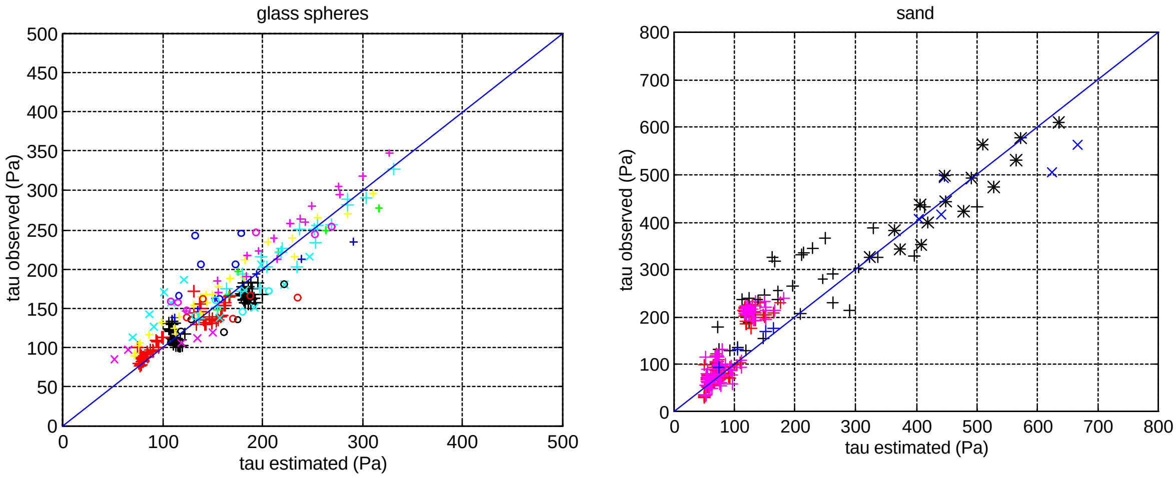


Fig.4 Shear stress observed -vs- estimated (eq.5) . (o) air (+) glycerine (x) water

## CONCLUSION

We have presented experimentally derived constitutive relation, for dense granular flow, which holds in any strain condition. Relation takes into account both quasi-static and dynamic stress component. The proposed model compare rather satisfactorily with experimental results obtained using sand and glass spheres shearing in air or newtonian fluids.

The expression of the dynamic stress component is consistent with that originally proposed by Bagnold for the asymptotic grain-inertia regime. In the macro-viscous regime for a dense granular flow, the quasi-static stress component is dominant and therefore our experimental asymptotic value scatter from that originally proposed by Bagnold .

The quasi static stress component is evaluated accounting for the static friction angle which is assumed to be constant and independent on the interstitial fluid.

The fitting parameters depend on the grain characteristics (mainly the shape) whereas the behaviour of weighting function F($\lambda$) is only slightly dependent on it.

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