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Exploring mathematics teacher educators' knowledge through the perceptions of in-service teachers

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Abstract

Mathematics Teacher Educators (MTEs) play a crucial role in advancing effective mathematics education, as they are responsible for preparing future mathematics teachers. In recent years, increasing attention has been paid to investigating MTEs' professional knowledge. However, most existing research has focused primarily on self-studies, wherein MTEs examine their own knowledge and practice, often without considering the perspectives of other professionals. This study addresses this gap by exploring how In-Service Mathematics Teachers (ISMTs) perceive the knowledge that characterizes "good" MTEs. Data were collected via open-ended written interviews with ISMTs working at various school levels and from different regions throughout Italy. The analysis was conducted using the Mathematics Teacher Educator Specialized Knowledge (MTESK) model and an inductive approach was employed to identify additional dimensions of MTEs' knowledge not initially included. Findings indicate that ISMTs value not only strong mathematical understanding and pedagogical content knowledge but also relational, communicative, and affective qualities such as empathy, motivation, and the ability to foster collaborative learning environments. Furthermore, professional teaching experience and familiarity with mathematics education research were identified as important factors contributing to MTE effectiveness. Notably, ISMTs' perceptions revealed some ambiguity in distinguishing knowledge intended to support teachers from that aimed at students, and highlighted tensions between the theoretical knowledge emphasized by MTEs and the practical, concrete support ISMTs seek. This study contributes to the understanding of MTEs' professional knowledge by incorporating the perspectives of those they train, thus illuminating often undervalued affective and experiential aspects. The findings offer significant implications for the design of teacher education programs to better address ISMTs' needs and enhance professional development.

Keywords: Mathematical Knowledge, Mathematics Teacher, Mathematics Teacher Educator, Pedagogical Content Knowledge, Perceptions

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Recent scholarship demonstrates significant advances in conceptualizing mathematics teachers' professional knowledge, recognizing that expertise in this field encompasses specialized domains extending well beyond foundational mathematical content knowledge (Ball et al., 2008; Carrillo et al., 2018; Shulman, 1986). Nevertheless, the professional knowledge possessed by Mathematics Teacher Educators (MTEs)—those responsible for preparing future educators—remains underexplored, necessitating further empirical investigation to address persistent gaps in understanding (Castro Superfine et al., 2024;

Chapman, 2021; Goos & Beswick, 2021; Malambo, 2023; Pérez-Montilla et al., 2025). Owing to their pivotal role in teacher preparation, MTEs occupy multifaceted positions that require them to simultaneously teach and exemplify effective instructional practices, thereby increasing the complexity of their professional responsibilities and required expertise (Rojas et al., 2021). Since the early 2000s, research has increasingly emphasized the imperative to clarify the specific forms and dimensions of knowledge needed by MTEs to fulfill these unique roles effectively (Jaworski, 2008; Perks et al., 2008; Zaslavsky & Leikin, 2004). Alternative theoretical perspectives have further suggested the utility of examining three distinct levels of teacher attention and focus within instructional practice (Mason, 1998). More contemporary international reports, including the TSG-35 Working Group (2024), have reasserted the necessity of investigating MTEs' beliefs and instructional practices while recognizing their central role in enhancing mathematics education at the school level (Rojas et al., 2023). Notably, the existing literature predominantly examines MTEs' professional knowledge from the standpoint of the educators themselves, with limited research integrating the perspectives of other educational stakeholders and professionals.

A diverse array of terminology and conceptual frameworks has been employed to characterize MTEs, illustrating the heterogeneity of their roles, professional backgrounds, and institutional contexts (Even, 2014; Giadas et al., 2024). Consistent with Escudero-Ávila et al. (2021), this study conceptualizes MTEs as professionals engaged in both pre-service and in-service teacher education, with the aim of enhancing the teaching and learning of mathematics across settings. The scope of MTEs' work varies considerably; some serve in higher education institutions where their responsibilities encompass both teaching and research, while others are practicing school teachers who support teacher preparation and ongoing professional learning (Ayalew, 2017; Beswick & Goos, 2018; Dengerink et al., 2015). This diversity generates a spectrum of professional development needs and necessitates the cultivation of distinct forms of knowledge tailored to their varied roles. Notably, many MTEs assume these positions without explicit formal preparation and encounter significant challenges in accessing professional development opportunities that effectively address the range of their professional trajectories and prior experiences (Rojas et al., 2023). Consequently, MTEs require access to learning opportunities that foster a robust understanding of mathematical content and equip them to support prospective teachers in analyzing and teaching complex material effectively (Masingila et al., 2018). Despite ongoing advances in the field, a universally accepted and comprehensive model that delineates the unique knowledge required by MTEs remains elusive (Castro Superfine et al., 2024). Accordingly, it is important to critically examine how the knowledge base of MTEs has been conceptualized in existing scholarship, with attention to the methodological approaches employed to investigate this domain.

A variety of methodological approaches have been leveraged to investigate the knowledge base of MTEs. Castro Superfine et al. (2024) delineate three primary methodological orientations: (a) self-studies, wherein MTEs systematically analyze their own instructional practices to illuminate the types of knowledge employed in teacher preparation; (b) research grounded in established frameworks of mathematics teachers' knowledge as scaffolds to extend or adapt these frameworks for MTEs; and (c) empirical investigations focusing on the instructional choices made by MTEs—such as task design or the formulation of learning goals—as a window into the knowledge enacted during these processes. Additional studies employ alternative methodologies, including classroom observation (Pascual, 2021), structured interviews with MTEs (Martignone et al., 2022), or systematic literature reviews foregrounding elements previously recognized in mathematics teachers' knowledge models (Escudero-Ávila et al., 2021).

Despite the range of available methodologies, Castro Superfine et al. (2024) underscore the need for more robust and diverse methodological strategies that provide richer access to the nuances of MTEs'

knowledge. Notably, self-study methodologies are increasingly prominent, as they afford MTEs opportunities to critically reflect upon and elucidate the forms of knowledge enacted in daily practice (Even, 2008). However, concerns persist regarding the generalizability of self-study findings, given their typically context-specific nature (Beswick & Goos, 2018). This raises important questions regarding the appropriate vantage points and participants in MTEs' knowledge research. In addressing these concerns, Castro Superfine et al. (2024) advocate for a critical, reflexive stance on the part of MTEs and emphasize the value of involving external observers to fortify the validity and credibility of research findings.

Parallel to discussions about methodology, scholarship has also considered which specific dimensions should be examined in the study of MTEs' knowledge. For example, Karsenty (2020) emphasizes aspects such as professional expectations, requisite knowledge, enacted practices, and established models of mathematics teachers' knowledge. Broadening this analytical lens, researchers such as Rojas et al. (2021) and Giadas et al. (2023) have included prospective and In-Service Mathematics Teachers (ISMTs) as research participants. Rojas et al. (2021), for example, analyzed MTEs' modelling practices from the perspective of prospective teachers and identified a potential disconnect between MTEs' instructional intentions and the prospective teachers' interpretations. Similarly, Giadas et al. (2023) engaged ISMTs in focus groups to elicit their conceptions of the professional knowledge that defines "good" MTEs. Including ISMTs' perspectives not only amplifies their voices within the literature but also uncovers facets of MTEs' knowledge that may otherwise remain invisible in dominant models.

Given the dearth of studies incorporating perspectives beyond those of MTEs, this research seeks to enrich current understanding by specifically integrating the viewpoints of ISMTs. The present study therefore aims to analyze how ISMTs conceptualize the knowledge characteristics of exemplary MTEs. Such analysis enables a systematic comparison of the general conceptions articulated by MTEs and those expressed by ISMTs, thus contributing a nuanced and complementary perspective to the evolving discourse on MTEs' knowledge.

MTEs' Knowledge Models

Various studies have opted to expand existing models of mathematics teachers' knowledge as a methodological strategy to identify key dimensions of MTEs' knowledge. The model featured in Figure 1, Mathematics Knowledge for Teaching Teachers (MKTT), proposed by Castro Superfine et al. (2020), is presented as an extension of the Mathematics Knowledge for Teaching (MKT) model developed by Ball et al. (2008), and originates from a reflective process by the authors grounded in their professional experiences as MTEs. Within their framework, it is posited that mathematics teachers' knowledge is subsumed within MTEs' knowledge; however, the latter is more complex, as it is required to support pre-service teachers in reconstructing their prior mathematical understanding. From this perspective, two broad categories of knowledge are identified as characteristic of MTEs. The first, content knowledge, includes: knowledge of content for teaching (as possessed by mathematics teachers), knowledge of the mathematical horizon in relation to future teachers, and specialized knowledge of the specific content of teacher education. The second, pedagogical content knowledge, encompasses: knowledge of content in relation to future teachers (specifically addressing the processes through which pre-service teachers engage with and understand content), knowledge of content and its teaching to prospective teachers, and knowledge of the teacher education curriculum.

Another example along these lines is presented in Figure 2, which features the Mathematics Teacher Educator Specialized Knowledge (MTESK) model, proposed by Martignone et al. (2022) as an

extension of the Mathematics Teacher Specialized Knowledge (MTSK) model developed by Carrillo et al. (2018). This model was developed from interviews with MTEs from diverse professional backgrounds, as reported in Ferretti et al. (2021). These interviews explored the knowledge considered essential for becoming a MTE, as well as the attributes associated with being a "good" MTE. Based on the analysis of these interviews, the authors proposed a model characterizing MTEs' knowledge, emphasizing that there is an intersection between the knowledge of mathematics teachers and MTEs, but that mathematics teachers' knowledge is not entirely encompassed by that of MTEs. This perspective identifies three main dimensions of MTEs' knowledge: First, Mathematical Knowledge (MK), which preserves the subdomains of the MTSK model, though with increased depth and solidity ascribed to MTEs. Second, Pedagogical Content Knowledge (PCK), which extends the MTSK structure by incorporating a dual focus: one towards prospective teachers, and another towards the students of these future teachers—a distinction later conceptualized as the "double gaze" by Reyes-Bravo et al. (2024). Additionally, the model incorporates Knowledge of Research in Mathematics Education (KoMER) as a new subdomain, which permeates both teaching and learning knowledge. Finally, the model recognizes the beliefs held by MTEs about mathematics and its teaching and learning as an integral component of their professional knowledge.

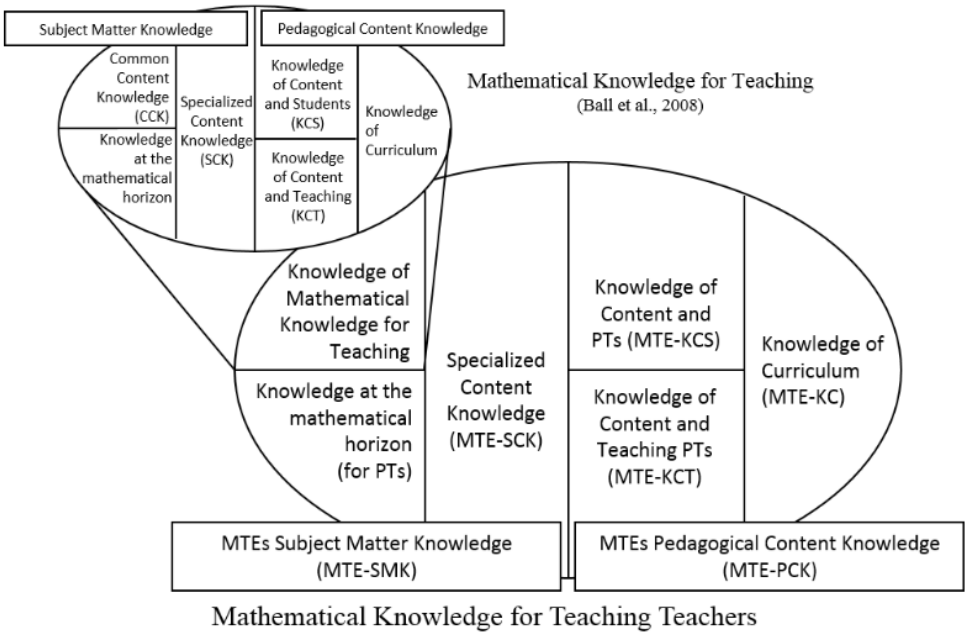


Figure 1. MKTT model (Castro Superfine et al., 2020, p. 372)

In addition to models that extend frameworks originally developed for mathematics teachers, other proposals have emerged. For instance, Leikin (2021) introduced a model that integrates mathematical potential and mathematical challenge with respect to both students and mathematics teachers; thus, the model encompasses both the knowledge and competencies of MTEs. Collectively, these contributions demonstrate that ongoing efforts to conceptualize MTEs' knowledge extend beyond adaptations of teachers' frameworks and increasingly reflect a more complex and specific understanding of what MTEs must know and be able to do.

These proposals indicate that, although many models are derived from conceptual frameworks focusing on mathematics teachers' knowledge, MTEs' knowledge assumes a distinctive configuration specifically oriented toward the education of prospective teachers. As argued by Zopf (2010), the

knowledge required by MTEs cannot be viewed as a mere extension of school teaching knowledge. The preparation of future teachers demands a qualitatively different type of knowledge—one that is attuned to the unique objectives, content, and learning experiences of teacher education. Accordingly, teaching mathematics as a MTE entails attention not only to disciplinary content, but also to the transformation of that content into teachable material for individuals who will themselves become mathematics teachers in school contexts.

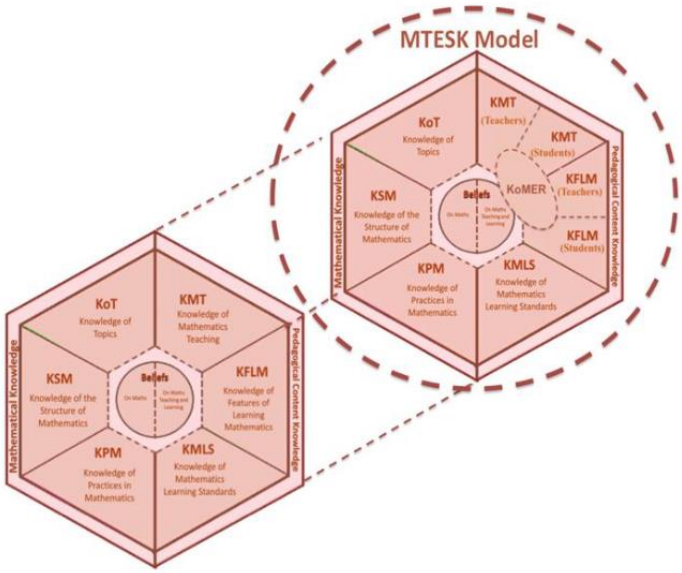


Figure 2. Mathematics Teachers Educator Specialized Knowledge model (Ferretti et al., 2021, p. 6)

METHODS

This research presents the results of an exploratory study conducted in March 2025 with 36 in-service mathematics teachers (ISMTs) from across Italy, who were selected through convenience sampling based on their previous contact with the research team. Specifically, 12 participants taught at the primary school level (grades 1–5), 20 at lower secondary school (grades 6–8), and the remaining 4 at upper secondary school (grades 9–13). This distribution resulted in a heterogeneous sample, enabling the exploration of a range of perspectives across different educational stages.

The research instrument consisted of a written interview comprising open-ended questions, specifically designed to elicit and explore participants' perceptions of MTEs' knowledge. The use of open-ended questions was chosen to capture the richness and depth of respondents' descriptions, as this format allows for more authentic and candid expression than closed or scaled alternatives. As noted by Cohen et al. (2000), open-ended items are particularly valuable in qualitative research because they elicit nuanced and honest insights that may otherwise remain inaccessible through more structured approaches. Consistent with this rationale, the instrument included three guiding questions, as summarized in Table 1.

The interviews were audio-recorded to enable subsequent transcription. Ethical clearance was secured from the relevant institutional review board, and all participants gave informed consent. The questions were conducted in Italian, the participants' native language, to facilitate engagement and ensure authenticity of responses. The research team transcribed and translated the responses into English for further analysis. These responses, treated as units of analysis, were examined using a

deductive approach (Patton, 2014). Initially, each unit was independently categorized by two authors according to the domains outlined in the MTESK model. The resulting categorizations were compared and discussed with the remaining authors until consensus was reached. For the third question, all responses were initially classified under the KoMER subdomain, as the question specifically referenced research findings widely regarded as significant. A subsequent level of categorization was applied, assigning each response to the relevant MTESK subdomain when the content pertained to specific aspects of teaching and learning processes. This two-stage procedure preserved the association with KoMER while situating each response within the most appropriate theoretical domain. This preliminary phase allowed for a clear and consistent classification of responses within the relevant theoretical domains, as detailed in the following section.

Table 1. Rationale of the interview questions

Question	Rationale
Q1. What are the most important characteristics you think "good" MTEs should have?	This question replicates the one used by Ferretti et al. (2021) in the development of the MTESK model. It was deliberately adopted to allow a direct comparison between the responses collected in our study and those obtained in the original one. The question explores, in a broad and unconstrained way, the characteristics that, according to teachers, "good" MTEs should possess, without requiring participants to frame their reflections within predefined categories such as competences or knowledge.
Q2. What types of knowledge do you think "good" MTEs should have?	While not identical, this question partially draws on Ferretti et al. (2021), where it was phrased as: "What knowledge and skills do you think someone needs to become a MTE?" (p. 4). In adapting it, we focused participants' attention specifically on the knowledge dimension of "good" MTEs, enabling a more targeted and in-depth exploration aligned with the knowledge domains outlined in the MTESK model.
Q3. Is it necessary for "good" MTEs to be familiar with the main research results in mathematics education? If so, which ones?	This question was designed to explore more explicitly the KoMER subdomain, which emerged in Ferretti et al. (2021), where participants identified knowledge of mathematics education research as a necessary type of MTE knowledge. Although Martignone et al. (2022) brought valuable attention to this subdomain, earlier work (e.g., Jaworski, 2008) had already highlighted the importance of such research-based knowledge in defining MTEs' professional identity. More recent studies (e.g., Malambo, 2023) have also reinforced its relevance, yet KoMER remains underexplored in the literature—hence its explicit inclusion in our study.

RESULTS AND DISCUSSION

Table 2 summarizes, by school level, the proportion of participants whose responses provided evidence for each MTESK knowledge subdomain. For each subdomain and group, the percentage was calculated as the number of ISMTs in that group who provided at least one excerpt coded in that subdomain divided by the total number of ISMTs in the group. This is followed by a general description of the main patterns emerging from the data, while the final part of the section offers a detailed analysis of the responses to each of the questions posed.

Furthermore, Table 2 reveals that, particularly within the domain of MK, the majority of responses categorizable within this domain were associated with the subdomain KoT. The other two MK subdomains, however, were less frequently—or not at all—considered by ISMTs across all educational levels. In this context, it is unsurprising that the most frequently mentioned subdomain within MK was KoT, since this pertains to disciplinary knowledge that holds direct relevance and meaning in professional teaching contexts, making it more readily recognized by ISMTs. One possible explanation for the limited emphasis on other MK subdomains is that, in the context of teacher education, ISMTs may prioritize knowledge related to the teaching process over content knowledge when reflecting on the core knowledge MTEs should possess (Ferretti et al., 2022). As a consequence, the disciplinary dimension of mathematics may tend to be underestimated.

Table 2. Percentages of participants showing evidence of MTESK subdomains by school level

Domain	Subdomain	Primary school teachers (N=12)	Low secondary school teachers (N=20)	High secondary school teachers (N=4)
Mathematical Knowledge (MK)	Knowledge of Topics (KoT)	83.3%	70%	25%
	Knowledge of Structure of Mathematics (KSM)	—	—	—
	Knowledge of Practices of Mathematics (KPM)	—	16.6%	—
Pedagogical Content Knowledge (PCK)	Knowledge of Mathematics Teaching – teachers (KMT – teachers)	41.6%	40%	25%
	Knowledge of Mathematics Teaching – students (KMT – students)	—	10%	—
	Knowledge of Features of Learning Mathematics – teachers (KFLM – teachers)	—	10%	25%
	Knowledge of Features of Learning Mathematics – students (KFLM – students)	—	25%	25%
	Knowledge of Mathematics Education Research (KoMER)	83.3%	85%	100%
	Knowledge of Mathematics Learning Standards (KMLS)	—	15%	50%

It is particularly noteworthy that, within the PCK domain, the subdomains predominantly surfaced among secondary school ISMTs. According to the observed frequencies, four of the six PCK subdomains were absent in the responses of primary ISMTs. This may be attributed to the fact that, in Italy, primary ISMTs generally do not receive highly specialised mathematical training; consequently, their responses tend to be more general and less specific, making them less easily classified within the subdomains defined by the MTESK model in this study. Furthermore, it should be emphasised that, irrespective of educational level, many responses in the PCK domain were articulated in general terms, often lacking the specificity to distinguish clearly between the dimensions related to teachers and those related to

students. KoMER was also frequently expressed in broad terms and, in some cases, could be categorised as general KoMER concerning other subdomains. This high frequency is not unexpected; it was anticipated that most responses would be classified as KoMER across educational levels (see Table 1), given that question Q3 had explicitly prompted reflection on this subdomain.

In the following paragraphs, the results are presented question by question. To facilitate reference to individual contributions, the notation ISMTx is used to indicate the response of participant number x to the corresponding question. Direct excerpts from the participants' written responses are provided in quotation marks to preserve the authenticity of their statements.

Q1 - What Are the Most Important Characteristics You Think "Good" MTEs Should Have?

One subdomain was identified within the MK domain of the MTESK model and five within the PCK domain. Additionally, several responses could not be classified within the considered categories, as they pertained to emotional or other affective aspects.

The evidence related to MK referred exclusively to the subdomain KoT, which includes knowledge of mathematical content (such as concepts, procedures, rules, and theorems), their meaning, and possible applications: "Disciplinary competence" (ISMT21).

As for the PCK domain, several pieces of evidence emerged. It is worth noting that, within the subdomain KMT, which encompasses specific knowledge related to the teaching of mathematics (such as effective instructional strategies for particular content, as well as potential challenges and limitations), responses were categorized exclusively within the component related to teaching practices, namely KMT–Teachers. This refers to the strategies and meta-strategies that enable MTEs to develop and enhance teachers' instructional skills, with the goal of fostering their ability to promote mathematical thinking among students. A clear example can be observed in the response from ISMT36: "Ability to apply monitoring and evaluation tools... and […] to have a plan A, B, C, D... that is, they should have answers to the likely questions from the course participants".

The subdomain Knowledge of Features of Learning Mathematics (KFLM), which refers to knowledge about how students learn mathematics, emerged in both of its facets: KFLM–Teachers and KFLM–Students. The former concerns the MTE's knowledge in relation to working with teachers as learners. This was evident in ISMT6's response: "Able to help future teachers understand how to identify the most common difficulties pupils face in mathematics (and language)". The KFLM–Students refers instead to knowledge about school students, i.e., those taught by the teachers in training. An illustrative example is provided by ISMT10: "Ability to assess students' prior and consolidated knowledge".

Evidence also emerged for KoMER subdomain, which encompasses the knowledge that MTEs must possess regarding constructs and theories derived from research. Notably, the findings revealed a need for teachers to be trained by MTEs who continuously update themselves on the latest research in the field: "Finally, they should be both a trainer and a trainee... that is, they should be trained through participation in multiple courses, conferences, and webinars in order to then provide training" (ISMT36). Additionally, one teacher (ISMT25) mentioned an aspect related to the Knowledge of Mathematics Learning Standards (KMLS): "Knowledge of the National Curriculum Guidelines". KMLS concerns the knowledge of the learning objectives students are expected to achieve.

Overall, these responses suggest that ISMTs conceptualize "good" MTEs as individuals who possess not only strong MK and PCK, but also relational, communicative, and affective qualities, which in this study are understood as relational qualities referring to the ability to establish supportive and trust-based interactions that foster collaboration and mutual respect; communicative qualities involving clear,

dialogic, and responsive exchanges that promote understanding and openness to diverse perspectives; and affective qualities encompassing empathy, enthusiasm, and the capacity to create an emotionally positive and motivating learning environment.

This view aligns with Dengerink et al.'s (2015) characterization of MTEs as "teachers of teachers" who possess deep expertise in both content and pedagogy. Studies by Giadas et al. (2023) and Rojas et al. (2021) similarly highlight the necessity of integrating disciplinary knowledge with pedagogical and interpersonal competencies to foster meaningful professional learning. The prominence of affective aspects in participants' responses can be interpreted through the lens provided by Escudero-Ávila et al. (2021), who argue that knowledge for teacher education is inherently interdependent, encompassing not only content and pedagogical strategies, but also practices, skills, and professional identity—including emotional and relational domains. These interconnected elements enable MTEs to establish learning environments in which prospective teachers can construct coherent, meaningful, and practical knowledge. As a result, affective qualities naturally emerge as a foundational component of effective mathematics teacher education.

Alongside these affective and relational aspects, participants also emphasised MTEs' disciplinary knowledge, particularly Knowledge of Topics (KoT), a point strongly underlined across all school levels, especially among teachers in primary and lower secondary education. This finding echoes Malambo (2023), who maintained that MTEs must possess a profound understanding of the mathematical content that school teachers are expected to teach, as well as insight into how such knowledge functions within school contexts. This expectation reflects an image of MTEs not only as content experts but also as those capable of applying disciplinary knowledge in ways that are relevant and useful in actual educational practice. This emphasis gains particular significance considering that, in Italy, primary teacher training has traditionally devoted comparatively less attention to advanced mathematical content (Ferretti et al., 2022), thereby reinforcing the notion that ISMTs value and seek such expertise in their professional development.

At the same time, ISMTs underscored fundamental pedagogical dimensions of MTEs' knowledge, particularly the ability to recommend effective instructional strategies, anticipate common learning difficulties, and prompt teachers to critically reflect on their own practices. This perspective reinforces the concept that MTEs' knowledge should be both practical and anchored in meaningful instructional experiences, aimed at developing the professional knowledge required for teaching (Perks et al., 2008). In support of this, many participants highlighted the value of concrete examples, modelling, and differentiated activities provided by MTEs, which echoes Oliveira et al. (2021), who suggest that the use of diverse representations and resources allows mathematics teachers to connect with students' prior knowledge. This emphasis on the practical and experiential qualities of MTE modelling aligns with findings by Rojas et al. (2021), who also observed that prospective teachers tend to appreciate and notice concrete instructional actions enacted by MTEs during their teaching.

Q2 - What Types of Knowledge Do You Think "Good" MTEs Should Have?

For this question, two subdomains within the MK domain were identified, as well as all six subdomains within PCK. Several answers were classified as reflecting KoT, such as ISMT10: "The fundamentals, the basic structures" and ISMT30: "Certainly in-depth knowledge of the founding principles of the discipline". These responses likely prompted participants to focus primarily on disciplinary knowledge. ISMT2 also referred to another MK subdomain, KPM, which encompasses essential mathematical processes, including communication, reasoning, proof, selection of representations, exploration, and generalization.

Specifically, ISMT2's reference to "alternative methods" suggests different ways of representing and working with mathematical objects.

Regarding PCK, evidence was collected for all its subdomains, with contributions reflecting both teacher and student dimensions. The KMT-Teacher's subdomain is evident in responses like ISMT23: "Effective teaching methodologies and strategies; knowledge in digital technologies; knowledge in assessment and evaluation," which points to the MTE's responsibility to equip future teachers with effective instructional tools, supporting the development of their didactic skills. Similarly, the KMT-Students component is reflected in ISMT4's response: "Knowledge of educational workshops, peer education, and tools that foster students' curiosity," where explicit reference is made to student engagement strategies.

Evidence of the KFLM subdomain was found for both teachers and students. ISMT21's response addresses the design of teaching actions: "Concerning how a given concept can be structured, both effectively and problematically (e.g., through misconceptions), as a result of the specific instructional approach adopted". In contrast, ISMT3's answer—"How students learn"—focuses on the students' component. However, certain responses did not clearly distinguish between the teacher and student KFLM subdomains. For example, ISMT24's mention of "Knowledge of different learning styles" was categorized as general KFLM since it was unclear whether it referred to students' approaches or to the MTE's responsibility to discuss these styles with trainee teachers.

The KoMER subdomain appeared more frequently in this question, possibly due to the explicit presence of the term knowledge in the question wording, which may have influenced participants' reflections. A particularly noteworthy answer is from ISMT13: "Knowledge in the field of research in general and discipline didactics," referencing not only research in mathematics education but also educational research more broadly. Finally, several responses were categorized within the KMLS subdomain, such as ISMT34's: "Knowledge of national indications".

When comparing these findings with previous studies, particularly Ferretti et al. (2021), a notable difference emerges in the conceptualization of PCK. While the MTEs in Ferretti et al. (2021) explicitly mentioned knowledge related to both teachers and students in mathematics teaching and learning, the ISMTs in our study did not consistently articulate this distinction. Some focused exclusively on their own students, while others referred only to interactions with MTEs, omitting any mention of school learners. In several cases, both dimensions were blurred or absent. This pattern suggests that ISMTs tend to perceive MTEs primarily as professionals working with teachers, often in academic or university settings, rather than as figures directly supporting student learning. This perspective may shape which aspects of PCK are considered most relevant. Additionally, references to the KMLS subdomain, especially explicit mention of National Curriculum Guidelines, were more evident in the present study, perhaps reflecting the timing of data collection during the release of a new draft of these guidelines. These findings suggest that contextual and role-related factors significantly influence how ISMTs perceive MTEs' knowledge.

Q3 – Is It Necessary for "Good" MTEs to be Familiar with the Main Research Results in Mathematics Education? If so, Which Ones?

The aim of the third question was to investigate which aspects of mathematics education research ISMTs consider essential for the training of "good" MTEs. All responses were initially classified under the subdomain of KoMER, as the question explicitly required reference to research findings that are widely acknowledged as important. Nevertheless, we further categorized the responses according to the subdomains of the MTESK model, as many referenced specific research content related to teaching and

learning processes characteristic of the MK and PCK domains. KoT was the only MK subdomain identified by one teacher: ISMT14 explicitly mentioned the "transmission of knowledge," implying that research can yield content-related insights relevant to teaching and learning processes.

Regarding PCK, several responses provided evidence for KMT, in both its components. Some answers were clearly attributable to KMT–Teachers, discussing knowledge of various teaching methodologies and approaches (for example, ISMT20: "Data regarding the use of new technologies in teaching. The improvements highlighted by research in the use of laboratory-based instruction"). Others were categorized as KMT–Students, referring to research on classroom teacher-student interactions, such as ISMT30: "Yes, especially in the context [...] of the didactic contract and the construction of knowledge".

The KFLM subdomain emerged in a rather general manner, as with the second question, making it difficult to distinguish between the KFLM–Teachers and KFLM–Students components. For instance, ISMT18 mentioned, "Research on specific topics/areas that highlight common difficulties and misconceptions," but did not specify whether this refers to student learning processes or to preparing teachers to address them. Therefore, all KFLM-related responses were treated as general KFLM without further distinction.

Notably, KMLS emerged differently compared to previous questions. There were no explicit references to national curriculum guidelines; instead, the focus shifted to a broader understanding of curriculum, interpreted as knowledge of what and when students are ready to learn. For example, ISMT16 stated: "How to develop number sense, mental calculation, and the understanding of perimeter and area of plane figures".

Together, these findings indicate that ISMTs associate mathematics education research with both theoretical and practical dimensions of teaching. A key theme is the value attributed to MTEs' familiarity with research, corresponding to the KoMER subdomain in the MTESK model. This aspect did not frequently arise when participants were asked general questions about what characterizes "good" MTEs. However, when prompted directly, many ISMTs emphasized the importance of MTEs being knowledgeable about key research findings and able to use them to inform and enhance teaching practices. These results suggest that, while ISMTs may not always be fully aware of how MTEs engage with research professionally, they nonetheless recognize research-informed practice as a core aspect of MTE expertise. This insight is consistent with recent literature noting the increasing significance of KoMER in the field. As Malambo (2023) observes, despite growing attention, KoMER remains conceptually underdeveloped and infrequently addressed explicitly in professional development contexts. By directly focusing on KoMER, our study delves deeper into this subdomain and provides empirical evidence that complements Ferretti et al. (2021), demonstrating that ISMTs value research-informed practice even when its role in teaching is not immediately visible. Consistent with Ferretti et al. (2021), our findings also reinforce the view that KoMER should not be considered in isolation, but rather as deeply interconnected with all other subdomains of MTE knowledge.

Findings Beyond the MTESK Framework

During the coding process, several responses emerged relating to the affective domain, referring both to the emotional skills of MTEs and to the overall affective atmosphere they foster in interactions with mathematics teachers in training. Some responses specifically addressed motivational aspects, underscoring their significance in mathematics teacher education. Many participants emphasized the importance of cultivating enthusiasm and engagement—not only in students but also in MTEs

themselves. This perspective is reflected in ISMT23's statement: "In my opinion, a good teacher educator cannot simply transmit knowledge but should inspire, support, and guide teachers in developing an effective and engaging teaching method". Such responses suggest that teachers need to feel actively engaged in their own professional development, with motivation identified as a fundamental component of the teaching and learning process.

In addition to motivation, nearly half of the responses focused on personal characteristics that shape teaching identity and practice, such as empathy (as illustrated by ISMT19: "Empathy and willingness to listen") and reflective professionalism (as indicated by ISMT27: "Able to question their own point of view, regularly updates their knowledge, and belongs to one or more communities of practice"). These responses can be associated with self-esteem, highlighting affective qualities central to professional identity and showing how personal and emotional dimensions contribute to effective teacher education. It means that research in mathematics education consistently underscores the major influence of affective factors—such as motivation, self-efficacy, and emotional support—on professional learning and mathematics instruction. Positive emotional climates, empathy, and reflective practices not only promote successful engagement for teachers and learners but are also recognized as essential elements in contemporary models of mathematics teacher education and professional development.

Responses also frequently highlighted the importance of creating stimulating, engaging, and collaborative learning environments. For instance, ISMT20 stated: "Being able to create a stimulating, engaging, and collaborative learning environment. Being able to communicate clearly and effectively, and to listen actively", while ISMT28 noted: "I believe that a good teacher educator should have strong communication skills, but above all should be open to dialogue. This would allow them to embrace divergent viewpoints while maintaining a clear and consistent vision of teaching that serves as a guide". Although these responses reference personal qualities such as communication and listening skills, their main emphasis is on developing a collaborative, interactive environment that supports professional growth for both MTEs and mathematics teachers in training.

Additionally, several responses referenced the importance of professional experience, whether in terms of direct school and classroom contexts or through sustained interactions with mathematics teachers. Some participants explicitly cited the value of classroom-based experience, such as ISMT13: "Having experience in the school context", while others focused more specifically on teacher education, as in ISMT34: "I believe that a good teacher educator should maintain a close relationship with mathematics teachers, not only during formal training sessions, but through a continuous cycle of exchange". These responses underscore the value of linking MTE practice to real classroom situations and fostering ongoing collaboration with ISMTs.

While these themes do not fit neatly within existing MTESK subdomains, they provide valuable insights into additional aspects of MTEs' knowledge that extend beyond the model's current boundaries. From an inductive perspective, this evidence suggests the potential inclusion of an additional category that addresses the interpersonal, emotional, and motivational dimensions of MTEs' professional practice. Such dimensions, highlighting the affective and relational dynamics underlying effective teacher education, may also be viewed as interrelated with existing subdomains—particularly KMT–Teachers and KFLM–Teachers—reflecting the fluidity between MTEs' beliefs and their practical knowledge. Participants frequently described ideal MTEs as empathetic, respectful, and supportive, capable of fostering trust-based and collaborative relationships with ISMTs.

These results underscore a dimension of professional practice that, though often underrepresented in the literature, appears highly valued and essential for a comprehensive understanding of MTEs'

professional knowledge. This resonates with the professional dispositions described by Ayalew (2017), who identifies metacognitive awareness, mentoring competence, and sensitivity to diversity as key qualities for teacher educators. Similarly, Giadas et al. (2023) found ISMTs to place significant importance on emotional and interpersonal aspects of MTEs' roles—even though these are often overlooked in favor of content knowledge or instructional expertise in the academic literature. Thus, our findings contribute to broadening the conceptual framework of MTEs' knowledge by reaffirming the centrality of these frequently underrepresented yet professionally significant dimensions.

Moreover, these findings prompt wider reflection on how MTEs' professional practices are perceived by the ISMTs they serve. Taken together, the evidence highlights a persistent tension between how MTEs conceptualize their roles and how those roles are perceived and valued by ISMTs. As noted by Rojas et al. (2023), there may be a gap between MTEs' intentions—often centered on transmitting theoretical and structured knowledge—and what ISMTs identify as most meaningful for their own professional development, such as modelling, relational engagement, and practical relevance. This divergence is echoed in our data, where ISMTs consistently prioritized concrete, supportive interactions over more abstract or theoretical facets of MTEs' practices. Rojas et al. (2021) contend that learning to teach is a situated, complex process, and recognizing how knowledge is interpreted by different actors is crucial for improving teacher education. Pérez-Montilla et al. (2025) further reinforce this point, demonstrating that MTEs activate a diverse body of knowledge, much of which may go unrecognized or be differently valued by the teachers they support.

CONCLUSION

This study advances the empirical understanding of MTEs' professional knowledge by foregrounding the perspectives of ISMTs. The integration of these external viewpoints offers a nuanced complement to self-reported accounts in the extant literature, illuminating key dimensions of MTE expertise most valued in teacher education settings. The findings underscore that effective MTE preparation should intentionally integrate research-informed insights with practical modelling, prioritize the dynamic interplay between disciplinary and pedagogical knowledge, and actively promote teacher motivation and reflective practice. Particularly salient are the affective and interpersonal elements—such as empathy, enthusiasm, and collaborative disposition—which emerged as essential for fostering meaningful, effective professional learning environments. These results highlight the necessity for MTEs to serve not merely as conveyors of content or pedagogical strategies, but as facilitators who inspire, support, and sustain the professional development of mathematics teachers across varied educational contexts.

Nevertheless, several limitations temper the generalizability and interpretive scope of the study. The brevity of certain responses and an uneven distribution of participants across educational levels constrained the depth and representativeness of the thematic findings. Additionally, the absence of detailed information regarding the specific types of secondary schools represented by some participants limited the granularity of cross-level comparisons. Future research should build on these findings by engaging larger, more diverse samples and employing qualitative designs capable of capturing richer, contextual accounts of practice and perspectives. Moreover, subsequent investigations should examine how contextual factors—including institutional structures, policy shifts, and evolving curricular frameworks—influence MTEs' knowledge and practice. Expanding research in this direction will sharpen the theoretical conceptualization of MTE knowledge while supporting the design of more effective, research-informed professional development programs in mathematics education.

Declarations

Author Contribution : PG: Conceptualization, Writing - Original Draft, Methodology, Formal analysis, Editing and Visualization.

MCC: Conceptualization, Writing - Original Draft, Methodology, Formal analysis, Editing and Visualization.

LM-R: Conceptualization, Writing – Review & Editing, Validation and Supervision.

FF: Resources, Writing – Review & Editing, Validation and Supervision.

LJR-M: Writing – Review & Editing, Validation, Supervision and Fundings acquisition.

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