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THE VALUE OF VISCOELASTICITY IN COMPUTATIONAL HEMODYNAMICS: UNCERTAINTY QUANTIFICATION AND COMPARISON WITH IN-VIVO DATA

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SUMMARY

To investigate the effects of uncertainties of parameters involved in computational hemodynamics, with particular concern on geometrical and mechanical parameters defining the viscoelastic vessel wall behavior, we propose a second-order stochastic asymptotic-preserving IMEX Finite Volume scheme, which guarantees spectral convergence in the stochastic space and ease of implementation, avoiding the risk of loss of hyperbolicity of the system of stochastic equations. The method is applied to solve the 1D a-FSI blood flow model, presenting numerical results of univariate and multivariate uncertainty quantification analyses concerning baseline and patient-specific single-artery tests. Computed pressure waveforms are compared with in-vivo records.

Key words: *blood flow modeling, fluid-structure interaction, uncertainty quantification*

1 INTRODUCTION

The acknowledgment of mathematical models as powerful support for the research in hemodynamics and cardiovascular medicine has been widely shared. In recent years, viscoelastic properties of vessel walls have also been recognized as one of the features which must be realistically included in a mathematical model to obtain accurate numerical results [1]. In fact, when characterizing the fluid-structure interaction (FSI) occurring between vessel wall and inner blood by a simpler elastic model, the information about hysteresis, i.e. energy dissipated by viscoelastic effects, is lost and pressure peaks may be wrongly estimated [2]. Among the existing linear viscoelastic models, the Standard Linear Solid (SLS) model provides a good characterization of the mechanical behavior of vessels, better than the frequently adopted Kelvin-Voigt model [2, 3], as the latter is not able to describe the exponential decay of the vessel wall stress over time [4].

When applying computational blood flow models to patient-specific simulations for clinical decision-making, the geometrical and mechanical parameters underlying FSI effects, as well as others computational inputs that need to be personalized, constitute possible sources of errors, given the large biological variability and the uncertainty underlying all measurements [5]. Therefore, the development and application of efficient computational methods for the assessment of the impact of parametric fluctuations on numerical solutions is essential for a correct interpretation of numerical simulations of cardiovascular hemodynamics [6, 7, 8].

2 METHODOLOGY

2.1 Mathematical model

The classical 1D non-linear, non-conservative system of incompressible blood flow in a compliant vessel is composed of the well-established equations of conservation of mass and momentum [2, 3].

To close the system, a constitutive law correlating the pressure inside the vessel to the cross-sectional area is needed. If this law is derived from the SLS model and is included in the system of equations in PDE form, the here discussed augmented fluid-structure interaction (a-FSI) system is obtained [10, 11]. When choosing to transmit statistical information to the problem, related to uncertain input parameters $z_1, \ldots, z_n$ which may be collected in a vector $z = (z_1, \ldots, z_n)^T \in \Omega \subseteq \mathbb{R}^{d_z}$, the solution of the system not only depends on the physical variables x and t (space and time, respectively) but also on the random vector. Hence, the stochastic a-FSI hyperbolic system, valid for both arteries and veins, reads [9]:

$$\partial_t A(x,t,z) + \partial_x q(x,t,z) = 0 \tag{1a}$$

$$\partial_t q(x,t,z) + \partial_x \left(\frac{q^2(x,t,z)}{A(x,t,z)} \right) + \frac{A(x,t,z)}{\rho} \partial_x p(x,t,z) = f(x,t,z) \tag{1b}$$

$$\partial_t p(x,t,z) + d(x,t,z)\, \partial_x q(x,t,z) = S(x,t,z), \tag{1c}$$

where A is the cross-sectional area, q is the flow rate, p is the internal pressure, ρ is the blood density, f is the friction loss term, d is the parameter depending on the elastic contribution of the wall and S is the source term accounting for viscoelastic damping effects [10, 11].

2.2 Numerical method

To solve system (1), a stochastic collocation IMEX Finite Volume (FV) method is adopted [9]. The scheme combines: the stochastic collocation method, which guarantees spectral convergence in the stochastic space and ease of implementation compared to intrusive methods, avoiding the risk of loss of hyperbolicity of the approximated stochastic system of governing equations [6, 8]; a second-order stiffly-accurate IMEX Runge-Kutta scheme that satisfies the asymptotic-preserving (AP) property in the stiff limit, i.e. the scheme is consistent with the equilibrium limit, which corresponds to the asymptotic elastic behavior [9, 12]; a second-order FV solver, which guarantees the correct treatment of non-conservative terms of the hyperbolic model when computing numerical fluxes [13].

To define boundary conditions, at the inlet of the 1D domain a flow rate or velocity waveform is prescribed, based on available in-vivo data; while at the outlet, to simulate the effects of resistance and capacity of peripheral vessels on pulse wave propagation, the 3-element Windkessel model is considered [2].

3 RESULTS AND CONCLUSIONS

We aim at investigating the effects of uncertainty of parameters involved in the viscoelastic constitutive equation, on which the a-FSI system (1) is based. Therefore, we assume as random inputs the equilibrium area $A_0 = A_0(z)$, the reference celerity $c_0 = c_0(z)$ and the viscosity coefficient of the wall $\eta = \eta(z)$. With this choice, all the uncertainty enclosed in the 3 parameters characterizing the viscoelastic SLS model, namely instantaneous Young modulus $E_0(z)$, asymptotic Young modulus $E_\infty(z)$ and relaxation time $\tau_r(z)$, is captured [9]. The chosen uncertain parameters are modeled as random Gaussian-distributed variables. Given the different sources of error related to the estimate procedures of these inputs, different degrees of uncertainty are associated to the three parameters of interest: 10% for A_0 and c_0 and 50% for η, which is affected by a more significant uncertainty.

To assess the impact of these parametric uncertainties on the numerical solution of the a-FSI blood flow model, two single-artery tests are presented: the first concerning a baseline thoracic aorta (TA) and the second considering a patient-specific common carotid artery (CCA). To evaluate the relevance of viscous damping effects in terms of sensitivity of the model, both the elastic and the viscoelastic characterization of the vessel wall behavior are considered for the TA simulation; while for the CCA test only the more realistic viscoelastic law is taken into account. For each elastic (resp. viscoelastic) simulation, 2 (resp. 3) univariate analyses are performed, followed by a multivariate analysis.

Elastic and viscoelastic results of the TA test are shown in Fig. 1. A different sensitivity emerges when comparing the variability of flow rate to the one of pressure, the latter resulting much more sensitive to the uncertainties considered. When comparing pressure waveforms predicted adopting the elastic

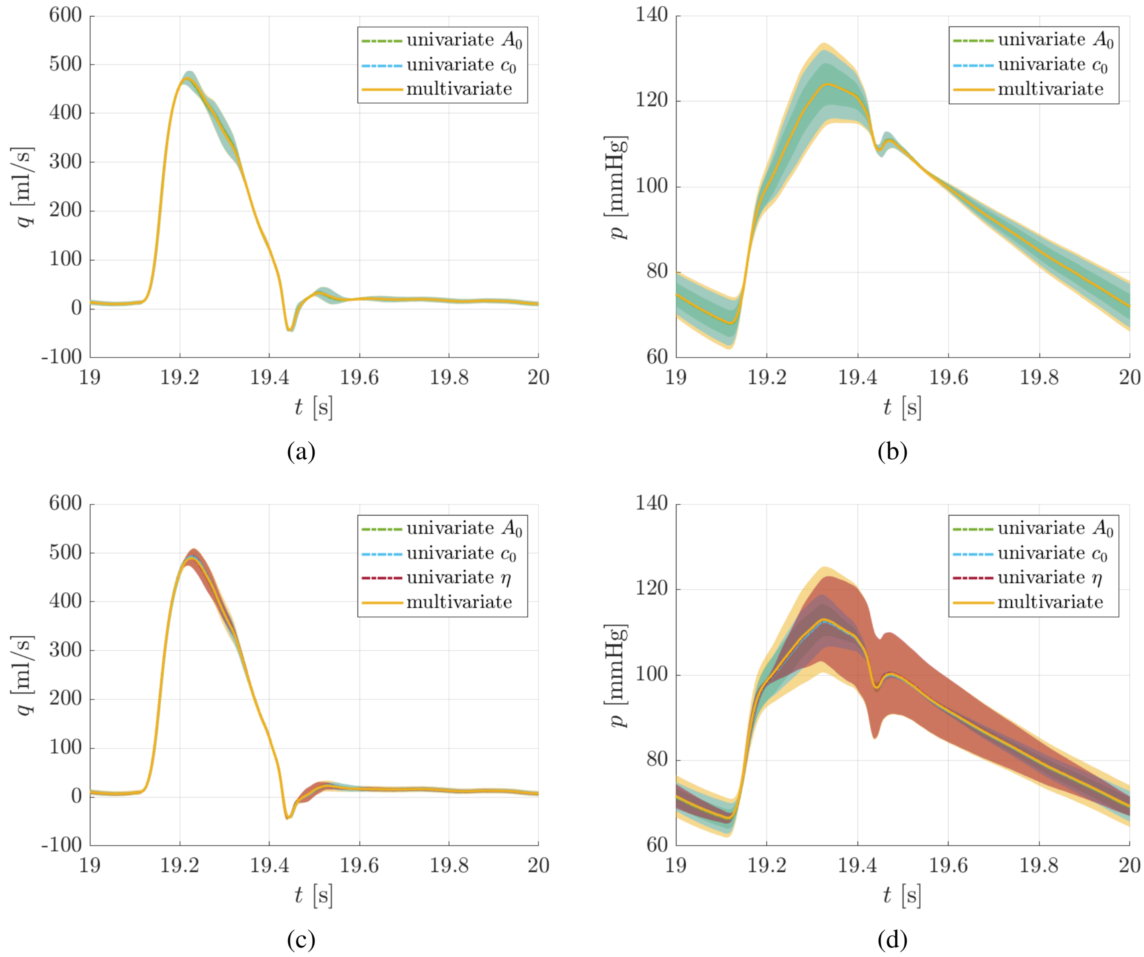

Figure 1: Numerical results representative of one cardiac cycle obtained in the baseline TA test when characterizing the mechanical behavior of the vessel wall through an elastic (first row) or a viscoelastic (second row) law. Results are presented in terms of flow rate (a,c) and pressure (b,d) at the midpoint of the domain, for 95% confidence intervals (colored area) and corresponding expectations (colored line).

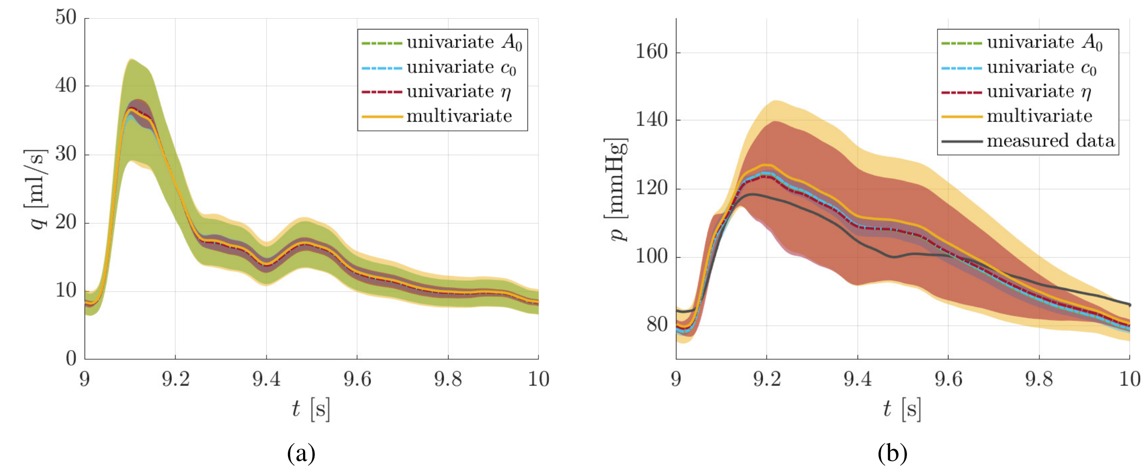

Figure 2: Numerical results representative of one cardiac cycle of a patient-specific CCA test, presented in terms of flow rate (a) and pressure (b) at the midpoint of the domain, for 95% confidence intervals (colored area) and corresponding expectations (colored line). Computed pressures are compared to the patient-specific waveform measured in-vivo with the PulsePen tonometer, as described in [11].

tube law with respect to those obtained with the viscoelastic characterization, it can be noticed that the great uncertainty of the viscosity parameter η plays a major role in the output, enlarging the confidence interval. Nevertheless, the impact of the wall viscosity is principally visible in the systolic peak and dicrotic limb and not in the anacrotic limb. The relevance of considering viscoelastic effects of arterial walls (and not only elastic ones) is confirmed when comparing expectations of computed pressures, being the systolic value of the viscoelastic simulation consistently damped with respect to the one obtained in the elastic run.

Results of the patient-specific CCA test, presented in Fig. 2, show a similar sensitivity. Furthermore, confidence intervals well capture the in-vivo signal and the expected pressure trend is similar to the measured one, confirming the validity of the proposed methodology.

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