PME44 VIRTUAL

THE 44th CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION

July 19-22, 2021

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Proceedings

of the 44th Conference of the International Group for the Psychology of Mathematics Education

VOLUME 2

Research Reports (A-G)

Editors:
Maitree Inprasitha, Narumon Changsri
and Nisakorn Boonsena

Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education

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PREFACE

We are pleased to welcome you to PME 44. PME is one of the most important international conferences in mathematics education and draws educators, researchers, and mathematicians from all over the world. The PME 44 Virtual Conference is hosted by Khon Kaen University and technically assisted by Technion Israel Institute of Technology. The COVID-19 pandemic made massive changes in countries' economic, political, transport, communication, and education environment including the 44th PME Conference which was postponed from 2020. The PME International Committee / Board of Trustees decided against an on-site conference in 2021, in accordance with the Thailand team of PME 44 will therefore go completely online, hosted by the Technion - Israel Institute of Technology, Israel, and takes place by July 19-22, 2021. A national presentation of PME-related activities in Thailand is part of the conference program.

This is the first time such a conference is being held in Thailand together with CLMV (Cambodia, Laos, Myanmar, Vietnam) countries, where mathematics education is underrepresented in the community. Hence, this conference will provide chances to facilitate the activities and network associated with mathematics education in the region. Besides, we all know this pandemic has made significant impacts on every aspect of life and provides challenges for society, but the research production should not be stopped, and these studies needed an avenue for public presentation. In this line of reasoning, we have hosted the IGPME annual meetings for the consecutive year, July 21 to 22, 2020, and 19 to 22 July 2021, respectively by halting "on-site" activities and shift to a new paradigm that is fully online. Therefore, we would like to thank you for your support and opportunity were given to us twice.

"Mathematics Education in the 4th Industrial Revolution: Thinking Skills for the Future" has been chosen as the theme of the conference, which is very timely for this era. The theme offers opportunities to reflect on the importance of thinking skills using AI and Big Data as promoted by APEC to accelerate our movement for regional reform in education under the 4th industrial revolution. Computational Thinking and Statistical Thinking skills are the two essential competencies for Digital Society. For example, Computational Thinking is related to using AI and coding while Statistical Thinking is related to using Big Data. Therefore, Computational Thinking is mostly associated with computer science, and Statistical Thinking is mostly associated with statistics and probability on academic subjects. However, the way of thinking is not limited to be used in specific academic subjects such as informatics at the senior secondary school level but used in daily life.

For the PME 44 Thailand 2021, we have 661 participants from 55 different countries. We are particularly proud of broadening the base of participation in mathematics education research across the globe. The papers in the four proceedings are organized according to the type of presentation. Volume 1 contains the presentation of our Plenary Lectures, Plenary Panel, Working Group, the Seminar, National Presentation, the Oral Communication presentations, the Poster Presentations, the Colloquium. Volume 2 contains the Research Reports (A-G). Volume 3 contains Research Reports (H-R), and Volume 4 contains Research Reports (S-Z).

The organization of PME 44 is a collaborative effort involving staff of Center for Research in Mathematics Education (CRME), Centre of Excellence in Mathematics (CEM), Thailand

Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession (IRDTP) for ASEAN Khon Kaen University, The Educational Foundation for Development of Thinking Skills (EDTS) and The Institute for the Promotion of Teaching Science and Technology (IPST). Moreover, all the members of the Local Organizing Committee are also supported by the International Program Committee. I acknowledge the support of all involved in making the conference possible. I thank each and every one of them for their efforts. Finally, I thank PME 44 participants for their contributions to this conference.

Thank you

Best regards

Associate Professor Dr. Maitree Inprasitha
PME 44 the Year 2021
Conference Chair

TABLE OF CONTENTS

VOLUME 2

RESEARCH REPORTS (A-G)

DIFFERENT CULTURE, DIFFERENT BELIEFS: THE CASE OF PRE-SERVICE TEACHERS' BELIEFS ABOUT MATHEMATICS

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Aiming at unfolding possible cultural differences concerning pre-service teachers' beliefs towards mathematics and its teaching, this paper presents a comparative study between nearby regions: Italy and Germany. The sample is composed by 460 pre-service teachers from three universities, one in Germany and two in Italy, one of them close to the Austrian border and multilingual. Using a clustering technique, we analyse responses to two multiple-answer questions, and we compare the composition of the obtained clusters in terms of linguistic background and origin. Relevant differences are evidenced and explained, at least partially, in terms of cultural differences.

INTRODUCTION

The importance of the effects of teachers' beliefs on their practice was the motivation for the extensive research on teachers' beliefs (cf. Fives & Gill, 2014). However, a major obstacle in interpreting locally obtained results about teachers' beliefs from an international perspective is given by the impact of local culture on teachers' beliefs (Felbrich, Kaiser, & Schmotz, 2012; Hofstede, 1986; Romijn, Slot, Leseman, & Pagani, 2020). Whereas Hofstede (1986) refers to cultural differences from a global perspective including the effect of high or low individualism that divides, for example, Europe from Asian countries, Romijn et al. (2020) refer to differences in beliefs of teachers from European countries. Thus, a seemingly homogenous cultural region may comprise cultural differences that are apparent in teacher' beliefs. Following this line of research, our research aims at providing a contribution to unfold possible cultural differences concerning teachers' beliefs towards mathematics and its teaching in two nearby regions, Italy and Germany.

Following Felbrich and colleagues (2012) we understand common experiences of a group of people that are shared through generations as the basis of culture. Furthermore, according to Hofstede (1986, p. 314), we conceive language as "the vehicle of culture". We investigate Italian and German pre-service teachers' beliefs as a specific expression of culture. Taken the linguistical influence into account, we also consider Italian pre-service teachers from a border region where some teachers use German language and other teachers speak Italian at school and in their daily life.

THEORETICAL LENSES ON TEACHERS' BELIEFS

We refer to beliefs on the basis of two aspects: First, teachers' beliefs as part of teachers' mathematics related affect (Hannula, 2012) play an important role in

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teachers' professional lives (Calderhead, 1996). For example, Eichler and Erens (2014), starting from the definitions by Pajares (1992) and Philipp (2007), understand the term beliefs as an individual's personal conviction concerning a specific subject, which shapes an individual's ways of both receiving information about a subject and acting in a specific situation. Thus, beliefs strongly impact on the way teachers learn mathematics at universities and teach mathematics at school (cf. Philipp, 2007). As pointed out by Pajares (1992), teachers' beliefs are often already developed during pre-service university courses; hence many studies focus on beliefs of perspective mathematics teachers (cf. Hannula, Liljedahl, Kaasila, & Roesken, 2007).

The second one concerns the impact of cultural aspect on pre-service teachers' beliefs towards mathematics and its teaching. One of the main obstacles to the general interpretation of results about to teachers' beliefs obtained at national level is the influence of social and cultural factors on teachers' beliefs (Felbrich et al., 2012). Some studies highlight that the process of learning and teaching of mathematics is dependent on the teachers' cultural background; this is evidenced both from global (Hofstede, 1986) and European (Romijn et al., 2020) perspectives.

Our research moves within this stream of thought and our aim is to investigate the cultural differences concerning teachers' beliefs towards mathematics and the teaching mathematics in two nearby regions, namely Italy and Germany. In details, in this paper we focus on pre-service teachers' beliefs about features that are decisive both for being successful in mathematics and performing well as teacher. As detailed below, we frame pre-service teachers' beliefs about mathematics within the model of *mathematical giftedness* by Pitta-Pantazi and colleagues (2011); we frame beliefs about mathematics teaching within the *Knowledge Quartet* (Rowland et al., 2005). Our research question is: What differences can we observe in beliefs manifested by primary pre-service teachers from different cultural and linguistic backgrounds?

METHODOLOGY

Sample

Our sample consists of students of the university of Bologna and the University of Bozen-Bolzano (Italy) and the university of Kassel (Germany). The University of Bologna is an historical big university in the northern part of Italy, attended by students coming from many different Italian regions. The University of Kassel, in Germany, is younger and is attended mainly by students from the surrounding area. The University of Bozen-Bolzano is a small university located in the South Tyrol region (Italy), at the border with Austria. This region, originally Austrian, was annexed to Italy after World War I and it still is a bilingual region. There are both German and Italian schools for any school level. The university of Bozen-Bolzano provides two versions of each course, in Italian and in German. Among our 460 respondents, 40% are from Bologna and 39% from Kassel. Pre-service teachers who attended their courses in Italian (we will refer to this group as Bozen ITA) are 15% of the sample; the remaining ones attended classes in German (Bozen GER). Respondents received the text of the questionnaire in the same language of their

courses. Translation of the questionnaire has been checked by all the authors and by a consultant speaking both languages.

Questionnaire

The data analyzed in this paper refer to two questions from a wider questionnaire (Ciani et al., 2019). We analyze the answers to two multiple-answer questions (Maffia et al., in press), corresponding to questions B2 (Fig. 1) and B4 (Fig.2) in the original questionnaire.

B2. Select THREE features that, according to you, are important for having success in mathematics.

A. Fluency	G. Predisposition
B. Organized working	H. Analytic thinking
C. Language appropriateness	I. Confidence
D. Flexible thinking	L. Memory
E. Motivation	M. Control
F. Perseverance	N. Originality

Figure 1: Question B2 on beliefs about success in mathematics

We selected the answer-options to question B2 according to the model of mathematical giftedness described by Pitta-Pantazi, Christou, Kontoyianni and Kattou (2011). Following this model, mathematical ability is the result of *Learned Abilities* (like verbal, spatial, quantitative abilities, etc. – options B, C, and H) and *Creativity* (defined as a combination of fluency, flexibility, and originality – options A, D, and N). Both Learned Abilities and Creativity are supported by *Natural Abilities* (including working memory, control, and speed of processing – options G, L, and M). We integrate this model adding the dimension of *affect* (options E, F, and I).

B4. Select THREE features that, according to you, are important for being a "good" teacher of mathematics.

A. Planning with awareness	G. Giving feedback about errors
B. Knowing several teaching methods	H. Adapting lessons to contingencies
C. Giving effective explanations	I. Knowing students' abilities
D. Valorising students' interventions	L. Selecting appropriate examples
E. Knowing mathematics	M. Relating different topics
F. Using several representations	N. Using technical terms

Figure 2: Question B4 on beliefs about mathematics teaching

Answer-options for question B4 were established according to the model of the Knowledge Quartet by Rowland and colleagues (2005). It is a theoretical framework for the analysis and development of mathematics teaching. From the perspective of the Knowledge Quartet, knowledge and beliefs evidenced in mathematics teaching can be seen in four dimensions: *Foundation* (options B, E, and N), *Connection* (A, I, and M), *Transformation* (C, F, and L) and *Contingency* (D, G, and H).

Data analysis

Answers to the multiple-answer questions have been clustered using an agglomerative hierarchical clustering algorithm on the whole sample of 460 respondents. In terms of the method, single linkage may function to determine the outliers in the data, and then performing the Ward algorithm classifies the remaining elements. While this algorithm usually results in a valid clustering, in this work its performance was reduced, due to the lack of isolated data points (Maffia et al., in press). The complete linkage rule was then chosen aiming to find compact clusters of similar diameters, avoiding chaining phenomena (Everitt, Landau, Leese & Sthal, 2011). The number of clusters is established minimizing the absolute maximum deviation from the median of the number of respondents per cluster (Maffia et al., in press).

RESULTS

In presenting our results, we dedicate a sub-section to each of the two abovementioned questions, that is B2 and B4, providing information about the obtained clusters and comparing the composition of clusters in terms of respondents having different origin.

Beliefs about success in mathematics

For question B2 we obtained six clusters and, even if they differ one from the other, their characterization depends on a few answer-options. In general, we can notice that Natural Abilities are considered as not important for succeeding in mathematics, while attention to affective factors is high. Clusters differ mostly in terms of the percentage of selection of affective factors, being 'Motivation' and 'Perseverance' some of the most selected options in many clusters. Creativity is represented in the largest clusters by 'Flexible thinking', while 'Originality' is usually undervalued. In the same fashion, the most representative Learned Ability is 'Analytic thinking', while 'Language appropriateness' is rarely considered.

As it is shown in figure 3, the composition of the six clusters differ in terms of the origin of respondents having some clusters mainly composed by Italian-speaking pre-service teachers and other more populated by German ones.

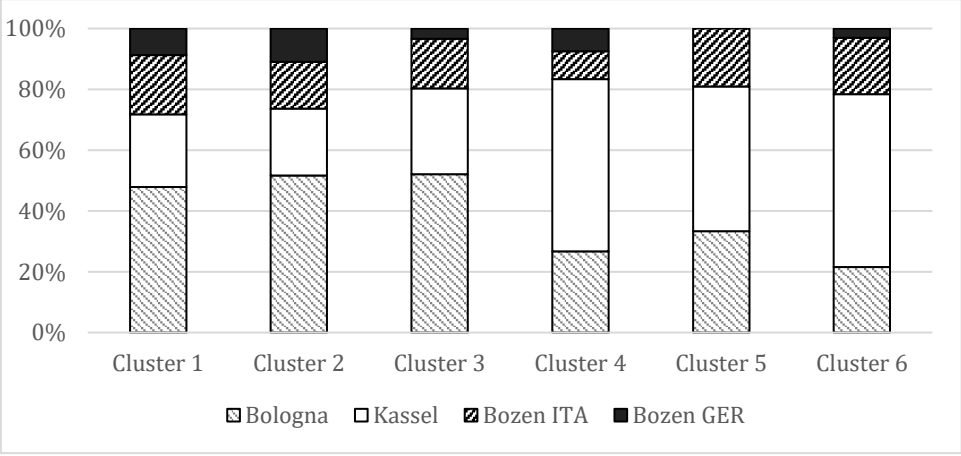

Figure 3: Composition of clusters (question B2) according to respondents' origin.

Respondents from Bologna are highly represented in the first three clusters where Affective factors are strongly considered. Clusters 1 and 2 comprehend respondents paying strong attention to 'Motivation'. While respondents belonging to the first

cluster (10% of the sample) also often select 'Organized working', those in the second one (20% of the sample) selects more often 'Flexible thinking', showing a preference for Creativity over Learned Abilities. The third cluster (25% of the sample) gives high credit to 'Flexible thinking' and 'Perseverance' (Fig. 4).

The percentage of pre-service teachers from Kassel is higher in clusters 4, 5, and 6 (respectively 26%, 14%. and 5% of the respondents), characterized by a high selection rate of options related to Learned Abilities (mainly options H or B, e.g. Fig. 4). Cluster 6 is the only one having a high percentage of members opting for 'Predisposition'.

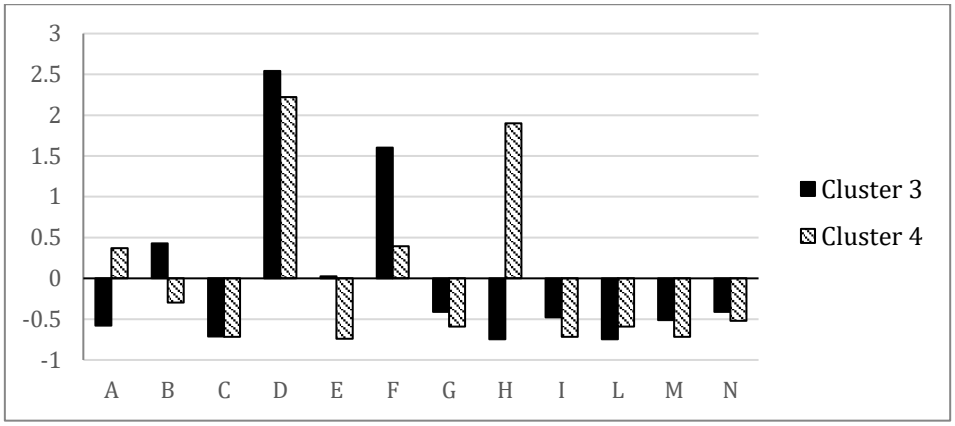

Figure 4: Standardized frequencies (1 unit corresponds to a difference of 1 SD from the average) for the answer-options to question B2 for the two largest clusters.

Respondents from Bozen-Bolzano are represented more evenly in the clusters, but we can notice that, among them, German-speaking pre-service teachers are more strongly represented in clusters 1 and 2. The percentage of Italian-speaking students from Bozen-Bolzano is higher in clusters where 'Perseverance' is considered one of the most important features. More generally, there is not a correspondence between clusters having the higher percentage of respondents from Kassel or Bologna and those having the higher number of respondents from Bozen-Bolzano speaking the same language. The only exception is cluster 3 that is composted by a large majority of Italian speakers.

Beliefs about mathematics teaching

The number of clusters obtained for question B4 is 11, much higher than the previous question. This result may suggest that per-service primary teachers' beliefs about mathematics teaching are more various than those towards mathematics itself. Participants are distributed unevenly in the clusters having the smallest ones representing each 4% of the sample (clusters 1 and 6) and the larger ones comprehending almost 15% of the sample (cluster 2 and 5). Clusters 3 and 4 count each 8% of the sample while other clusters include the 10% of the participants circa.

Even if the number of clusters is quite high, it is interesting to notice that most of them are characterized by four answer-options belonging to three of Rowland and colleagues' (2005) dimensions: Foundations (knowledge about teaching methods and/or mathematics in particular), Transformation (mainly the effectiveness of explanations), and Contingency (mainly feedback on students' errors) have a more

relevant role than Connections in characterizing our pre-service primary teachers' beliefs about the teaching of mathematics.

Figure 5 shows the composition of the eleven clusters in terms of the origin of respondents. We can see that there are extreme cases, where the cluster is almost entirely composed by pre-service teachers speaking the same language, while other clusters are more evenly composed.

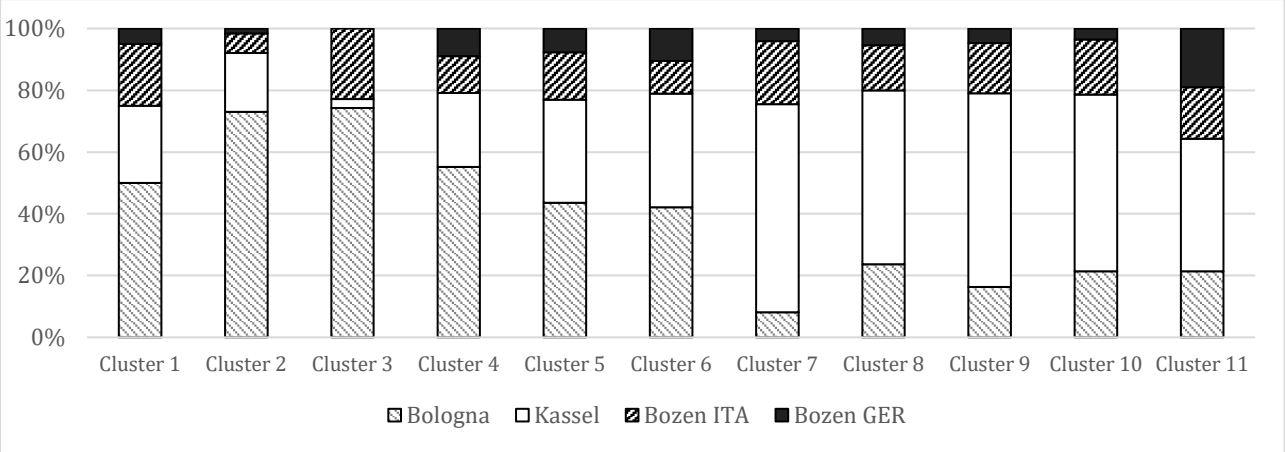

Figure 5: Composition of clusters (question B4) according to respondents' origin.

The first four clusters are all characterized by a high percentage of respondents from Bologna. In these clusters, there is a high rate of selection for the option 'Knowing several teaching methods', while other aspects of Foundation are often ignored. Cluster 3, the most "Italian" cluster, differs from the others since 'Knowing mathematics' is the most chosen option. Members of clusters 1 and 2 often select 'Giving feedback about errors'. However, these two clusters differ in their attention for Transformation: cluster 1 believes that 'Giving effective explanations' is as important as 'Knowing several teaching methods'. A high attention to effective explanations characterizes cluster 4 as well, but this cluster does not have a particular preference for options belonging to the categories of Connection and Contingency. On the contrary, Contingency is the focus for the last three clusters, where students from Kassel are more present. Members of clusters 9 and 10 often refer to 'Giving feedback about errors', while cluster 11 selects mostly 'Valorising students' interventions'. Cluster 7, 8, and 9 pay strong attention to effective explanations. Cluster 7 – the one with the highest percentage of respondents form Kassel – often opts for 'Planning with awareness' and its attention to Foundation is lower than many other clusters.

Clusters 5 and 6 reflects the composition of the whole sample. Cluster 6 is the one giving more credit to knowledge about mathematics (option B) and it is one of the two smallest clusters. Cluster 11 is characterized by a high presence of respondents from Bozen-Bolzano and by a high rate of selection for option D; this is also the only cluster paying a certain attention to answer-options related to Connections.

DISCUSSION AND CONCLUSION

Our analysis allowed to observe relevant differences in pre-service teachers' beliefs about features that are decisive both for performing well in mathematics and for a

successful mathematics teacher. We obtained six different clusters related to mathematics (question B2) and 11 clusters concerning its teaching (question B4).

Our results show that pre-service teachers from the three universities give less importance to natural abilities as basis of success in mathematics (B2). The clusters seem to show cultural differences. For example, the clusters more populated by students from Kassel are characterized by a strong attention to analytical thinking and creativity. By contrast, the percentage of pre-service teachers from Bologna is higher in clusters characterized by attention to flexible thinking and affective factors. Students from Bozen-Bolzano are almost distributed equally in all the clusters, suggesting a mix of beliefs. In relation to the basis for successfully teaching mathematics(B4), we observe that Italian speakers have a stronger attention to foundations than the German ones. Also, pre-service teachers from Bozen-Bolzano are characterized by the strongest attention to connections; a peculiarity of this border-location differing both from the German context and the context of another Italian university.

Considering together the results of both questions, we can state there are common features to all the linguistic and cultural contexts, but also peculiarities. These differences may depend on many factors related to the common experiences of the group of people attending the same university in the same city, that is what we have considered as their culture (Felbrich et al., 2012). Among these experiences we must certainly consider schooling and, in particular, the university courses attended by the pre-service teachers participating in the research. The organization of their university degree cannot be the only source of the observed differences. Indeed, in Italy, Primary Education degrees are regulated at national level and so pre-services teachers from the University of Bolzano-Bozen attend a degree course that is structurally similar to the course of the University of Bologna – the main difference being multilingualism. We are not assuming that all the observed differences could be explained in terms of the spoken language but, assuming that language is the vehicle of culture (Hofstede, 1986), we can claim that cultural factors can affect pre-service beliefs even in nearby regions, and not only at the global level as most of research has shown up to date (e.g. Felbrich et al., 2012). However, more research is needed to better clarify the nature of these factors. Furthermore, it is an open question if and how the observed differences correlate with other constructs that shape the teachers' professional lives, namely the teachers' knowledge, emotions, or motivation.

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