8th IAHR EUROPE CONGRESS

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Divisão de Divulgação Científica e Técnica
AV DO BRASIL 101 • 1700-066 LISBOA
e-mail: livraria@lnec.pt
www.lnec.pt

Publisher: LNEC

Collection: National and international Meetings

Series: RNI 117

1st. edition: 2024 (pdf)

Descriptors: Water resources management / Hydraulic engineering / Congress
Descritores: Gestão de recursos hídricos / Engenharia hidráulica / Congresso

CDU 626/628.1(063)(4)
ISBN 978-972-49-2330-7
DOI 10.34638/jv1h-bs60

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Dam break in power-law cross-section channels with different upstream/downstream widths

Alessandro VALIANI[1], Valerio CALEFFI[1]

[1]Department of Engineering, University of Ferrara, Italy
alessandro.valiani@unife.it
valerio.caleffi@unife.it

ABSTRACT

The dam break problem is studied in a channel characterised by a power-law cross-section, with different widths upstream and downstream of the dam. This research is the extension of a previous study by Valiani and Caleffi (2019), designed for rectangular cross-sections, to different and more complex geometries. A 1D a-SWE (one dimensional *augmented* Shallow Water Equation) system is developed, which has been shown to capture the full range of possible solutions.

To address abrupt changes in the section, the classic SWE system is augmented with a third equation consisting of the time-invariance of the width scale. The idea of the augmented system was introduced by LeFloch and Thanh (2011) for the unit-width SWE, using the bed elevation as an additional variable, and by Valiani and Caleffi (2019) for rectangular narrowing/widening cross-sections, using the channel width as an additional variable.

The numerical model is a Finite Volume Method, second-order accurate in space and time, using a path conservative scheme to evaluate the numerical flux at the cell interfaces. A nonlinear path is adopted, which is shown to be optimal for capturing both the sharp contact wave at the dam and the moving shock(s) downstream of the dam.

The comparison of the numerical results with the analytical results for different solution patterns of the solution supports the confidence in the reliability of the model.

1. Mathematical Model

We consider a channel with a power-law cross section, characterised by the following geometry:

$$b = b_0\left(\frac{Y}{Y_0}\right)^m ;\ A = \frac{b_0 Y_0}{1+m}\left(\frac{Y}{Y_0}\right)^{1+m} ;\ I = \frac{b_0 Y_0^2}{(1+m)(2+m)}\left(\frac{Y}{Y_0}\right)^{2+m} \tag{1}$$

where b_0 is the width scale, Y_0 is the depth scale, m is the shape coefficient, b is the current width, Y is the current depth, A is the wetted area, I is the static moment of the cross section (Valiani and Caleffi, 2009).

A discontinuous width scale is considered, specifically b_{0L} upstream of the dam and b_{0R} downstream. Both cases of contraction and expansion cases are analysed. Y_L and Y_R ($<Y_L$) are the initial upstream and downstream depths. The water is initially at rest everywhere. The 1D a-SWE system governing the problem is:

$$\begin{gathered}\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \\ \frac{\partial Q}{\partial t} + \left[\left(g\frac{A}{b}\right) - \left(\frac{Q^2}{A^2}\right)\right]\frac{\partial A}{\partial x} + \left(2\frac{Q}{A}\right)\frac{\partial Q}{\partial x} - \left(g\frac{A}{b}\right)\Lambda\frac{\partial K}{\partial x} = 0 \\ \frac{\partial K}{\partial t} = 0\end{gathered} \tag{2}$$

where (x, t) are space and time, respectively; Q is the discharge, g is the acceleration due to gravity, K is the additional geometrical variable (b_0 in this specific case), and Λ is a geometrical variable (A/b_0 in this specific case), whose definition (η is the water surface elevation) is:

$$\Lambda \frac{\partial K}{\partial x} = \left(\frac{\partial A}{\partial x}\right)_{\eta=const} \tag{3}$$

Such a term is crucial when geometric discontinuities occur but vanishes in smooth cases.

2. Numerical Model

Numerical integration is performed using a second-order accurate Finite Volume Method, with a DOT (Dumbser and Toro, 2011) Riemann solver to compute the flux at the cell boundaries, where the solution is discontinuous (Toro, 2009). A path-conservative scheme (Dal Maso et al., 1995; Castro et al., 2008) is employed; the linear path introduced by *Parés* (2006) is modified here as proposed in *Valiani & Caleffi* (2019), introducing an original nonlinear path that can be adopted for cross-sections of arbitrary shape.

Referring to cell i, its boundaries are $x_{i-1/2}$ and $x_{i+1/2}$. At this last boundary, each state variable W has two different values, $W^-_{i+1/2}, W^+_{i+1/2}$, on the left and right respectively. We assume a linear path in the variables H, Q, K, where H is the total head of the flow:

$$\begin{aligned} \Psi_H\left(H^-_{i+1/2}, H^+_{i+1/2}; s\right) &= H^-_{i+1/2} + \left(H^+_{i+1/2} - H^-_{i+1/2}\right) s \\ \Psi_Q\left(Q^-_{i+1/2}, Q^+_{i+1/2}; s\right) &= Q^-_{i+1/2} + \left(Q^+_{i+1/2} - Q^-_{i+1/2}\right) s \qquad 0 \le s \le 1 \\ \Psi_K\left(K^-_{i+1/2}, K^+_{i+1/2}; s\right) &= K^-_{i+1/2} + \left(K^+_{i+1/2} - K^-_{i+1/2}\right) s \end{aligned} \tag{4}$$

and a nonlinear path for the cross-sectional area, $\Psi_A\left(A^-_{i+1/2}, A^+_{i+1/2}; s\right)$, which must be derived from:

$$H(s) = \eta(s) + \frac{[Q(s)]^2}{2g[A(s)]^2}, \quad 0 \le s \le 1 \tag{5}$$

3. Results

Depending on the ratios b_{0R}/b_{0L} and Y_R/Y_L, 6 different solution patterns are possible (2 for contractions and 4 for expansions). There is a contact wave occurs at the dam position, and one (or more) shocks downstream of the dam. The augmented system approach is crucial to capture the correct solution when the resonant case (two eigenvalues are both zero) occurs at the dam position. The proposed numerical method gives excellent results for the whole range of cases considered. Examples are shown in Fig. 1.

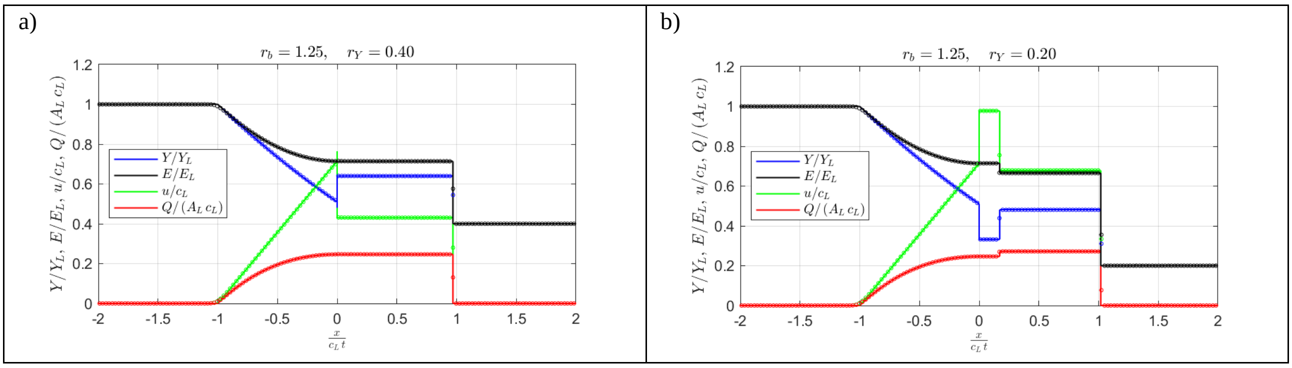

Fig. 1. Dam break in power-law cross-sections for a sudden expansion, $b_{0R}/b_{0L} = 1.25$, for different initial depth ratios: a) $Y_R/Y_L = 0.40$; b) $Y_R/Y_L = 0.20$. Lines/dots are analytical/numerical solutions. c is the relative celerity of small waves, E is the specific energy (=H for horizontal flat bed), u is the mean velocity; the subscript L is the initial reference value upstream of the dam.

References

Castro MJ , LeFloch PG , Muñoz Ruiz ML , Parés C (2008) Why many theories of shock waves are necessary: convergence error in formally path-consistent schemes. Journal of Computational Physics 227 (17), 8107-8129.

Dal Maso GD, LeFloch P, Murat F (1995) Definition and weak stability of nonconservative products. J. de Mathématiques Pures et Appliquées 74, 483-548.

Dumbser M, Toro EF (2011) A simple extension of the Osher Riemann solver to non-conservative hyperbolic systems, Journal of Scientific Computing 48 (1–3), 70-88.

LeFloch PG, Thanh, MD (2011) A Godunov-type method for the shallow water equations with discontinuous topography in the resonant regime, Journal of Computational Physics 230 (20), 7631-7660.

Pares C. (2006) Numerical methods for nonconservative hyperbolic systems: A theoretical framework, SIAM Journal on Numerical Analysis 44 (1), 300-321.

Toro EF (2009) Riemann Solvers and Numerical Methods for Fluid Dynamics, Springer-Verlag, 724 pp.

Valiani A, Caleffi V (2009) Depth–energy and depth–force relationships in open channel flows. II: Analytical findings for power-law cross-sections, Advances in Water Resources 32, 213-224.

Valiani A, Caleffi V (2019) Dam break in rectangular channels with different upstream-downstream widths, Advances in Water Resources 132, 103389.