

Proceedings of the Nineteenth ERME Topic Conference

Mosvold Reidar, Fauskanger Janne, Ferretti Federica, Vondrová Naďa

▶ **To cite this version:**

Mosvold Reidar, Fauskanger Janne, Ferretti Federica, Vondrová Naďa. Proceedings of the Nineteenth ERME Topic Conference. ETC19, Feb 2026, Prague, Czech Republic. 2026, 978-80-7603-673-4. hal-05463964

HAL Id: hal-05463964
https://hal.science/hal-05463964v1

Submitted on 18 Jan 2026

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY-NC 4.0 - Attribution - Non-commercial use - International License



Proceedings of the Nineteenth ERME Topic Conference

Connecting the Learning of Mathematics Teaching to Practice

Charles University, Faculty of Education
Prague (Czech Republic)
4–7 February 2026

Edited by

Reidar Mosvold
Janne Fauskanger
Federica Ferretti
Naďa Vondrová

erme
european society for research in mathematics education

FACULTY
OF EDUCATION
Charles University

Editor Committee

Fanni Biro (Hungary)
Ove Gunnar Drageset (Norway)
Janne Fauskanger (Norway)
Federica Ferretti (Italy)
Marita Friesen (Germany)
Mark Hoover (USA)
Miguel Montes (Spain)
Reidar Mosvold (Norway)
Naďa Vondrová (Czech Republic)

Organizing Committee

Libuše Samková (Czech Republic) – Lead
Miguel Montes (Spain) – Co-Lead
Mark Hoover (USA) – Co-Lead
Fanni Biro (Hungary)
Janne Fauskanger (Norway)
Federica Ferretti (Italy)
Marita Friesen (Germany)
Reidar Mosvold (Norway)
Naďa Vondrová (Czech Republic)

International Program Committee

Stefan Zehetmeier (Austria) – Lead
Miguel Montes (Spain)
Andreas Stylianides (United Kingdom) – ERME Board
Janne Fauskanger (Norway)
Libuše Samková (Czech Republic)

Local Organizing Committee

Naďa Vondrová (Czech Republic) – Lead
Maria Chiara Cibien (Italy)
Federica Ferretti (Italy)
Radka Havlíčková (Czech Republic)
David Janda (Czech Republic)
Jakub Michal (Czech Republic)
Sophia Pantelaki (Czech Republic and Greece)
Libuše Samková (Czech Republic)
Jana Slezáková (Czech Republic)

Conference webpage: https://etc2026.pedf.cuni.cz/

Proceedings of the Nineteenth ERME Topic Conference
Connecting the Learning of Mathematics Teaching to Practice
held on 4–7 February 2026 in Prague, Czech Republic

Publisher: Charles University, Faculty of Education, and ERME
Place: Prague (Czech Republic)
Year: 2026
ISBN: 978-80-7603-673-4

All contributions were peer-reviewed.
© Copyright left to authors.

Recommended citation for the proceedings

Mosvold, R., Fauskanger, J., Ferretti, F., & Vondrová, N. (Eds.) (2026). *Proceedings of the Nineteenth ERME Topic Conference: Connecting the Learning of Mathematics Teaching to Practice*. Charles University, Faculty of Education, and ERME.

Recommended citation for single entries in the proceedings

Authors (2026). Title of the contribution. In R. Mosvold, J. Fauskanger, F. Ferretti, & N. Vondrová (Eds.), *Proceedings of the Nineteenth ERME Topic Conference: Connecting the Learning of Mathematics Teaching to Practice* (pp. xx–yy). Charles University, Faculty of Education, and ERME.

Primary teachers' reflections on student errors in mathematics: Insights from standardized assessment results

Maria Chiara Cibien[1], Francesca Martignone[2], Marta Saccoletto[2] and Camilla Spagnolo[1]

[1]University of Ferrara, Italy; camilla.spagnolo@unife.it

[2]University of Eastern Piedmont, Italy

This paper explores Italian primary teachers' reflections on students' errors in standardized mathematics assessments, specifically using an item from the Italian National standardized test. This qualitative study investigates how teachers mobilize their mathematical and pedagogical content knowledge when analyzing possible student errors. A questionnaire was administered to 128 in-service teachers. The results highlight which aspects of mathematics teachers' specialised knowledge can emerge from their response to this questionnaire.

Keywords: Standardized assessment, teacher professional development, specialised content knowledge, in-service primary teachers.

Introduction

Alongside major international surveys such as OECD-PISA, IEA-TIMSS, and IEA-PIRLS, most countries have in recent decades implemented national standardized assessment systems in primary and secondary education (Eurydice, 2016). Over time, such assessments have gradually influenced classroom practices, affecting both teaching and learning processes (Doig, 2006), and they have gained increasing attention in mathematics education research (Suurtamm et al., 2016).

In Italy, the National Institute for the Evaluation of the Education and Training System (INVALSI) carries out national assessments with the main purpose of evaluating the Italian educational system. The INVALSI national standardized tests have been administered since 2008 and cover both primary and secondary education, specifically for grades 2, 5, 8, 10, and 13 (INVALSI, 2018). These tests are aligned with the Italian National Guidelines (MIUR, 2012) and are designed to balance the demands of standardized assessment with contemporary research paradigms in mathematics education (Ferretti et al., 2020). Such characteristics allow them to be fruitfully used in mathematics education research, for instance for the study of didactical phenomena (Spagnolo et al., 2021). When thoughtfully integrated into teacher education, findings from standardized assessments can provide teachers with tools and benchmarks for diagnostic assessment, indicate directions for formative assessment, and enhance their knowledge and practice (Ferretti et al., 2022). For the context of this study, we focused on the analysis of students' errors. Research has long shown that errors may be valuable opportunities for teaching and learning, when they are opportunely explored (Brodie, 2014). Teachers' ability to recognize and interpret students' errors has been identified as an essential component of their professional knowledge (Larraín & Kaiser, 2019). However, how best to support teachers in developing this interpretative ability remains under discussion (Larraín & Kaiser, 2019). Recent studies show how INVALSI data offer a powerful tool to investigate such ability (e.g., Viola et al., 2024).

In this paper, we investigate Mathematics Teachers' Specialised Knowledge (Carrillo-Yañez et al., 2018) in investigating teachers' reflections on students' errors in answering an INVALSI item.

The study is part of the national research project *Mathematics Standardized Assessment as a Tool for Teachers' Professional Development* (*MaSt*). The MaSt project aims to develop theoretical and operational tools to leverage the use of national mathematics assessment materials for the professional growth of teachers at all school levels. Specifically, the project seeks to promote the formative use, at both institutional and classroom levels, of INVALSI assessment materials of the effective teaching practices that may emerge from them.

Mathematics Teacher Specialised Knowledge

The study of teachers' professional knowledge in mathematics education has been significantly influenced by Shulman's (1986) foundational work, which delineated two interrelated dimensions of knowledge: Subject Matter Knowledge and Pedagogical Content Knowledge. Building upon this dichotomy, the Mathematics Teacher Specialised Knowledge (MTSK) model was subsequently developed by Carrillo-Yañez and colleagues (2018), offering a nuanced framework for understanding the specialised knowledge of mathematics teachers.

The MTSK model conceptualizes that teachers' professional knowledge is comprised of two interconnected domains: Mathematics Knowledge (MK) and Pedagogical Content Knowledge (PCK), which converge and intersect within the teacher's belief system. A distinctive feature of this model is its recognition that teaching practices are shaped not only by teachers' mathematical knowledge, but also by their conceptions and beliefs (Thompson, 1992) about the discipline, student learning, and instructional approaches. These core beliefs permeate and interact with each subdomain of knowledge, influencing both the development and classroom implementation of teachers' knowledge (Carrillo-Yañez et al., 2018).

The subdomains of Mathematics Knowledge (MK) include:

- KoT – Knowledge of Topics (such as knowledge of definitions, properties, procedures, and connections among mathematical topics).
- KSM – Knowledge of the Structure of Mathematics (knowledge of how mathematics is organized, including intra- and interconnections).
- KPM – Knowledge of Practices in Mathematics (knowledge of key mathematical processes and practices, such as proving, modelling, problem solving, and representing).

The subdomains of Pedagogical Content Knowledge (PCK) are:

- KMT – Knowledge of Mathematics Teaching (knowledge of approaches, resources, technologies, and strategies for presenting and representing mathematical content).
- KFLM – Knowledge of Features of Learning Mathematics (knowledge of learning theories, of the ways in which students interact with mathematics, and of common misconceptions or errors);
- KMLS – Knowledge of Mathematics Learning Standards (knowledge of curriculum guidelines, expected outcomes, and objectives at different school levels).

Teachers' ability to interpret and use students' errors in their teaching is influenced by both their subject knowledge and their pedagogical content knowledge (Brodie, 2014). Our research investigates which domains of teacher Content Knowledge emerge from teachers' reflections on INVALSI item. We use the MTSK model (Carrillo-Yañez et al., 2018) to address this purpose and answer the following research question: Which domains of Mathematics Teacher Specialised Knowledge (MTSK) emerge from teachers' reflections on students' errors?

Method

Questionnaire

As part of the MaSt project, a questionnaire was developed and administered to in-service primary teachers. Its' aim was to capture how teachers approach and make sense of standardized assessments within the framework of professional development. After a pilot phase, the final version was distributed to participants online. Answers were anonymously collected. This study considers the responses of 128 in-service primary school teachers (grades 1–5) from across Italy (convenience sample). In this context, mathematics education at primary school level is commonly provided by generalist teachers, who frequently undergo a process of subject-specific specialization as they gain experience and advance in their professional roles.

The questionnaire comprised three sections. The first section collected demographic and professional background information. The second focused on INVALSI mathematics items, which were selected for specific characteristics, such as being suitable to illustrate some national macro-phenomena or the percentage of correct answers. In the questionnaire for primary school teachers, only primary-level tasks were included. Each item was followed by questions regarding teachers' awareness of common students' errors, possible pedagogical and assessment uses, curricular alignment, and the influence of wording. The final section addressed broader issues, examining teachers' perceptions of INVALSI resources and their relation to everyday teaching and assessment practices.

This work focuses on teachers' reflections on a INVALSI item, described in the following paragraph.

Analysis of INVALSI item

Figure 1 represents one INVALSI item that was included in the questionnaire. The item was administered by INVALSI to Grade 5 students in Italian schools in 2018. This item has been selected as illustrates common misconceptions related to the representation of rational numbers on the number line, which have been documented in the literature (e.g., De Wolf et al., 2014). Such errors are often linked to semantic and syntactic challenges in interpreting the fraction symbol. In particular, option A may reflect students who place the fraction at the midpoint of the segment between 2 and 3; option B may indicate those who interpret $\frac{3}{2}$ as "three and a half"; and option D may correspond to students who misread $\frac{3}{2}$ as 3.2.

Analysis of national response data shows that around 17% of the students answered the item correctly, making it the item with the lowest correct response rate in the entire mathematics test that year. Each of the incorrect options were selected by approximately 23–33% of students.

Difficulties in ordering and placing rational numbers on the number line, especially when represented as fractions, have been widely reported. These challenges are often related to interpreting the value of digits and the fraction bar (Behr & Bright, 1984; Ni & Zhou, 2005; Saxe et al., 2007), as reflected in options A, B, and D. Furthermore, the item presents numbers on a graduated line, requiring students to interpret and navigate the scale, which adds further complexity. As noted by De Wolf et al. (2014), the number line makes visible the difficulties students encounter when handling different representations of rational numbers, particularly in terms of comparing and ordering fractions and decimals.

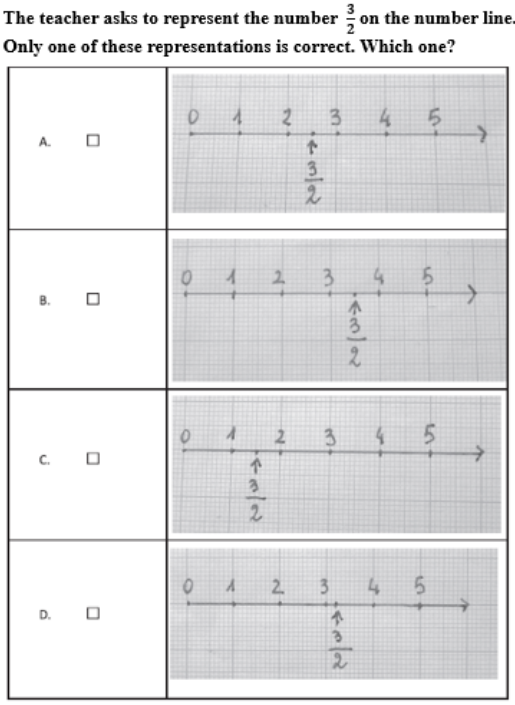


Figure 1: Item D25, INVALSI Grade 5 Mathematics Test, 2018 (translation by the authors)

As noted earlier, this item was included in the questionnaire for primary school teachers, who were asked to identify which incorrect option they believed was most frequently chosen nationally and to justify their choice.

Data analysis

Data analysis involved two phases. First, teachers' motivations for their selected answers were categorized by means of thematic analysis (Cohen et al., 2018) to study which answer the teacher selected, and which students' errors were identified. Second, answers were analyzed in order to identify whether and which subcategories of the MTSK model emerge in teachers' motivations for their selected answer. Findings from these two phases were then triangulated to gain a more comprehensive understanding of teachers' knowledge.

Results

A total of 128 teachers completed the questionnaire, of whom 86 reported having taught mathematics in a grade 5 class. Among respondents, 67 teachers (52.3%) identified option B as the most frequently

chosen, 32 teachers (25%) indicated option D, and 24 teachers (18.75%) selected option A. Seven teachers either responded "I don't know" or selected multiple options.

Analysis of teachers' reflections led to the identification of seven overarching categories of possible explanations of student errors. The largest category, "**Conversion between fraction and number line**", included 41 responses, representing about one-third of the sample. These responses highlighted students' difficulties in translating a fraction into a corresponding position on the number line. Teachers frequently referred to misinterpretations of the numerator and denominator in relation to segments of the line. For example: "D because students may associate the denominator with the number of segments and the numerator with the whole number on the line." Another teacher states: "B, because unfortunately, in Italy, even reading fractions can cause misunderstandings. When children read 'three halves,' they first look for 3 on the number line and then for its half. If we read it (as in some other countries) as 'halves of three,' the concept might be more intuitive." The category "**Conversion between fraction and decimal**" was indicated by 13 teachers. Responses in this category reflected errors arising from incorrect decimal conversion or confusion between fractional and decimal representations, in which the fraction line is not interpreted as division. Illustrative examples include: "D; many students transform $\frac{3}{2}$ into 3.2 without recalling the steps or strategies to convert fractions to decimals." The category "**Partial reading of the fraction**" included 15 responses across all distractors. Teachers in this group noted that students often focus on only one part of the fraction—typically the numerator or denominator—without recognizing the relationship between the two. Examples are: "I think the most frequently chosen option is B because the whole number 3 corresponds to the numerator." The category "**Similarity between fractions and integers on the line**" included 11 responses. Here, teachers emphasized that students often concentrate on individual digits rather than the fractional relationship, for instance: "D, because the student takes into account the numbers, they 'see' but not their ratio." The category "**Rushed reading**" contained nine responses in which teachers attributed errors to hasty or unconsidered reading of the question. An example response is: "A. A hasty reading can be misleading." The category, "**Knowledge of fractions**" comprised three responses in which teachers linked students' errors to incomplete understanding of fraction concepts, without further specification. For example: "I think D, because most students know 'everything' about fractions but do not grasp their true meaning." The final category, "**Non-standard situation**", contained a single response noting students' unfamiliarity with atypical or non-routine problems: "A, because students are not accustomed to this type of problem and textbook exercises are not presented in this way, leaving them confused." Finally, 27 teachers did not provide justifications for their choice or responded: "I don't know."

Analysis of teachers' responses considering the MTSK framework revealed links between their explanations of students' errors and their content and pedagogical knowledge. Knowledge of Topics (KoT) was evident in references to the definition and representation of fractions. When reflecting on students' errors, teachers explicitly drew on their knowledge of fractions, for example, by identifying the numerator and denominator of the fraction, as in the answer: "A, because the numerator is 3 and the denominator is 2". KoT appeared in the categories "Conversion between fraction and number," "Conversion between fraction and decimal," "Partial reading of the fraction," and "Similarity between fraction and integers." In several of these responses, teachers' topic knowledge was clearly

expressed and appeared to guide their analysis of the incorrect answers. As expected, Knowledge of Features of Learning Mathematics (KFLM) was strongly represented, as teachers recognized typical errors related to reading fractions, converting to decimals, and handling the number line. In this sense, all the over-mentioned categories are related to the KFLM. In some responses, teachers explicitly recalled typical student errors or behaviors, which helped them identify the incorrect option most frequently chosen by pupils. For example, in the response: "D, because many pupils convert $\frac{3}{2}$ into 3.2 without remembering the steps or strategies needed to convert a fraction into a decimal number and vice versa."

Knowledge of Mathematical Standard (KMS) and Knowledge of Mathematics Teaching (KMT) did not emerge explicitly, however, we think teachers' answers show some insight into these knowledge subcategories, for example, in noticing the familiarity/unfamiliarity of students with such problematic situations, and the knowledge mobilized by students in such an item.

Discussion and conclusions

The study highlights how teachers mobilize their KoT and KFLM to describe errors and name plausible causes. Given that this study draws on INVALSI test materials, it is worth noting that such insights are made possible by the careful design of distractors. The development of INVALSI tests is informed by expert knowledge and a thorough understanding of the relevant literature in mathematics education and National Guidelines. Particular attention is devoted to the construction of distractors, which play a key role in revealing students' reasoning and misconceptions. Considering the Features of Learning Mathematics, teachers identify possible misconceptions or mistakes at different levels. Some teachers attribute students' wrong choices to inappropriate behaviors, such as not reading the text carefully or focusing solely on the numerical similarity between the number in the text and the number in the figures. However, most teachers link errors to difficulties within the domain of fractions and rational numbers. Among these, some responses indicate a lack of understanding of the meaning of fractions—encompassing a cognitive dimension—while others highlight challenges in recalling the steps or strategies needed to convert fractions to decimals, thus mainly considering the procedural aspects.

In summary, this exploratory analysis of primary teachers' reflections on students' answers to an INVALSI item, indicates that teachers mainly rely on KoT and KFLM when analyzing student errors. This reflects their awareness of different student behaviors, misconceptions, and difficulties.

To foster a formative use of INVALSI materials and results—and to translate insights from students' incorrect responses into enhanced classroom practice—we believe teacher education should explicitly link error analysis to instructional decision-making and curriculum standards (KMT, KMLS). Moreover, discussing and reflecting on research results can enhance teachers' knowledge regarding the teaching-learning process (KFLM and KMT), providing teachers with information that may influence their students' error analysis. These reflections may provide valuable guidance for the development and implementation of a model for teacher educational programs. The results could also be connected to other research strands, such as those on teacher noticing and interpretative knowledge (Ribeiro et al., 2016), which would require further theoretical networking among these frameworks. Future research will focus on investigating these potential connections.

Acknowledgment

Study part of the project "2022CKWY77 - *Mathematics standardized assessment as a tool for teachers' professional development cup.* F53D23006210001" funded by the European Union - NextGenerationEU under the National Recovery and Resilience Plan (PNRR) - Mission 4 Education and research - Component 2 From research to business - Investment 1.1 Notice Prin 2022 - DD N. 104 of 2/2/2022.

References

Behr, M., & Bright, G. (1984). *Identifying fractions on number lines*. Paper presented at the meeting of the American Educational Association, New Orleans.

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics*, *85*(2), 221–239. https://doi.org/10.1007/s10649-013-9507-1

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., … Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, *20*(3), 236–253. https://doi.org/10.1080/14794802.2018.1479981

Cibien, M. C., Ferretti, F., & Spagnolo, C. (2026*).* The Gestinv database as a tool to develop mathematics teachers' hermeneutical competence. In C. Bokhove & A. Clark-Wilson (Eds.), *Proceedings of the 17th International Conference on Technology in Mathematics Teaching* (ICTMT17), London UK. University College London.

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (8th ed.). Routledge.

De Lange, J. (2007). Large-scale assessment and mathematics education. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 1111–1144). NCTM.

De Wolf, M., Rapp, M., Bassok, M., & Holyoak, K. J. (2014). Semantic alignment of fractions and decimals with discrete versus continuous entities: A textbook analysis. In B. Bello et al. (Eds.), *Proceedings of the Cognitive Science Society* (pp. 2133–2138). Austin: Cognitive Science Society.

Doig, B. (2006). Large-scale mathematics assessment: looking globally to act locally. *Assessment in Education: Principles, Policy & Practice*, *13*(3), 265–288. https://doi.org/10.1080/09695940601035403

Eurydice, I. U. I. (2016). *La valutazione delle scuole in Europa: politiche e approcci in alcuni Paesi europei* [School evaluation in Europe: Policies and approaches in some European countries]. Eurydice Italia, 83.

Ferretti, F., Funghi, S., & Martignone, F. (2020). How Standardised Tests Impact on Teacher Practices: An Exploratory Study of Teachers' Beliefs. In C. Andrà, D. Brunetto, & F. Martignone (Eds.), *Theorizing and Measuring Affect in Mathematics Teaching and Learning* (pp. 139–146). Springer. https://doi.org/10.1007/978-3-030-50526-4_14

Ferretti, F., Martignone, F., & Santi, G. (2022). Analysis of standardized tests and pre-service teacher education: reflections on developed teachers' specialized knowledge. In J. Hodgen, E., Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of CERME12* (pp. 3569–3576). ERME/Free University of Bozen-Bolzano.

INVALSI (2018). *Quadro di riferimento delle prove di INVALSI di matematica*. [Framework of the INVALSI mathematics tests]. Retrieved from https://invalsi-areaprove.cineca.it/docs/file/QdR_MATEMATICA.pdf

Ministero dell'Istruzione, dell'Università e della Ricerca (2012). *Indicazioni Nazionali per il Curricolo della Scuola dell'Infanzia e del Primo Ciclo d'Istruzione*. [National guidelines for the curriculum of preschool and the first cycle of education]. Retrieved from https://www.mim.gov.it/documents/20182/51310/DM+254_2012.pdf

Larrain, M., & Kaiser, G. (2019). Analysis of students' mathematical errors as a means to promote future primary school teachers' diagnostic competence. *Uni-pluriversidad*, *19*(2), 17–39. https://doi.org/10.17533/udea.unipluri.19.2.02

Ni, Y., & Zhou, Y.-D. (2005). Teaching and learning fractions and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Ribeiro, M., Mellone, M., & Jakobsen, A. (2016). Interpreting students' non-standard reasoning: insights for mathematics teacher education. *For the Learning of Mathematics*, *36*(2), 8–13.

Saxe, G. B., Shaughnessy, M. M., Shannon, A., Langer-Osuna, J., Chinn, R., & Gearhart, M. (2007). Learning about fractions as points on a number line. In P. Elliot et al. (Eds.), *NCTM 2007, 53,* 221–237.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4–14. https://doi.org/10.2307/1175860

Spagnolo, C., Capone, R., & Gambini, A. (2021). Where do students focus their attention on solving mathematical tasks? An eye tracker explorative study. In Inprasitha, M., Changsri, N., & Boonsena, N. (Eds.), *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 89–96). PME

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., & Vos, P. (2016). *Assessment in mathematics education: Large-scale assessment and classroom assessment*. Springer.

Thompson, A. G. (1992). Teacher's beliefs and conceptions: A synthesis of the research. In D. A. Grows (Ed.), *Handbook on mathematics teaching and learning* (pp. 127–146). MacMillan.

Viola, G., Ferretti, F., Gambini, A., Francesca, M., & Spagnolo, C. (2024). Teachers' professional development and mathematics LSA: first result of national project. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.), *Proceedings of the 47th International Group for the Psychology of Mathematics Education (*Vol. 1, pp. 223–224). PME